Evolution of stress intensity factors in mixed mode with presence of mismatch effect

TAWFIK TAMINE^{1,a}, NABIL KAZI TANI² AND GUY PLUVINAGE³

¹ Laboratoire LCGE, Faculté de Génie Mécanique, USTO M.B, 1505 EL M'Naouer, 31000 Oran, Algérie

² Laboratoire LM2SC, Faculté d'Architecture et de Génie Civil, USTO M.B, 1505 El M'Naouer, 31000 Oran, Algérie

³ Fiabilité Mécanique Conseil, 57530 Silly sur Nied, France

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Abstract – In this study, numerical models based on finite-element method are developed for several applications of fracture mechanics. The use of cracked specimens formed by different material properties has allowed us to analyse the effect of the crack position to the interface and to have a well understanding of mismatch effect for the case of mixed mode of fracture. In other hand, we have noticed that it is very interesting to see the effect of the interface position and loading for the case of symmetrically cracked ring specimen under compression loading.

Key words: Crack / interface / bi-material / mixed mode / stress intensity factor / mismatch effect

Résumé – **Évolution des facteurs d'intensité de contraintes en mode mixte en présence de l'effet mismatch.** Dans cette étude, des modélisations numériques par la méthode des éléments-finis ont été développées pour des applications en mécanique de la rupture. L'utilisation d'éprouvettes fissurées constituées de matériaux de propriétés mécaniques différentes ont permis de mettre en exergue l'influence de la position de la fissure par rapport à l'interface et de comprendre l'effet du mismatch en mode mixte. D'autre part, il a été intéressant de voir aussi l'influence de la position de l'interface par rapport au chargement pour des fissures symétriques dans une structure en anneau.

Mots clés : Fissure / interface / bi-matériaux / mode mixte / facteur d'intensité de contraintes / effet mismatch

1 Introduction

Actually, most of the structures and mechanical components are formed within at least two dissimilar materials. For almost all cases, the use of heterogeneous materials is due to strength issues, protection against corrosion, rubbing, etc. The difficulty is to describe the stress field which is perturbed at the crack tip near the interface. So many researchers have tried to analyse stress fields and to evaluate fracture parameters of cracked bi-material specimens. In the mid 1960, various research works have been carried out by Erdogan [1], Rice and Sih [2] to verify William's [3] finding of complex singularities and oscillating stresses near the crack tip. The well know Dundurs [4] parameters were used by many researchers to predict the stress intensity factors for bimaterials when the crack was perpendicular to the interface. Comninou [5] resolved the crack interface problem by assuming that there is a small

contact zone near the crack tip which is extremely small. For interface crack in tension, the contact zone model provides a contact zone of the order 10^{-4} of the crack length. Recently, some formulations and new techniques were developed such as Wang and Stahle [6], Marur and Tippur [7] who have solved the interfacial crack problems in bi-material plates analytically by developing the stress field relationships as analytical series. Stress intensity factor for crack in bi-material interface has been studied by Bjerken and Persson [8] and Ikeda and Sun [9]. The first authors have applied integral method for the determination of complex relationships of stress intensity factors, the second authors have used the virtual crack extension technique for cracked bi-material specimens under thermal loading. Kang Yilan and Lu Hua [10] have developed a calculation procedure for displacement field near the crack tip emanating from interface. Experimental methods have been used by Jiang et al. [11] to describe the evolution of J-R curves and to study the propagation of

^a Corresponding author: tamine63@yahoo.fr

Nomenclature

K [MPa.m ^{1/2}]	Stress intensity factor
K_1 [MPa.m ^{1/2}]	Stress intensity factor for the first mode of fracture
K_2 [MPa.m ^{1/2}]	Stress intensity factor for the second mode of fracture
ε	Bi-elastic constant of material
E_j	Young modulus of material j
ν_j	Poisson ratio of material j
G_j	Shear modulus of material j
u^i	Displacement of node i with respect to x axis
v^i	Displacement of node i with respect to y axis
s	Distance between nodes a and c or a and e
a [m]	Crack length
W [m]	Plate wide
K_{eq} [MPa.m ^{1/2}]	Equivalent stress intensity factor
$\bar{K}_{eq}$ [MPa.m ^{1/2}]	Normalized equivalent stress intensity factor
$\bar{K}_{eqA}$ [MPa.m ^{1/2}]	Normalized equivalent stress intensity factor at crack tip A
$\bar{K}_{eqB}$ [MPa.m ^{1/2}]	Normalized equivalent stress intensity factor at crack tip B
R_1 [m]	Internal radius of cracked ring specimen
R_2 [m]	External radius of cracked ring specimen
σ_g [MPa]	Global applied stress
σ_{g1} [MPa], σ_{g2} [MPa]	Applied stress in y direction for material 1 and 2 respectively
α [°]	Inclination angle of interface with respect to x axis
θ [°]	Inclination angle of the crack with respect to x axis
SIF	Stress Intensity Factor
RGB	Red Green Blue
PSM1/Al	Polycarbonate/aluminium

cracks emanating from interface of bi-material specimens under three points flexion loading. Another experimental method based on RGB photoelasticity has been carried out by Cirello and Zuccarelo [12] to analyse the propagation of cracks perpendicular to interface of bi-material plates. Numerical models have been developed by Madani et al. [13], Ouinas et al. [14] and Kazi Tani et al. [15] to show the effect of interfaces on crack propagation in bi-material specimen. The restitution energy rate G and the stress intensity factor (SIF) have been determined by those authors for several crack positions between dissimilar materials.

In this present paper, two important points are discussed; the first one is to describe the effect of mixed mode on the decreased curves of stress intensity factor obtained by Cirello and Zuccarelo using RGB photoelasticity technique [12] and confirmed numerically by Kazi Tani et al. [15] on bi-material plates PSM1/Al polycarbonate/aluminium, for the first mode of fracture. The second point concerns the influence of the interface position and loading direction on the stress field near the crack tip of bi-material ring specimen where cracks are symmetrically located.

2 Stress intensity factor at bi-material interface

For a crack of $2a$ length located at the interface between two dissimilar materials subjected to a uniform

tensile load, the complex relationship of the stress intensity factors are given by Rice [16] as:

$$K_1 + iK_2 = \sigma_y (1 + 2i\varepsilon) (2a)^{-i\varepsilon} (\pi a)^{0.5} \quad (1)$$

Bi-elastic constant is given by:

$$\varepsilon = \frac{1}{2\pi} \ln(\gamma) \quad (2)$$

with

$$\gamma = \frac{G_1 + \kappa_1 G_2}{G_2 + \kappa_2 G_1} \quad (3)$$

where

$$\kappa_j = 3 - 4\nu_j \quad \text{in plane strain;}$$

$$\kappa_j = (3 - \nu_j) / (1 + \nu_j) \quad \text{in plane stress;}$$

and

$$G_j = \frac{E_j}{2(1 + \nu_j)} \quad (4)$$

G_j , E_j and ν_j are respectively shear modulus, Young modulus and Poisson ratio of material j .

For the case where the crack is at the interface (Fig. 1), Tan and Gao [17], Emin Ergun et al. [18] expressed SIF relationships from displacement correlation method.

From displacements values of nodes a , b , c , d , e obtained by finite-element method, it is possible to express

Fig. 1. Interface crack in bi-material specimen.

opening and shear fracture modes by the following equations given by references [17, 18]:

$$K_1 = \sqrt{\frac{2\pi}{s}} [C_1 (-v^e + 4v^d - 3v^a) - C_2 (-v^c + 4v^b - 3v^a)] \quad (5)$$

$$K_2 = \sqrt{\frac{2\pi}{s}} [C_1 (-u^e + 4u^d - 3u^a) - C_2 (-u^c + 4u^b - 3u^a)] \quad (6)$$

s is the distance between nodes a and c or a and e . u^i and v^i are respectively the node displacements with respect to x and y axis.

$$C_1 = \frac{(1 + \gamma)\zeta}{\cosh(\pi\epsilon)} \frac{G_1}{\kappa_1 e^{\pi\epsilon} + \gamma e^{-\pi\epsilon}} \quad (7)$$

$$C_2 = \frac{(1 + \gamma)\zeta}{\cosh(\pi\epsilon)} \frac{G_1}{\kappa_2 \gamma e^{-\pi\epsilon} + e^{\pi\epsilon}} \quad (8)$$

with

$$\zeta = \left[\frac{1}{4} + \epsilon^2 \right]^{1/2} \quad (9)$$

3 Numerical examples

3.1 Mismatch effect in mixed mode

To have a well understanding of mismatch effect, we have taken back the numerical example which was in analyzed in references [12, 15]. In our case, an inclined central crack of $2a$ length with an angle θ from horizontal axis x as shown in Figure 2 is studied. The cracked bi-material formed by polycarbonate PSM-1 (Material 1) and aluminium (Material 2) with dimensions equal to are $W_2/W_1 = 1.5$, $H/W_1 = 12.5$, $W_1 = 12$ mm and the thickness is equal to 5 mm. The bi-material plate is subjected to a tensile load in y direction, $\sigma_g = 20$ MPa. It is possible to determine the applied stress on each material by applying the principle of matrix-fiber system where the applied stress is given by [12] as follows:

$$\sigma_g = \sigma_{g1} = \left[\nu_1 + (1 - \nu_1) \frac{E_2}{E_1} \right] \quad (10)$$

If we suppose that both materials 1 and 2 behave elastically and the global deformation is the same for each side of material 1 and 2, then we can write:

$$\frac{\sigma_{g1}}{\sigma_{g2}} = \frac{E_1}{E_2} \quad (11)$$

From Equations (10) and (11), we can split the total stress σ_Y into two components σ_{g1} and σ_{g2} .

Numerical model carried out by using ANSYS software finite-element have allowed us to determine the displacement field and stress intensity factors for mode I and mode II. The solution of the problem has been calculated by using 8 node quadratic elements in plane stress by computing K_1 , K_2 from the nodal displacements on opposite sides of the crack plane [19] with singular element size around 5% of crack length. For symmetric configuration of the specimen, only the quarter of the plate is considered in finite-elements model (Fig. 2)

The numerical results of stress intensity factors obtained in mode I and II are plotted in Figures 3a and 4. For well representing those results, the values of SIF are normalized by $K_0 = \sigma_{g1} \sqrt{\pi a}$. Results of $\bar{K}_I$ in Figure 3a are compared with the case of mode I Figure 3b obtained by [15].

According to the results plotted in Figures 3a and 4, we have noticed that for the most of θ inclination angles of crack, the normalized stress intensity factors $\bar{K}_I$ and $\bar{K}_{II}$ are decreasing as the crack becomes longer. In the case of mode I, the decreased values are more significant when θ varies between 0° and 30° . The values of $\bar{K}_I$ seems constant for the case when $\theta > 45^\circ$. This decreased curve is well described by [15] for a central crack in mode I. The same evolution has been noticed for mode II (Fig. 4) except for the case $\theta = 10^\circ$ where the normalized SIF increases with crack lengths $a/W > 0.6$.

In order to compare the numerical results with first mode of fracture mode I ($\theta = 0$) [15], we have represented in Figure 5, the equivalent stress intensity factor for the mixed mode $\bar{K}_{eq}$, which it is given by the following relationship:

$$\bar{K}_{eq} = \sqrt{\bar{K}_I^2 + \bar{K}_{II}^2} \quad (12)$$

In Figure 5, the equivalent stress intensity factor $\bar{K}_{eq}$ decreases when inclination angle θ varies between 0° and 45° . For angles $\theta = 60^\circ$ and 75° , $\bar{K}_{eq}$ values seem to be constant and do not depend on the crack length. This can be explained by the presence of mismatch effect which is more significant when the crack tip is near the interface. The presence of the crack in weakest material and mismatch effect yield a “closing” bridging stress intensity factor that decreases the stress intensity factor resulting from external loading, especially when the crack tip is close to the interface.

Elsewhere, we have taken back the same model by changing the position of materials where material 1 is Aluminium and material 2 is PSM-1.

The numerical results of stress intensity factors in mode I and II are shown in Figures 6 and 7.

Fig. 2. Numerical model of bi-material plate containing a central inclined crack.

Fig. 3. (a) Evolution of normalized stress intensity factor $\bar{K}_I$ with crack length and inclination angle θ , $E_1/E_2 = 0.047$. (b) Evolution of normalized stress intensity factor in mode I ($\theta = 0^\circ$) [15].

Fig. 4. Evolution of normalized stress intensity factor $\bar{K}_{II}$ with crack length and inclination angle θ , $E_1/E_2 = 0.047$.

Fig. 5. Evolution of normalized equivalent stress intensity factor $\bar{K}_{eq}$ with crack length and inclination angle θ , $E_1/E_2 = 0.047$.

Fig. 6. Evolution of normalized stress intensity factor $\bar{K}_I$ with crack length and inclination angle θ , $E_1/E_2 = 21.276$.

In Figure 6, the increasing curves of $\bar{K}_I$ are more significant when θ angles vary between $\theta = 0^\circ$ and $\theta = 45^\circ$. For $\theta = 60^\circ$ and 75° , the values of normalized SIF are very close.

In Figure 7, which represents the second mode we notice that $\bar{K}_{II}$ values increase with the crack length for all angles except $\theta = 10^\circ$ where $\bar{K}_{II}$ data finishes by decreasing from $a/W_1 > 0.6$.

To have a well understanding of the influence of the mixed mode (I + II), we have represent the $\bar{K}_{eq}$ data in Figure 8.

Graphs in Figure 8 show the increase of normalized stress intensity factor with crack length and this for all the inclination angles of the crack except for $\theta = 75^\circ$ where $\bar{K}_{eq}$ remains constant for all geometric ratios a/W_1 .

From the results, the influence of the interface on the evolution of $\bar{K}_{eq}$ is significant for $a/W_1 > 0.3$ and the slope evolution of graphs seems higher especially for inclination angles between $\theta = 0^\circ$ and $\theta = 60^\circ$. This slope is less pronounced for $\theta > 60^\circ$.

These results show that the increase of $\bar{K}_{eq}$ with the ratio of crack size to the distance to the interface depends strongly on the ratio of Young's moduli of the two materials: under uniform applied displacement, $\bar{K}_{eq}$ decreases when the crack inclination becomes closer to the interface direction if the crack is located in the weakest material.

Also it is important to insist that the width ratio W_1/W_2 has no effect on the $\bar{K}_{eq}$ results and this is connected to the boundary conditions which states that $\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$.

Fig. 7. Evolution of normalized stress intensity factor $\bar{K}_{II}$ with crack length and inclination angle θ , $E_1/E_2 = 21.276$.

Fig. 8. Evolution of normalized equivalent stress intensity factor $\bar{K}_{eq}$ with crack length and inclination angle θ , $E_1/E_2 = 21.276$.

The mismatch effect concerns also the mixed mode of fracture and depends on crack inclination angle. It is important to established that normalized equivalent stress intensity factors values in mixed mode will never exceed the ones obtained for the case of the first mode of fracture, i.e. $\theta = 0^\circ$.

The fact to consider geometric ratio of specimen $W_1/W_2 = 1.5$ in mixed mode don't have any influence on the decreasing tendency of the stress intensity factor which is confirmed by [15] for geometric ratio $W_1/W_2 = 1$ in mode I.

3.2 Symmetric cracks in bi-material ring specimen

3.2.1 Influence of interface position on the stress intensity factors

Figure 9 shows a bi-material cracked ring specimen where the edge cracks are located symmetrically with a length and subjected to a compression load F . Mechanical properties of material 1 are: Young modulus

$E_1 = 3050$ MPa, Poisson ratio $\nu_1 = 0.39$ and for material 2, we have: Young modulus $E_2 = 64\,300$ MPa, Poisson ratio $\nu_2 = 0.33$.

Geometric ratio of the crack is defined by $a/(R_2 - R_1)$ which varies between 0.1 and 0.7, radius ratio of ring is $R_2/R_1 = 4.0$ and the thickness of the ring specimen is equal to 5 mm.

Numerical model of the cracked specimen is considered with a flat edge where a pressure of $P = F/S$ is applied (S is the flat cross section). The resolution is performed in plane stress using 8 nodes quadratic elements, (Fig. 9). The global stress for this case can be expressed with the following relationship given by [20]:

$$\sigma_g = \frac{3F(R_2 + R_1)}{\pi B (R_2 - R_1)^2} \quad (13)$$

The calculation of the normalized stress intensity factors is done by using the formulation $K_0 = \sigma_g \sqrt{\pi a}$ at crack tips A and B. Then, we obtain on the following graphs, Figures 10 and 11 respectively for mode I and mode II at crack tip A.

Fig. 9. Numerical model of bi-material ring with symmetric edge cracks.

Fig. 10. Evolution of normalized stress intensity factor in mode I (opening) at crack tip A, $E_1/E_2 = 0.047$.

Fig. 11. Evolution of normalized stress intensity factor in mode II (shear) at crack tip A.

Fig. 12. Evolution of normalized stress intensity factor in mode I (opening) at crack tip B.

Fig. 13. Evolution of normalized stress intensity factor in mode II (shear) at crack tip B.

Figures 12 and 13 show the same factors for the crack tip B. In each case, the SIF data decrease when the interface angle increases. A minimum appears more or less pronounced for a normalized crack length of 0.4.

Figures 14 and 15 show the evolution of normalized equivalent stress intensity $\bar{K}_{eqA}$ and $\bar{K}_{eqB}$.

For inclination angles α between 0° and 85° (Fig. 14), we notice that normalized equivalent SIF $\bar{K}_{eqA}$ data are very small and present an overlap when the crack tip A grows in the material with lower Young modulus. For the case of crack tip B which is located in the material 2, the values of normalized equivalent stress intensity factors $\bar{K}_{eqB}$ (Fig. 15) are very high and their evolution is on progress especially for crack lengths $a/(R_2 - R_1) \geq 0.4$. When α varies between 95° and 165° , the crack tip A is

located in the most stiffer material (aluminium) and $\bar{K}_{eqB}$ values are close and quite small.

Also, it is very interesting to represent in Figures 16 and 17 the evolution of the same stress intensity factors $\bar{K}_{eqA}$ and $\bar{K}_{eqB}$ with inclination angle α .

Figures 16 and 17 show the both zones of crack tip evolution where the interface is marked by a vertical dashed line which corresponds to $\alpha = 90^\circ$. For the case where α is below 90° (Fig. 16), the crack tip A is located in the material 1 and the stress intensity factor values $\bar{K}_{eqA}$ are low and look the same until α is near 85° where the tendency to decrease is more significant. Near $\alpha = 90^\circ$, we notice that $\bar{K}_{eqA}$ increases due to the higher stiffness. This evolution is even more significant with crack size $a/(R_2 - R_1)$. Figure 17 shows the opposite evolution for crack tip B.

Fig. 14. Evolution of normalized equivalent stress intensity factor at crack tip A.

Fig. 15. Evolution of normalized equivalent stress intensity factor at crack tip B.

Fig. 16. Effect of interface angle α on normalized equivalent stress intensity factor at crack tip A.

Fig. 17. Effect of interface angle α on normalized equivalent stress intensity factor at crack tip B.

Fig. 18. (a) Evolution of normalized equivalent stress intensity factor with crack length and material ratio E_2/E_1 for $\alpha = 75^\circ$. (b) Evolution of normalized equivalent stress intensity factor with crack length and material ratio E_2/E_1 for $\alpha = 120^\circ$.

This is due to fact where the crack tip is located in the weakest material.

3.2.2 Influence of material properties on stress intensity factors

The influence of material properties on normalized stress intensity factors has been study for $\alpha = 75^\circ$ and 120° , and material ratio $E_2/E_1 = 2, 4$ and 6 (see Figs. 18a and b). For the case where $\alpha = 75^\circ$, the graphs seem very close for any material ratio.

When $\alpha = 120^\circ$, the crack tip A is located in the stiffer material, $\bar{K}_{eqA}$ increases more significantly as crack

length ratio increases. The growth tendency has been presented in the paper of Madani et al. [13] for the case of bi-material plate with an edge crack in term of energy release rate variation with crack length and material ratio E_2/E_1 .

Finally it is interesting to study the variation of $\bar{K}_{eqA}$ according to E_2/E_1 for different values of α and for a fixed crack length $a/(R_2 - R_1) = 0.4$ (Fig. 19). In this figure, the corresponding values to $\alpha = 90^\circ$ have been calculated using Excel program based on node displacements obtained by finite element method and Equations (1) to (9). The results show that some of the graphs follow a growth evolution when $\alpha > 90^\circ$ and the crack tip A is located in the

Fig. 19. Variation of normalized equivalent stress intensity factor at crack tip A with material ratio E_2/E_1 and α for $a/(R_2 - R_1) = 0.4$.

stiffer material. For $\alpha \leq 90^\circ$, the $\bar{K}_{eqA}$ values decrease especially when $\alpha = 0^\circ$, $\alpha = 30^\circ$ and $\alpha = 85^\circ$ where the $\bar{K}_{eqA}$ values are very close.

Most of the numerical results show that compression test on the cracked ring specimen formed by bi-material presents a continuity of normalized equivalent stress intensity factors evolution in term of orientation angle, stiffness ratio and normalized crack length. Then, it is much easier to characterize the fracture strength of cracked interface.

The fracture toughness of a brittle biomaterial can be evaluated from Figure 19 according to Young's modulus ratio E_2/E_1 and inclination angle α . Due to the fact that we are in elastic case, fracture toughness is given by multiplying K_{eqA} by the ratio F_c/F_{comp} where F_c is the critical load and F_{comp} the computing load.

4 Conclusion

This study has extended the mismatch effect analyses for the mixed mode of fracture as the published results were only concerning the case of opening mode. In this paper, it has been noticed that the mismatch effect can be present in the mixed mode of fracture for both cases such as: bi-material plates with inclined crack under tensile loading and bimaterial cracked ring specimen subjected to compression loading.

The results show that the mismatch effect depends on the crack length for both opening mode and on the crack inclination angle i.e. the bimodality crack loading. This effect is characterized by the tendency to decrease the equivalent SIF in mixed mode when the crack tip is located in the weakest material. Hence, we confirm the results obtained by Kazi Tani et al. [15] and experimentally by Cirello et al. [12] who have used the same specimen in the first mode of fracture with different width ratios.

In order to develop toughness tests for cracked bi-material specimens, we have studied the mismatch effect by using a cracked bi-material ring. The choice of this specimen is based on the fact that its manufacture is easy and cheaper. Furthermore, its shape allows us to change the bimodality by a simple rotation of the interface to loading direction.

The results show that it can be easy to obtain specimen calibration function in order to determine the critical stress intensity factor. The low cost of the test specimen and the execution celerity can allow easily getting the intrinsic curve of the interface fracture for bi-material.

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