

REGULAR ARTICLE

# Mixed $H_2/H_\infty$ guaranteed cost control for high speed elevator active guide shoe with parametric uncertainties

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**Abstract.** Aiming at the phenomenon that the elevator car system generates horizontal vibration due to the unevenness of the guide rail and the guide shoe modeling uncertainty caused by friction, wear and spring aging between the rolling guide shoe and the guide rail, a mixed  $H_2/H_\infty$  optimal guaranteed cost state feedback control strategy is proposed. Firstly, as the high-speed elevator car system always exist the phenomenon of stiffness and damping uncertainty in the guide shoe, the LFT method is adopted to construct the state space equation of the car system with parameter uncertainty. Secondly, considering the performance indexes of horizontal acceleration at the center of the car floor and the guide shoe vibration displacement system, an optimal guaranteed performance state feedback controller is designed based on the linear convex optimization method, which to minimize  $H_2$  performance index and achieve the specified  $H_\infty$  performance level. Thirdly, the free matrix is introduced to reduce the conservatism of the controller. Finally, by comparing the simulation results with other control methods under the same conditions, it is verified that the control strategy can make the car system have better vibration suppression ability, and can significantly improve the ride comfort of the elevator.

**Keywords:** Active guide shoes / parameter uncertainty / LMIs convex optimization / mixed  $H_2/H_\infty$  optimal guaranteed cost control / vibration suppression

## 1 Introduction

With the acceleration of urbanization, elevators, as modern vertical transportation tools, are developing towards high speed and high comfort. The increase of elevator running speed will bring a series of vibration problems, which makes passengers take adverse physiological reactions when taking the elevator [1,2]. Therefore, it is an imperative trend to effectively control the elevator vibration and ensure the passengers superior riding comfort.

During the operation of high-speed elevator, the parameters of elevator system will various due to the change of load, the disturbance of external excitation, the friction and wear of components, as well as the aging of elastic elements and so on, thus affecting the operation stability and ride comfort of elevator car system. Therefore, in order to reflect the real problems more accurately and reliably, it is necessary to consider the influence of parameter uncertainty on the performance of elevator system. Feng et al. [3] considering the uncertainty of external load, combined with Lyapunov method, designed

position controller and attitude controller based on translation model and rotation model to reduce the horizontal vibration of elevator car. Rijanto et al. [4] combined with the characteristics of elevator dynamics parameters, designed an elevator damping control system. The LPV output feedback controller is designed to improve the performance of the elevator by the vibration problem of the parameter variation caused by the deflection of the guide rail. Hu et al. [5] aimed at the influence of load disturbance and parameter uncertainty of electromagnetic guide actuator of linear elevator, adopted robust  $H_\infty$  control to realize fast tracking strong robustness and effectively suppress horizontal vibration of car. Wang et al. [6] presented a starting torque strategy of weightless sensor based on unbiased model predictive control to solve the problem of car fallback of gearless elevator with incremental encoder. The electromagnetic torque can quickly balance the uncertain load torque to minimize the sliding distance of the elevator car, and realize the superior ride comfort of the car. Yu et al. [7] presented an adaptive controller for the uncomfortable operation caused by the change of external load and disturbance, and designed the servo error observer and disturbance rejection control. By identifying the parameters of the linearized system, the

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observer coefficients and gain values of the system are self-adaptive, thus reducing the influence of parameter changes and uncertainties. Wolszczak et al. [8] studied the nonlinear mechanical problem of elevator braking system under the condition of uncertainty. Based on the condition of mechanical balance, the determinate model between braking force and uncertain parameters was deduced. The influence of the spring reaction force uncertainty on the braking force is analyzed quantitatively, and the influence of the parameter uncertainty on the operation efficiency of the elevator braking system is explored by solving the robust optimization problem. In the above researches, scholars mainly consider the influence of load uncertainty and external excitation disturbance on the ride comfort and operation efficiency of elevator car system. Venkatesh et al. [9] proposed a practical method of designing a high-performance LFT controller for elevator vertical motion of high-rise buildings. The simulation results show that the method improves the quality of vertical transportation and is not sensitive to parameter uncertainty caused by normal wear and tear. Hu et al. [10] designed the vertical motion control system of the super high-rise/high-speed elevator, described the actual elevator system with a linear model, and then generated the controller using LMIs processing method. The simulation results show that the controller is robust to the change of system parameters caused by component wear and dynamic change of elevator wire rope. The above researches have been carried out on the influence of the change of stiffness and damping between elastic elements on the performance of elevator system. Some scholars have also comprehensively studied the influence of load uncertainty, external excitation disturbance and the uncertainty of stiffness and damping between elastic elements on the performance of car system [11,12]. For example, Santd et al. [11] using the state-dependent Riccati Equation (SDRE) control method, studied the horizontal nonlinear response of the three-degree-of-freedom vertical transport model under the deformation of the guide rail. Also, the robustness of the SDRE control technique to parameter errors due to parameter uncertainties is tested.

The uncertainty of the parameters studied above mainly emphasize the influence of the uncertainty of the load outside the elevator car, the stiffness and damping of the elastic elements on the ride comfort of the elevator. It may be noticed that even though there are number of researches synthetically studies on the influence of the uncertainty of elevator system stiffness and damping on the ride comfort of the car under the external excitation disturbance. Nevertheless, the influence of the stiffness and damping uncertainty of the guide shoe under the excitation of the guide rail unevenness on the control performance of the elevator car system, has not been brought out in the above studies. In the process of vibration response analysis of high-speed elevator car system, as an important dynamic component, the guide shoe is usually simplified as the spring-damper model [13]. The stiffness and damping of the rolling guide shoe can not only withstand the impact of the elevator in high-speed operation, but also attenuate the vibration amplitude of the human body, which is much sensitive to vibration between 1 and 2 HZ in horizontal direction. The close contact between the guide shoe and the

guide rail limits the deflection movement and vibration displacement of the elevator car in the horizontal direction during the up and down operation of the elevator, which can significantly reduce the horizontal vibration of the elevator car system caused by the guide rail excitation. Therefore, as a crucial connecting part between the guide rail and the car, the rolling guide shoe plays an important role in ensuring the stable operation of the car and improving the ride comfort. However, under the actual working condition, the parameters of rolling guide shoe will change due to the disturbance of external excitation, friction and wear of components and aging of spring, thus it may tremendously deteriorate the control performance on the car system in the horizontal direction. Therefore, it is a theoretical significance for studied the influence that the rolling guide shoe parameters uncertainty on the horizontal vibration control performance of the elevator car system, which can improve the ride comfort of the high-speed elevator. Confronting the uncertainty and external excitation disturbance, we combine the mixed  $H_2/H_\infty$  optimal guaranteed cost control algorithm to design the active guide shoe controller, which can realize the active vibration reduction of the elevator car system. The mixed  $H_2/H_\infty$  optimal guaranteed cost control algorithm is a comprehensive algorithm. It integrates the characteristics that  $H_\infty$  control can guarantee the robust stability of the system under external excitation disturbance [14,15] and  $H_2$  can guarantee the optimal controlled performance of the system [16,17]. On the other hand, the state feedback controller is obtained by solving LMIs [18], which provides an impressive flexibility to tune the controller to compromise between  $H_\infty$  performance and  $H_2$  performance [19].

Based on the above discussion, the main contribution of this paper is that an LFT structure state-space uncertainty model of the high-speed elevator car with diagonal structure. Secondly, a model-based mixed  $H_2/H_\infty$  optimal guaranteed cost control strategy is proposed. Meanwhile, in the design process of the control strategy, a free matrix is introduced to reduce the conservativeness of uncertain structure and further improve the robust stability of the system.

This paper is organized as follows: the state-space uncertainty model of the horizontal vibration of the high-speed elevator car is established by using the LFT in section two. In Section 3, robust performance analysis and mixed  $H_2/H_\infty$  optimal guaranteed cost control strategy by linear convex optimization method are proposed. The simulation results of frequency domain and time domain response of 4m/s high-speed elevator car in Section 4. Section 5 concludes this paper.

## 2 Establishment of elevator car system model considering uncertainty of guide shoe parameters

### 2.1 Active control model for horizontal vibration of elevator car system

By adding controllers to each guide shoe, the horizontal vibration of elevator car system can be reduced by active

guide shoe. Assuming that the structural parameters of each guide shoe are identical, it is simplified as a mass-spring-damper system. The tangential creep force between the guide shoe and the guide rail is neglected, and the normal contact force between the guide shoe and the guide rail is only considered. Taking the point O at the center of the car floor as the observation point of the system, the horizontal vibration of elevator car system is established as shown in Figure 1. The elevator model established in this paper has 6-DOF, which are the degree of freedom of movement of the car in the  $X$ -axis direction, the degree of freedom of rotation of the car around the centroid, and the freedom of movement of each shoe in the  $X$ -axis direction.

Where  $m_c$  and  $m_{ui}(i=1\sim 4)$  represent the mass of the car and each guide shoe, respectively;  $k_{sf}$  and  $c_{sf}$  represent the equivalent stiffness and damping coefficient between the guide shoes and the car, respectively;  $k_{tf}$  and  $c_{tf}$  represent the normal contact stiffness coefficient and the normal contact damping coefficient between the guide shoe and the guide rail, respectively;  $x_c$  represents the horizontal vibration displacement for car centroid and  $x_{ui}$  is the vibration displacement of the guide shoe system with  $i=1\sim 4$ ;  $\theta$  and  $I_c$  are the deflection angles and moment of inertia of elevator car centroid, respectively;  $l_1$  and  $l_2$  are the vertical distances from the active shoe centroid to the

car centroid;  $u_i$  represents active control force applied to the car system of each shoe with  $i=1\sim 4$ ;  $x_{ri}$  is the guide rail excitation of the each guide shoes with  $i=1\sim 4$ .

According to Lagrange equation [20], the dynamic equation for this 6 DOF high-speed elevator car system model shown in Figure 1 can be modelled as

$$[\mathbf{M}]\ddot{\mathbf{Y}} + [\mathbf{C}_X]\dot{\mathbf{Y}} + [\mathbf{K}_X]\mathbf{Y} = \mathbf{F}_X \quad (1)$$

where  $\mathbf{M}$  is the generalized mass matrix of the system,  $\mathbf{Y}$  is the degree of freedom of the horizontal vibration of the system,  $\mathbf{F}_X$  is the generalized force of the system,  $\mathbf{K}_X$  is the generalized stiffness matrix of the system,  $\mathbf{C}_X$  is the generalized damping matrix of the system.

*See equation below.*

## 2.2 Problem statement and establish equation of state

The main objective of active guide shoe control is to minimize the horizontal vibration from the guide rail unevenness excitation to the car body and improve passenger ride comfort as well. As an important index to evaluate the ride comfort of high-speed elevator, the horizontal acceleration at the center of the car floor should

$$\Delta l = l_2 - l_1, \quad \Delta l' = l_1^2 + l_2^2, \quad \mathbf{M} = \text{diag}[m_c \quad I_c \quad m_{u1} \quad m_{u2} \quad m_{u3} \quad m_{u4}],$$

$$\mathbf{Y} = \begin{bmatrix} x_c \\ \theta \\ x_{u1} \\ x_{u2} \\ x_{u3} \\ x_{u4} \end{bmatrix}, \quad \mathbf{F}_X = \begin{bmatrix} \sum_{i=1}^4 u_i \\ u_1 l_1 - u_2 l_2 + u_3 l_1 - u_4 l_2 \\ k_{tf} x_{r1} + c_{tf} \dot{x}_{r1} - u_1 \\ k_{tf} x_{r2} + c_{tf} \dot{x}_{r2} - u_2 \\ k_{tf} x_{r3} + c_{tf} \dot{x}_{r3} - u_3 \\ k_{tf} x_{r4} + c_{tf} \dot{x}_{r4} - u_4 \end{bmatrix},$$

$$\mathbf{K}_X = \begin{bmatrix} 4k_{sf} & -2k_{sf}\Delta l & -k_{sf} & -k_{sf} & -k_{sf} & -k_{sf} \\ -2k_{sf}\Delta l & 2k_{sf}\Delta l' & -k_{sf}l_1 & k_{sf}l_2 & -k_{sf}l_1 & k_{sf}l_2 \\ -k_{sf} & -k_{sf}l_1 & (k_{sf} + k_{tf}) & 0 & 0 & 0 \\ -k_{sf} & k_{sf}l_2 & 0 & (k_{sf} + k_{tf}) & 0 & 0 \\ -k_{sf} & -k_{sf}l_1 & 0 & 0 & (k_{sf} + k_{tf}) & 0 \\ -k_{sf} & k_{sf}l_2 & 0 & 0 & 0 & (k_{sf} + k_{tf}) \end{bmatrix},$$

$$\mathbf{C}_X = \begin{bmatrix} 4c_{sf} & -2c_{sf}\Delta l & -c_{sf} & -c_{sf} & -c_{sf} & -c_{sf} \\ -2c_{sf}\Delta l & 2c_{sf}\Delta l' & -c_{sf}l_1 & c_{sf}l_2 & -c_{sf}l_1 & c_{sf}l_2 \\ -c_{sf} & -c_{sf}l_1 & (c_{sf} + c_{tf}) & 0 & 0 & 0 \\ -c_{sf} & c_{sf}l_2 & 0 & (c_{sf} + c_{tf}) & 0 & 0 \\ -c_{sf} & -c_{sf}l_1 & 0 & 0 & (c_{sf} + c_{tf}) & 0 \\ -c_{sf} & c_{sf}l_2 & 0 & 0 & 0 & (c_{sf} + c_{tf}) \end{bmatrix}.$$

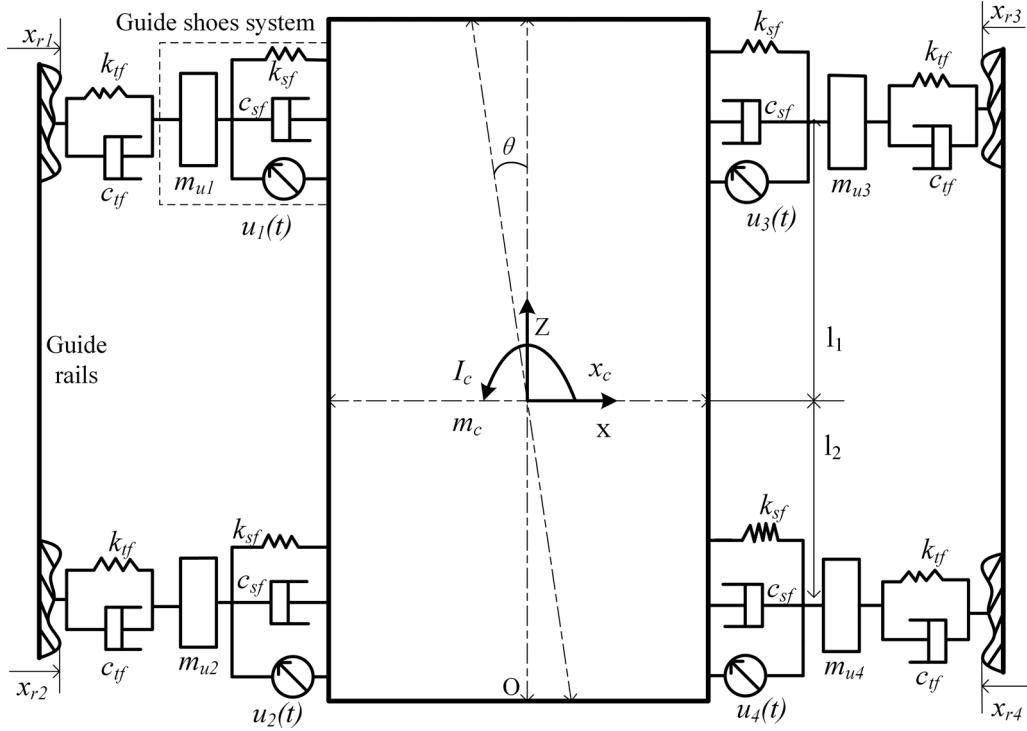


Fig. 1. Horizontal vibration active control model of 6 DOF high-speed elevator car system.

be strictly controlled to be the minimum under the disturbance of guide rail unevenness. At the same time, in order to ensure the safety of the car system when the elevator is running at high speed and avoid the phenomenon of “derailment” caused by the excessive vibration displacement of each guide shoe during the vibration process, the vibration displacement of each guide shoe should be controlled within the displacement constraint between the guide rail and the guide shoe. Therefore, the horizontal acceleration at the center of the car floor is taken as the  $H_2$  performance index, and its expression can be denoted as follows

$$\ddot{x}_{oc} = \ddot{x}_c + l_2 \ddot{\theta}. \quad (2)$$

The vibration displacement constraints of each guide shoe is taken as  $H_\infty$  performance index, and its expression can be denoted as follows

$$\frac{x_{ui}}{x_{\max}} \leq 1 \quad (3)$$

where  $x_{\max}$  is the maximum overlapping size between guide rail and guide shoe.

In order to achieve the above performances, the controlled perform outputs of car system are defined as

$$\mathbf{Z}_2 = [\lambda \ddot{x}_{oc}] \quad (4)$$

$$\mathbf{Z}_\infty = \begin{bmatrix} \frac{x_{u1}}{x_{\max}} & \frac{x_{u2}}{x_{\max}} & \frac{x_{u3}}{x_{\max}} & \frac{x_{u4}}{x_{\max}} \end{bmatrix}^T \quad (5)$$

where  $\lambda$  is define as a weighting coefficient, which represents the weighting proportion of state variables in the system outputs.  $\ddot{x}_{oc}$  is the controlled performance output,  $\frac{x_{ui}}{x_{\max}}$  is the constrained performance output with  $i = 1 \sim 4$ . Then the system model can be written as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}(t) + \mathbf{B}_2\boldsymbol{\varpi}(t) \\ \mathbf{Z}_2(t) &= \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_2\mathbf{u}(t) \\ \mathbf{Z}_\infty(t) &= \mathbf{C}_1\mathbf{x}(t) \end{aligned} \quad (6)$$

Defining the control input  $\mathbf{u}(t) = [u_1(t) \ u_2(t) \ u_3(t) \ u_4(t)]^T$ ,  $u_i(t) (i = 1, 2, 3, 4)$  is the active control force applied to the car system by each guide shoe in Figure 1.

The guide rail excitation  $\boldsymbol{\varpi}(t) = [x_{r1} \ x_{r2} \ x_{r3} \ x_{r4} \ \dot{x}_{r1} \ \dot{x}_{r2} \ \dot{x}_{r3} \ \dot{x}_{r4}]^T$ , and the state vector  $\mathbf{x}(t) = [\dot{x}_c \ \theta \ \dot{x}_{u1} \ \dot{x}_{u2} \ \dot{x}_{u3} \ \dot{x}_{u4} \ x_c \ \theta \ x_{u1} \ x_{u2} \ x_{u3} \ x_{u4}]^T$ ,  $\mathbf{A}$ ,  $\mathbf{B}_1$ ,  $\mathbf{B}_2$ ,  $\mathbf{C}_2$ ,  $\mathbf{D}_2$ ,  $\mathbf{C}_1$  are matrices with proper dimension and the specific numerical matrix are shown in Appendix A.

The vibration velocity of the four guide shoes and the car center of mass in the above state equation can be measured by the speed sensor, and the vibration displacement can be obtained by integrating the output value of the speed sensor. Similarly, the car deflection angle speed can be measured by the angle speed sensor, and the deflection angle is obtained by the output value integration of the angle speed sensor.

### 2.3 Car system model with parameter uncertainty

When the elevator is running at high speed, the rolling guide shoe will change the damping and rigidity of its

elastic elements due to the change of working conditions, friction and wear between guide wheel and guide rail, aging of spring, etc., which makes it difficult to describe the guide shoe system with accurate mathematical model. The uncertainty of damping and stiffness will exist in the online model, which will lead to the model uncertainty in the elevator car system and affect the control accuracy of the system. Parameter uncertainty is related to the difference between the actual physical values of the system and the numerical parameters used in the system [21]. Therefore, in order to express the mathematical model of elevator car system more accurately, the stiffness and damping of guide shoe system can be defined as

$$k_{sf} = \bar{k}(1 + d_k \delta_k(t)) \quad (7)$$

$$c_{sf} = \bar{c}(1 + d_c \delta_c(t)) \quad (8)$$

where  $\bar{k}$  and  $\bar{c}$  represent nominal values of stiffness and damping of guide shoe, respectively;  $d_k$  and  $d_c$  are perturbation values of parameters, which are used to describe the difference between theoretical model and actual dynamic model of guide shoe.  $\delta_k$  and  $\delta_c$  is used to describe the perturbation range of parameter uncertainty,  $-1 \leq \delta_k, \delta_c \leq 1$ .

In order to establish a state-space uncertainty model of LFT structure with diagonal structure of high-speed elevator car horizontal vibration, we introduce the LFT method. This method is a modeling method that separates the determined part and the uncertain part of the system, and then connects the determined part and the uncertain part of the system by feedback. According to the concept of LFT, equation (7) and equation (8) can be represented as [22]:

$$k_{sf} = \Gamma_U(\Phi_k, \delta_k(t)) \quad (9)$$

$$c_{sf} = \Gamma_U(\Phi_c, \delta_c(t)) \quad (10)$$

where  $\Gamma_U$  is the representation of the upper LFT,  $\Phi_k = \begin{bmatrix} 0 & \bar{k} \\ d_k & \bar{k} \end{bmatrix}$ ,  $\Phi_c = \begin{bmatrix} 0 & \bar{c} \\ d_c & \bar{c} \end{bmatrix}$ .

The model of the guide shoe parameter uncertainty is shown in Figure 2.  $\delta(t)$  is the uncertain time-varying matrix with bounded norm,  $\delta(t) = \begin{bmatrix} \delta_k(t) & 0 \\ 0 & \delta_c(t) \end{bmatrix}$ .  $q_j$  and  $p_j$  are the input and output of uncertain time-varying matrix  $\delta(t)$ .  $j$  is the number of state variables with  $j=1, 2, \dots, 12$ .  $\psi$  is nominal model of guide shoe without parameter uncertainty, and  $\psi = \begin{bmatrix} \Phi_k & 0 \\ 0 & \Phi_c \end{bmatrix} \cdot u(t)$  is the active control force applied by the guide shoe to the car system, and  $\varpi(t)$  is the random excitation of the guide rail unevenness.

Hence, the state space equation of high-speed elevator car system considering the uncertainty of guide shoe

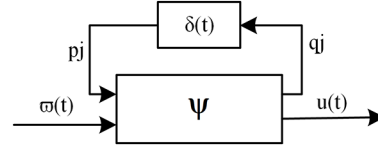


Fig. 2. LFT model representation of guide shoe parameter uncertainty.

parameters can be described by LFT as follows

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{q} \\ \mathbf{Z}_2 \\ \mathbf{Z}_\infty \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_{p0} & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{C}_{p0} & \mathbf{D}_{p0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_2 & \mathbf{D}_{p2} & \mathbf{D}_2 & \mathbf{0} \\ \mathbf{C}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \\ \mathbf{u} \\ \varpi \end{bmatrix} \quad (11)$$

$$\mathbf{q} = [q_1, q_2, \dots, q_{12}]^T$$

$$\mathbf{p} = [p_1, p_2, \dots, p_{12}]^T$$

$$\mathbf{p} = \delta(t)\mathbf{q}$$

where  $\mathbf{B}_{p0} \in R^{12 \times 12}$ ,  $\mathbf{D}_{p2} \in R^{1 \times 12}$  defines the relationship between the input of parameter uncertainty matrix and the system state variables and performance output.

$\mathbf{C}_{p0} \in R^{12 \times 12}$ ,  $\mathbf{D}_{p0} \in R^{12 \times 12}$  defines the relationship between the input and output of parameter uncertainty matrix.

Consequently, the state equation of car system considering the uncertainty of guide shoe parameters can be further expressed as follows

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A} + \Delta\mathbf{A})\mathbf{x}(t) + (\mathbf{B}_1 + \Delta\mathbf{B}_1)\mathbf{u}(t) + \mathbf{B}_2\varpi(t) \\ \mathbf{Z}_2(t) &= \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_2\mathbf{u}(t) \\ \mathbf{Z}_\infty(t) &= \mathbf{C}_1\mathbf{x}(t) \end{aligned} \quad (12)$$

$\Delta\mathbf{A}$ ,  $\Delta\mathbf{B}$  are an uncertain matrix function with appropriate dimension, it can be represented by

$$[\Delta\mathbf{A} \quad \Delta\mathbf{B}_1] = \mathbf{H}\Delta(t)[\mathbf{E}_1 \quad \mathbf{E}_2] \quad (13)$$

where  $\mathbf{H}$ ,  $\mathbf{E}_1$ ,  $\mathbf{E}_2$  are known matrices to represent the structure of uncertain matrices.  $\Delta(t) = \text{diag}(\delta_N(t))$  is a norm-bounded time-varying uncertain matrix with  $N = \underbrace{c, c, \dots, c}_6, \underbrace{k, k, \dots, k}_6$ ,  $\delta_N(t)$  is the time-varying value of the parameter uncertainty of the  $j$ -th state variable, and  $|\delta_N(t)| < 1$ .

**Remark 1.** The state equation (12) of high-speed elevator car system with diagonal structure constructed by LFT has the disadvantage of great conservatism. we only know the uncertain matrix  $\Delta(t)$  constructed by LFT is time-varying diagonal matrix block, but the internal structure is not clear. Therefore, it is necessary to describe the uncertainty matrix  $\Delta(t)$  with known matrix to reduce the conservatism of the system.

### 3 Robust performance analysis and controller design of car system

#### 3.1 Robust performance analysis of car system

The purpose of this paper is to design a mixed  $H_2/H_\infty$  optimal guaranteed cost controller for the car system with parameters uncertain guide shoe. Thus, the following conditions should be satisfied:

(a) The system has quadratic stability and all eigenvalues of the state matrix closed-loop system are in the left half-open complex plane.

(b) The performance index  $J(K)$  of the closed-loop transfer function  $\|T_{Z\omega}(s)\|_2$  of  $H_2$  should be the smallest. The specific requirements are as follows

$$\min J(K) \quad (14)$$

where  $J(K)$  represents the upper bound of  $H_2$  performance index of elevator car system under the worst-case of guide rail excitation.

(c) The closed-loop transfer function  $\|T_{Z\omega}(s)\|_\infty$  of  $H_\infty$  does not exceed the given disturbance rejection  $\gamma$ . The specific requirements are as follows

$$\|T_{Z\omega}(s)\|_\infty \leq \gamma. \quad (15)$$

In order to achieve better controlled performance, a state feedback controller is designed as follows

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t) \quad (16)$$

where  $\mathbf{K}$  state-feedback gain matrix to be designed. Combining equation (12) with equation (16), the closed-loop system of elevator car can be rewritten as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \overline{\mathbf{A}}_c \mathbf{x}(t) + \mathbf{B}_2 \omega(t) \\ \mathbf{Z}_2(t) &= \mathbf{C}_{2c} \mathbf{x}(t) \\ \mathbf{Z}_\infty(t) &= \mathbf{C}_{1c} \mathbf{x}(t) \end{aligned} \quad (17)$$

where

$$\begin{aligned} \overline{\mathbf{A}}_c &= \mathbf{A}_c + \mathbf{H}\Delta(t)\mathbf{E}_c, \mathbf{C}_{2c} = \mathbf{C}_2 + \mathbf{D}_2\mathbf{K}, \mathbf{C}_{1c} = \mathbf{C}_1, \\ \mathbf{A}_c &= \mathbf{A} + \mathbf{B}_1\mathbf{K}, \mathbf{E}_c = \mathbf{E}_1 + \mathbf{E}_2\mathbf{K}. \end{aligned}$$

In a word, the vibration control diagram of car system considering the uncertainty of guide shoe parameters can be shown as Figure 3:  $\mathbf{G}$  represents the nominal model of generalized car system without uncertain parameter,  $\Delta(t)$  is the diagonal time-varying matrix with norm bounded,  $\mathbf{K}$  is the state feedback controller designed for the system, and  $\mathbf{Z}(t)$  is the performance output of the system.

#### 3.2 Design of mixed $H_2/H_\infty$ optimal guaranteed cost controller

Based on the literature [23], the desired performance of the closed-loop system of the elevator car is achieved. The proposed mixed  $H_2/H_\infty$  optimal guaranteed cost controller can be obtained by solving the optimal solution of the theorem.

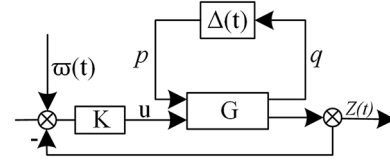


Fig. 3. Control diagram of elevator car system considering uncertainty of guide shoes parameters.

**Lemma** [24] Given matrix  $\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12}^T & \mathbf{S}_{22} \end{bmatrix} < 0$ ,

There are matrix  $\mathbf{S}_{11}$  and negative definite matrix  $\mathbf{S}_{22}$ , the following two matrices are equal.

- (1)  $\mathbf{S} < 0$
- (2)  $\mathbf{S}_{11} - \mathbf{S}_{12} \mathbf{S}_{22}^{-1} \mathbf{S}_{12}^T < 0$

**Theorem.** Considering the closed-loop elevator car system in (17) with uncertain matrix  $\Delta(t)$ : Given positive scalars  $\gamma$ , if there exist symmetric positive definite matrices  $\mathbf{X}$ , and general matrices  $\mathbf{V}$  satisfying the following inequalities. The closed-loop system in (17) is asymptotically stable and satisfies the desired conditions described in (14) and (15).

$$\begin{aligned} \min \mathbf{Z} &= \text{tr}(\mathbf{N}) \\ \begin{bmatrix} \tilde{\mathbf{W}} & \mathbf{S}_1 & \mathbf{S}_2 & \mathbf{S}_3 & \mathbf{H}\tilde{\mathbf{Q}} \\ * & -\tilde{\mathbf{Q}} & 0 & 0 & 0 \\ * & 0 & -\beta\mathbf{I} & 0 & 0 \\ * & * & 0 & -\mathbf{I} & 0 \\ * & * & 0 & 0 & -\tilde{\mathbf{Q}} \end{bmatrix} &< 0 \\ \begin{bmatrix} -\mathbf{N} & \mathbf{B}_2^T \\ * & -\mathbf{X} \end{bmatrix} &< 0 \end{aligned} \quad (18)$$

Moreover, the mixed  $H_2/H_\infty$  optimal guaranteed cost controller considering the uncertainty of guide shoe parameters can be obtained as:

$$\mathbf{u}(t) = \mathbf{V}\mathbf{X}^{-1}\mathbf{x}(t). \quad (19)$$

At the same time, equation (19) is substituted for equation (17) to calculate that the upper bound of  $H_2$  performance index is  $J(K) = \text{tr}(\mathbf{B}_2^T \mathbf{X}^{-1} \mathbf{B}_2)$ .

Where

$$\tilde{\mathbf{W}} = (\mathbf{A}\mathbf{X} + \mathbf{B}_1\mathbf{V})^T + \mathbf{A}\mathbf{X} + \mathbf{B}_1\mathbf{V} + \beta\gamma^{-2}\mathbf{B}_2\mathbf{B}_2^T$$

$$\mathbf{S}_1 = (\mathbf{E}_1\mathbf{X} + \mathbf{E}_2\mathbf{V})^T \quad \mathbf{S}_2 = (\mathbf{C}_1\mathbf{X})^T \quad \mathbf{S}_3 = (\mathbf{C}_2\mathbf{X} + \mathbf{D}_2\mathbf{V})^T$$

**Proof.** In order to further clarify the structure of the uncertain matrix  $\Delta(t)$  in the equation (13), reduce the conservatism in the design of the system controller, and improve the robust stability performance of the system, the following free matrix is applied to Theorem:

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_{12}], \quad \alpha_j > 0, \quad (20)$$

$$\tilde{\mathbf{Q}} = \text{diag}\{\alpha_1\mathbf{I}, \alpha_2\mathbf{I}, \dots, \alpha_{12}\mathbf{I}\} \quad (21)$$

$$\tilde{\mathbf{Q}}^{-1} = \text{diag}\{\alpha_1^{-1}I, \alpha_2^{-1}I, \dots, \alpha_{12}^{-1}I\} \quad (22)$$

Uncertainty matrix  $\Delta\mathbf{A}$  and  $\Delta\mathbf{B}$  can be further described as

$$\begin{aligned} [\Delta\mathbf{A} \quad \Delta\mathbf{B}] &= \mathbf{H}\Delta(t)[\mathbf{E}_1 \quad \mathbf{E}_2] \\ &= \mathbf{H}\tilde{\mathbf{Q}}\Delta(t)\left[\tilde{\mathbf{Q}}^{-1}\mathbf{E}_1 \quad \tilde{\mathbf{Q}}^{-1}\mathbf{E}_2\right] \end{aligned} \quad (23)$$

Based on the literature Theorem 2 [23], the matrix  $\mathbf{H}$  is represented by  $\mathbf{H}\tilde{\mathbf{Q}}$ , and  $\alpha I$  is represented by  $\tilde{\mathbf{Q}}$ , and matrix inequality (18) can be obtained by Lemma.

In matrix inequality (18),  $\text{tr}(\mathbf{B}_2^T \mathbf{X}^{-1} \mathbf{B}_2) < \text{tr}(\mathbf{N})$  is obtained by Lemma and the minimization of  $\text{tr}(\mathbf{N})$  guarantees the minimization of  $\text{tr}(\mathbf{B}_2^T \mathbf{X}^{-1} \mathbf{B}_2)$ , that is, the minimization of  $\|T_{z_2 w}(s)\|_2$ . Prove to be true.

**Remark 2.** By introducing the free matrix  $\tilde{\mathbf{Q}}$ , the structural information of the uncertain matrix  $\Delta(t)$  is further clarified to reduce the conservativeness of the controller design. If the Theorem has an optimal solution, the mixed  $H_2/H_\infty$  optimal guaranteed cost controller considering the uncertainties of guide shoe parameters can be achieved. That is to ensure that the vibration displacement of guide shoe has certain robustness under the excitation of guide rail, and at the same time, the horizontal vibration acceleration of car is minimized.

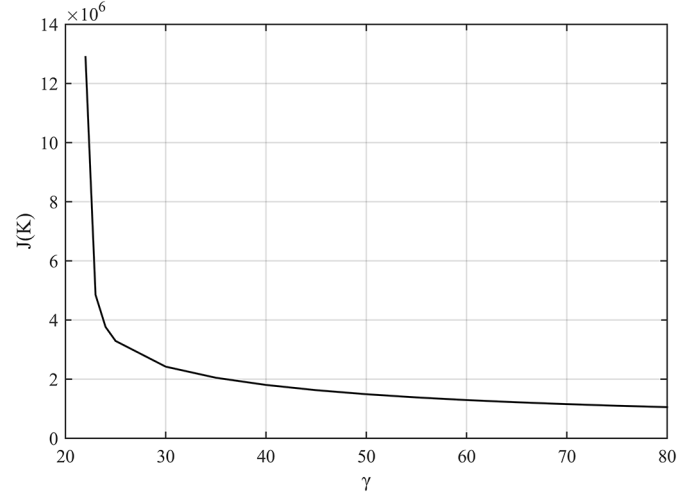
**Corollary.** Considering the closed-loop elevator car system in (17) with uncertain matrix  $\Delta(t)$ : Given positive scalars  $\gamma$ ,  $\varepsilon$  and  $\rho$ , if there exist symmetric positive definite matrices  $\mathbf{X}$ , and general matrices  $\mathbf{V}$  satisfying the following inequalities. The closed-loop system in (17) is asymptotically stable and satisfies the desired conditions described in (14) and (15).

See equation (24) below.

Moreover, the  $H_\infty$  control gain considering the uncertainty of guide shoe parameters can be obtained as:

$$\mathbf{K}_{H_\infty} = \mathbf{V}\mathbf{X}^{-1} \quad (25)$$

**Proof.** The proof of corollary can be easily completed by choosing matrices  $\mathbf{X}$ ,  $\mathbf{V}$  and following the similar manner of Theorem.



**Fig. 4.** The relationship between optimal performance and robust stability of car closed-loop system.

Theorem and Corollary are a linear convex optimization problem with matrix variables  $\mathbf{X}$  and  $\mathbf{V}$ , which needs to be solved by calling mincx function of Matlab. The corresponding value  $J(K)$  under different disturbance rejection degree  $\gamma$  is shown in Figure 4. As can be seen from Figure 4, the disturbance suppression degree  $\gamma$  of  $H_\infty$  and the  $H_2$  performance index  $J(K)$  in the Theorem is mutually constrained. Therefore, in order to ensure the minimum acceleration of horizontal vibration at the center of car bottom and the displacement of guide shoe vibration meet the limit of displacement constraint when the elevator is running at high speed,  $\gamma = 40$  is selected finally.

By solving Theorem with  $\gamma = 40$ , the closed-loop elevator car system in (17) controller under the proposed control method is obtained as

$$\mathbf{u}(t) = [\mathbf{K}_{11} \quad \mathbf{K}_{12} \quad \mathbf{K}_{13}] \mathbf{x}(t) \quad (26)$$

where

$$\mathbf{K}_{11} = 10^4 \times \begin{bmatrix} -10.353 & 10.903 & 0.3499 & -0.0468 \\ -5.7685 & -7.8267 & -0.0095 & 0.0886 \\ -10.353 & 10.903 & -0.1460 & -0.0468 \\ -5.7685 & -7.8267 & -0.0095 & -0.0567 \end{bmatrix},$$

$$\begin{aligned} & \min \gamma^2 \\ & \begin{bmatrix} \mathbf{X}\mathbf{A}^T + \mathbf{V}^T \mathbf{B}_1^T + \mathbf{A}\mathbf{X} + \mathbf{B}_1\mathbf{V} & \mathbf{H} & \mathbf{X}\mathbf{E}_1^T + \mathbf{V}^T \mathbf{E}_2^T & \mathbf{B}_2 & \mathbf{X}\mathbf{C}_1^T + \mathbf{V}^T \mathbf{D}_1^T \\ \mathbf{H}^T & -\varepsilon \mathbf{I} & 0 & 0 & 0 \\ \mathbf{E}_1\mathbf{X} + \mathbf{E}_2\mathbf{V} & 0 & -\varepsilon^{-1} \mathbf{I} & 0 & 0 \\ \mathbf{B}_2^T & 0 & 0 & -\gamma^2 \mathbf{I} & 0 \\ \mathbf{C}_1\mathbf{X} + \mathbf{D}_1\mathbf{V} & 0 & 0 & 0 & -\mathbf{I} \end{bmatrix} < 0 \\ & \begin{bmatrix} -\mathbf{I} & \sqrt{\rho}(\mathbf{C}_2\mathbf{X} + \mathbf{D}_2\mathbf{V}) \\ \sqrt{\rho}(\mathbf{X}\mathbf{C}_2^T + \mathbf{V}^T \mathbf{D}_2) & -\mathbf{X} \end{bmatrix} < 0 \end{aligned} \quad (24)$$

$$\mathbf{K}_{12} = 10^3 \times \begin{bmatrix} -1.4606 & -0.469 & -46.8560 & 588.35 \\ -0.0949 & -0.5672 & 59.221 & -109.50 \\ 3.4995 & -0.4684 & -468.56 & 588.35 \\ -0.0949 & 0.8867 & 59.221 & -109.50 \end{bmatrix},$$

$$\mathbf{K}_{13} = 10^5 \times \begin{bmatrix} 512.59 & -0.2975 & -507.81 & -0.2975 \\ -1.2781 & 666.72 & -1.2781 & -666.48 \\ -507.81 & -0.2975 & 512.59 & -0.2975 \\ -1.2781 & -666.48 & -1.2781 & 666.72 \end{bmatrix}.$$

By solving Corollary with  $\varepsilon = 1$  and  $\rho = 0.000125$ , the closed-loop elevator car system in (15) controller under the  $H_\infty$  control method is obtained as

$$\text{Where } \mathbf{u}(t)_{H_\infty} = [\mathbf{K}_{21} \quad \mathbf{K}_{22} \quad \mathbf{K}_{23}] \mathbf{x}(t) \quad (27)$$

$$\mathbf{K}_{21} = 10^8 \times \begin{bmatrix} -1602.9 & 1.8986 & -10.276 & -9.8712 \\ -36.603 & 0.4337 & -2.3467 & -2.2542 \\ -1602.9 & 1.8986 & -10.276 & -9.8712 \\ -366.03 & 0.4335 & -2.3467 & -2.2542 \end{bmatrix}$$

$$\mathbf{K}_{22} = 10^8 \times \begin{bmatrix} -10.276 & -9.8712 & 1380.9 & 508.36 \\ -2.3467 & -2.2542 & -315.34 & 116.09 \\ -10.276 & -9.8712 & -1380.9 & 508.36 \\ -2.3467 & -2.2542 & -315.34 & 116.09 \end{bmatrix}$$

$$\mathbf{K}_{23} = 10^9 \times \begin{bmatrix} 4.9437 & -15.683 & 4.9386 & 15.683 \\ 1.1282 & 3.5841 & 1.1282 & 3.5790 \\ 4.9386 & 156.83 & 4.9437 & 15.683 \\ 1.1282 & 3.5790 & 1.1282 & 3.5841 \end{bmatrix}$$

## 4 Simulation studies

In this section, taking a 4 m/s high-speed elevator as an example to evaluate and verify the performance of the proposed control strategy. The numerical values of the horizontal vibration model of a 6 DOF elevator car system, which are listed in Table 1. For the existing parameter uncertainties in  $k_{sf}$ ,  $c_{sf}$  we assume that the parameters vary between 0.8 and 1.2 times of the normal values, respectively. The maximum allowable overlap size of the guide rail is  $x_{\max} = 0.047m$ . Similarly, weight coefficient  $\lambda = 0.135$  is set. To highlight the effectiveness of the proposed strategy, the  $H_\infty$  control strategy is adopted for the comparison, which also considering the uncertainty of guide shoe stiffness and damping. The mixed  $H_2/H_\infty$  optimal guaranteed cost controller and the  $H_\infty$  control controller are obtained by solving Theorem and Corollary respectively.

The disturbance of horizontal vibration of high-speed elevator mainly comes from the unevenness of the guide rail [25]. Relevant research shows that the surface roughness of guideway obeys normal distribution approximately [26,27]. Therefore, the unevenness of the guide rail is taken on by

**Table 1.** Numerical values of elevator car system.

| Parameter | Value              | Unit                |
|-----------|--------------------|---------------------|
| $m_c$     | 2000               | kg                  |
| $I_c$     | 4048               | kg m <sup>2</sup>   |
| $\bar{k}$ | $3 \times 10^5$    | N m <sup>-1</sup>   |
| $\bar{c}$ | 920                | N s m <sup>-1</sup> |
| $k_{tf}$  | $6.71 \times 10^4$ | N m <sup>-1</sup>   |
| $c_{tf}$  | 134                | N s m <sup>-1</sup> |
| $l_1$     | 2.00               | m                   |
| $l_2$     | 1.50               | m                   |
| $m_u$     | 13                 | kg                  |

**Table 2.** Simulation setup of parameter uncertainty.

| Uncertainties | case 1 | case 2 | case 3 |
|---------------|--------|--------|--------|
| $d_k$         | 0      | 0.2    | -0.2   |
| $d_c$         | 0      | 0.2    | -0.2   |

the random excitation which has a mean value of 0 and standard deviation 4mm.

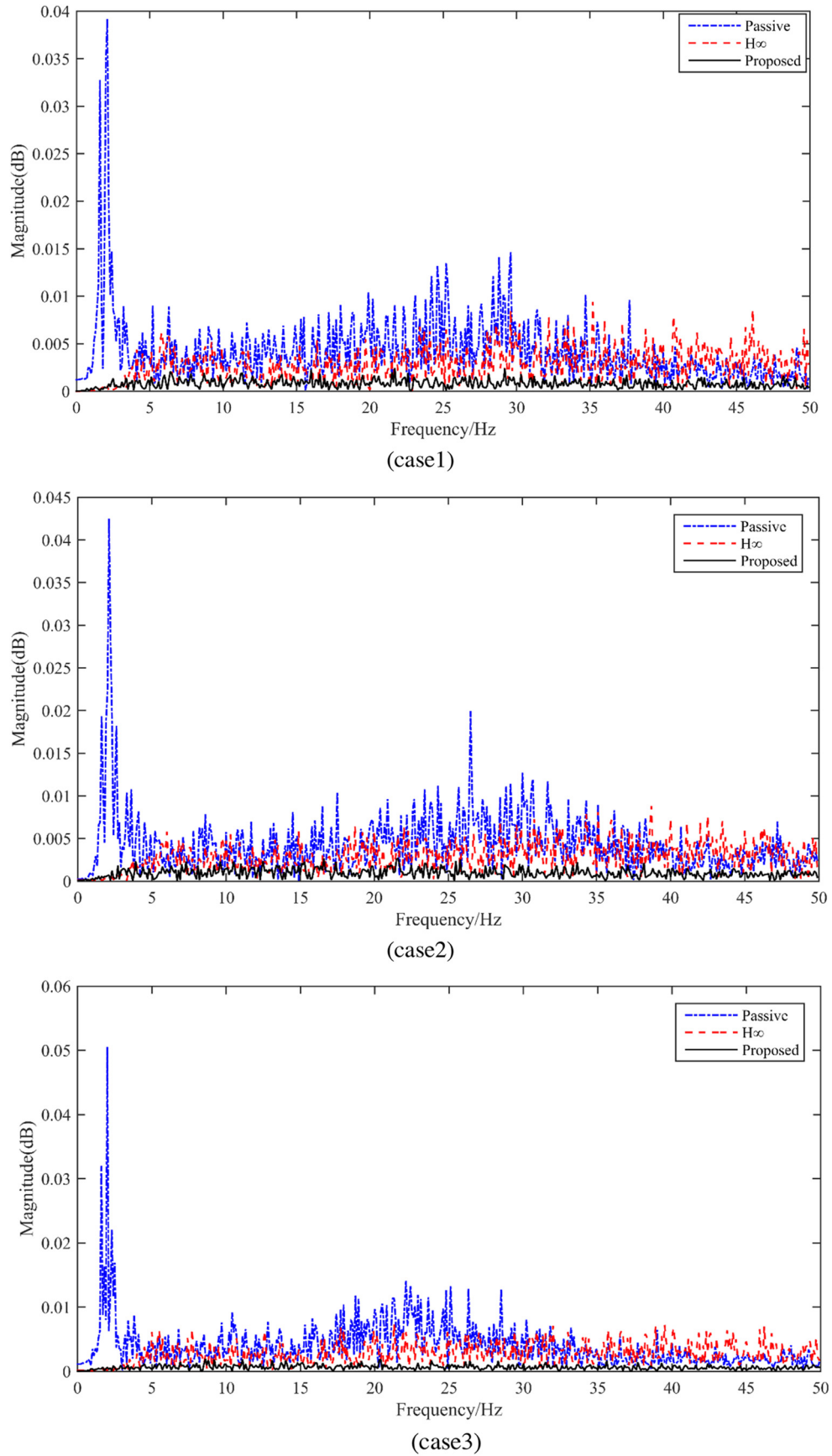
The parameter uncertainties in different cases are shown in Table 2. In case 1, there are no parameters variations in the guide shoes system. In case 2, the guide shoe parameters increased by 20% compared to the normal values. In case 3, the guide shoe parameters decreased by 20% than their normal values.

### 4.1 Frequency-domain response

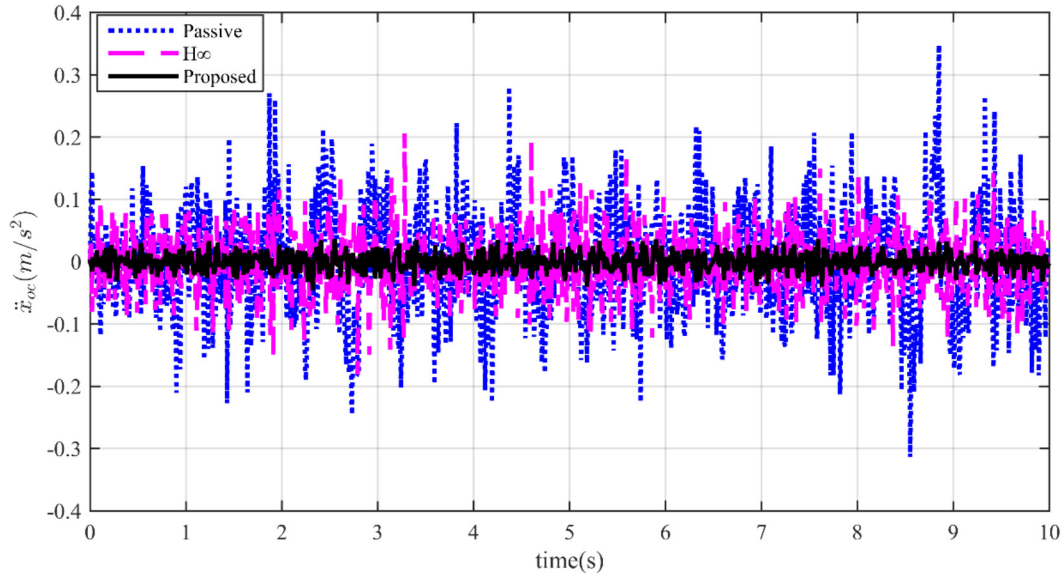
Frequency response of the elevator car horizontal vibration acceleration are shown in Figure 5. We can see that in the case of three changes of guide shoe stiffness and damping, the horizontal vibration amplitude of the car under the proposed control method is still smaller than that under the  $H_\infty$  control and passive control, and no significant change of the maximum amplitude in the sensitive frequency range of human body. The simulation results show that the proposed control method can effectively reduce the horizontal vibration of the elevator car and tolerant with the variations of stiffness and damping uncertainty. In addition, the simulation results have confirmed that the horizontal vibration of the elevator car in the low frequency range is mainly caused by the unevenness of the guide rail.

### 4.2 Time-domain response

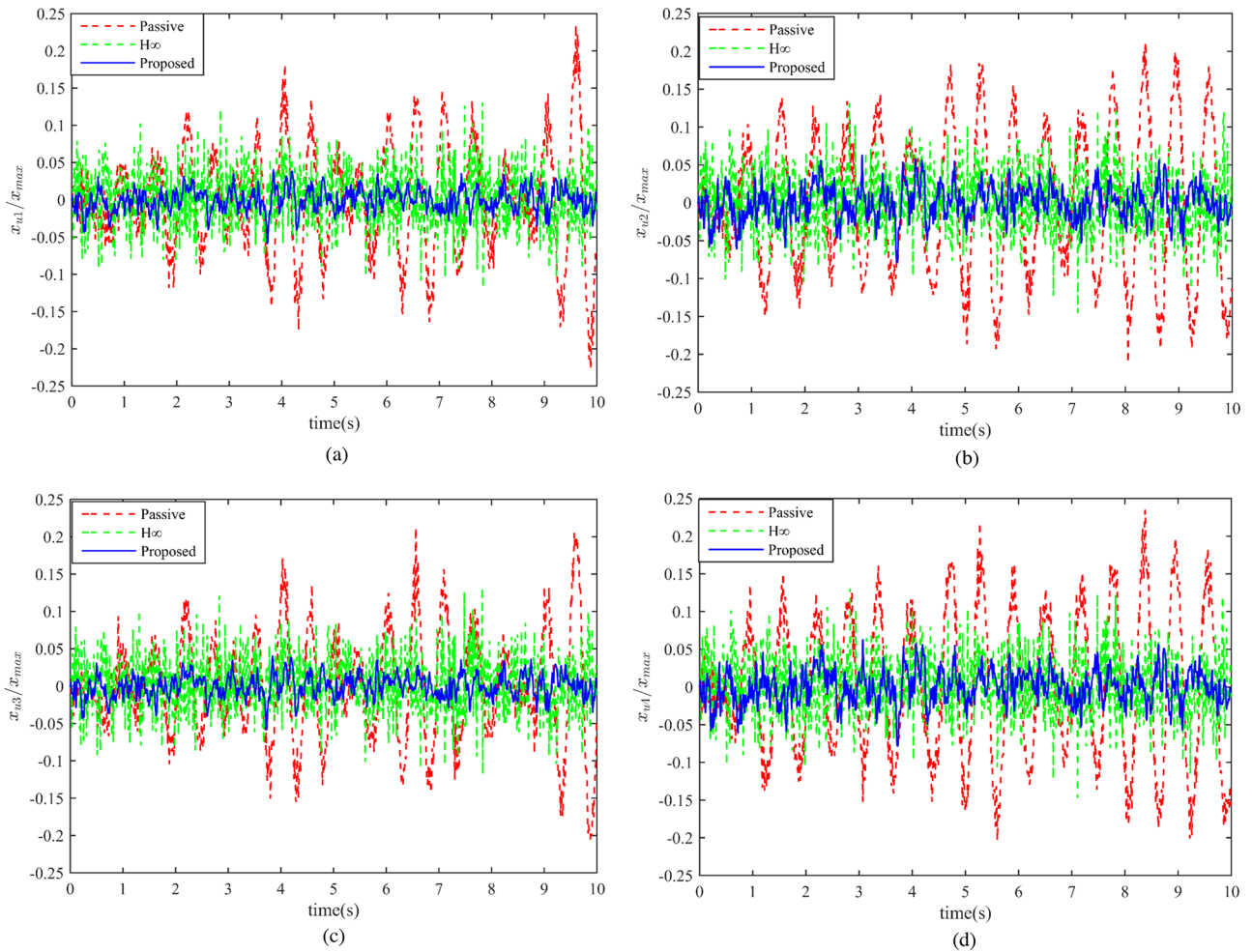
The time-domain response of the horizontal acceleration at the center of the elevator car floor in three cases is shown in Figures 6, 8 and 9. It can be seen that the vibration acceleration value of the car under the proposed control strategy is obviously smaller than that under the  $H_\infty$  control, and the change degree of acceleration vibration amplitude is far less than that under the  $H_\infty$  control, which shows that the proposed controller contributes largely to



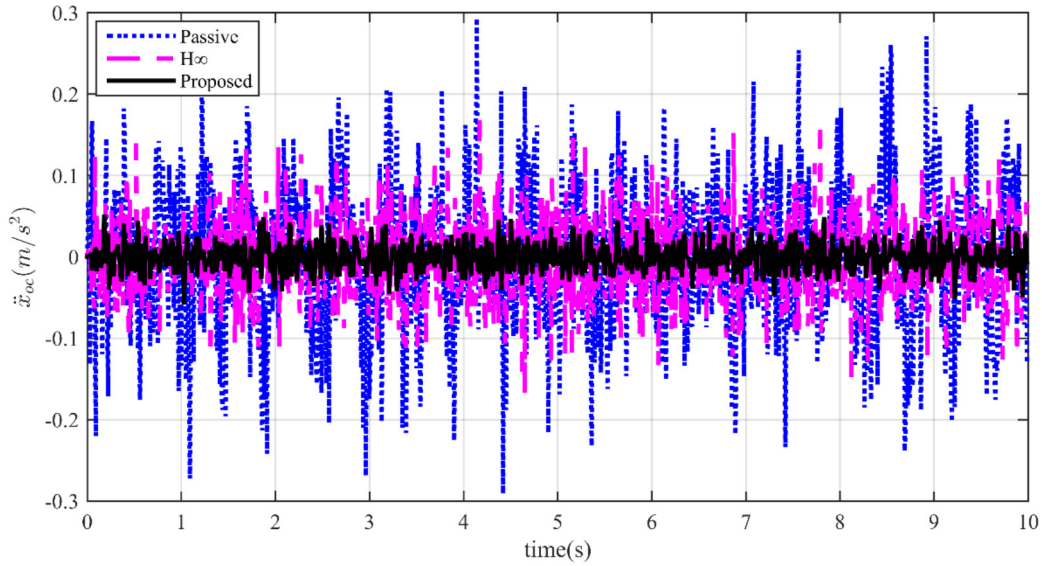
**Fig. 5.** Frequency response of the elevator car horizontal vibration acceleration.



**Fig. 6.** Time response of horizontal vibration acceleration at the center of car floor (case 1).



**Fig. 7.** Time response of vibration displacement constraint of each guide shoe (case 1). (a) Vibration displacement constraint of upper left guide shoe. (b) Vibration displacement constraint of left lower guide shoe. (c) Vibration displacement constraint of upper right guide shoe. (d) Vibration displacement constraint of right lower guide shoe.



**Fig. 8.** Time response of horizontal vibration acceleration at the center of car floor (case 2).

**Table 3.** Comparison of horizontal acceleration under three case.

|            | Case 1 |        | Case 2 |        | Case 3 |        |
|------------|--------|--------|--------|--------|--------|--------|
|            | Max    | RMS    | Max    | RMS    | Max    | RMS    |
| Passive    | 0.3479 | 0.0887 | 0.2937 | 0.0901 | 0.3220 | 0.0867 |
| $H_\infty$ | 0.2063 | 0.0520 | 0.1752 | 0.0528 | 0.1576 | 0.0515 |
| Proposed   | 0.0460 | 0.0145 | 0.0588 | 0.0175 | 0.0398 | 0.0114 |

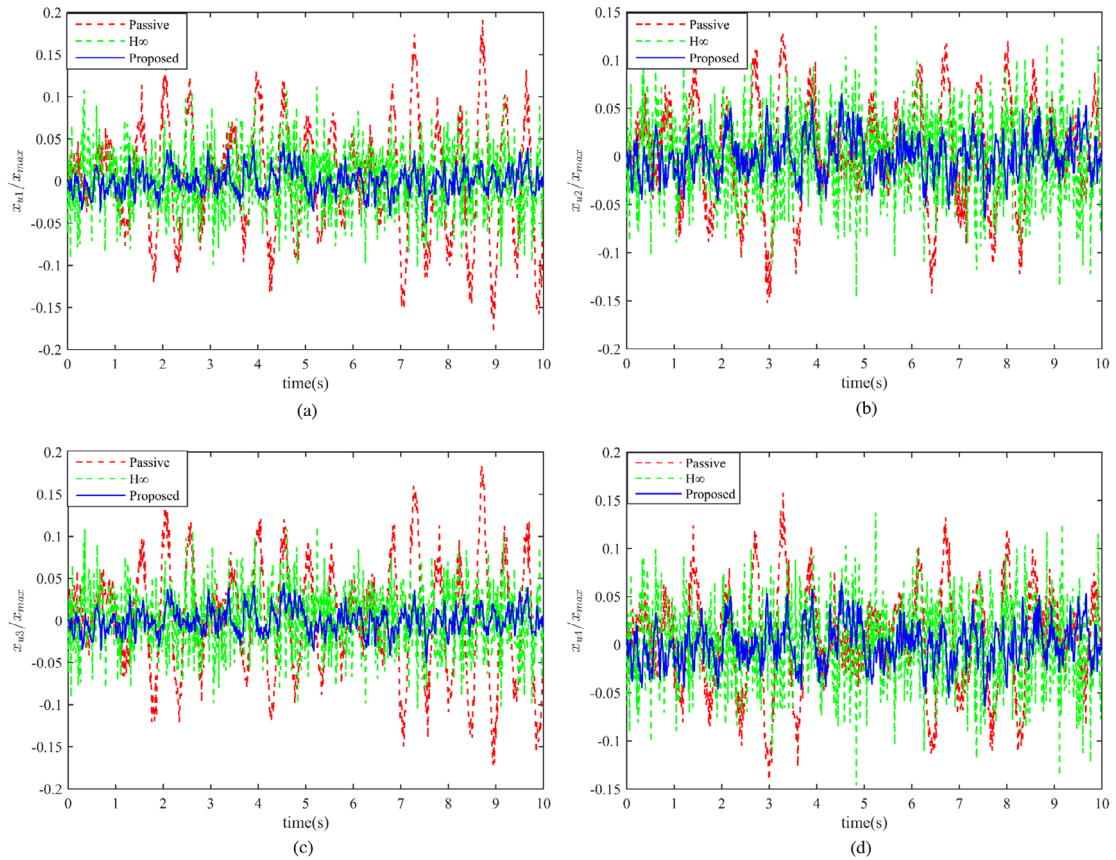
suppress the horizontal vibration and improve of ride comfort. The comparison between the maximum value of horizontal acceleration and the RMS value is shown in Table 3. Contrastive analysis shows, the horizontal vibration acceleration under  $H_\infty$  control is much higher than  $0.15\text{m/s}^2$  specified in the elevator industry standard, while the maximum value of horizontal acceleration under the controller proposed is basically about  $0.05\text{m/s}^2$ , and the RMS value is 72.11%, 66.85% and 77.96% lower than that under  $H_\infty$  control respectively.

The time-domain response of the guide shoe vibration displacement constraint is shown in Figures 7, 9 and 10. It can be seen that when the guide shoe stiffness and damping change, the vibration displacement constraint of each guide shoe under the proposed control strategy meets the displacement constraint condition, and the constraint value is far smaller than the vibration displacement constraint ratio of each guide shoe under the  $H_\infty$  control. This reveals that when the elevator is running at high speed, the proposed controller can meet the limit that the vibration displacement of the guide shoe does not exceed the maximum overlapping size of the guide rail, and ensure the safety of the elevator in operation.

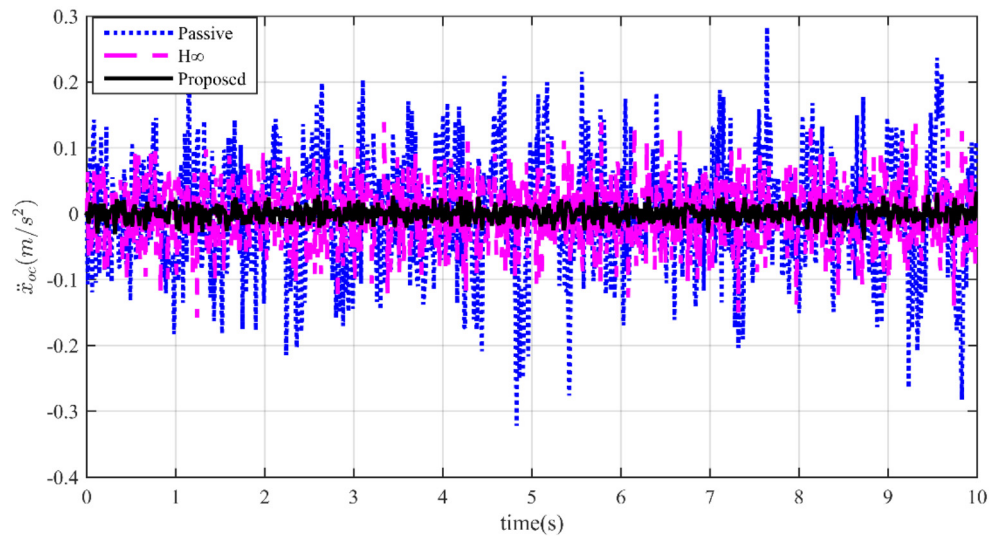
Obviously, the proposed controller has better performance on the horizontal vibration acceleration, while satisfying the displacement constraints of the guide shoe under the disturbance of guide rail unevenness and guide shoe stiffness and damping uncertainty. Meanwhile, it has good robustness to parameter uncertainty of guide shoe system, which further illustrate that the design active guide shoe controller can improve elevator car system ride quality. Thus, the simulation results verify the effectiveness and robustness of the proposed controller.

## 5 Conclusion

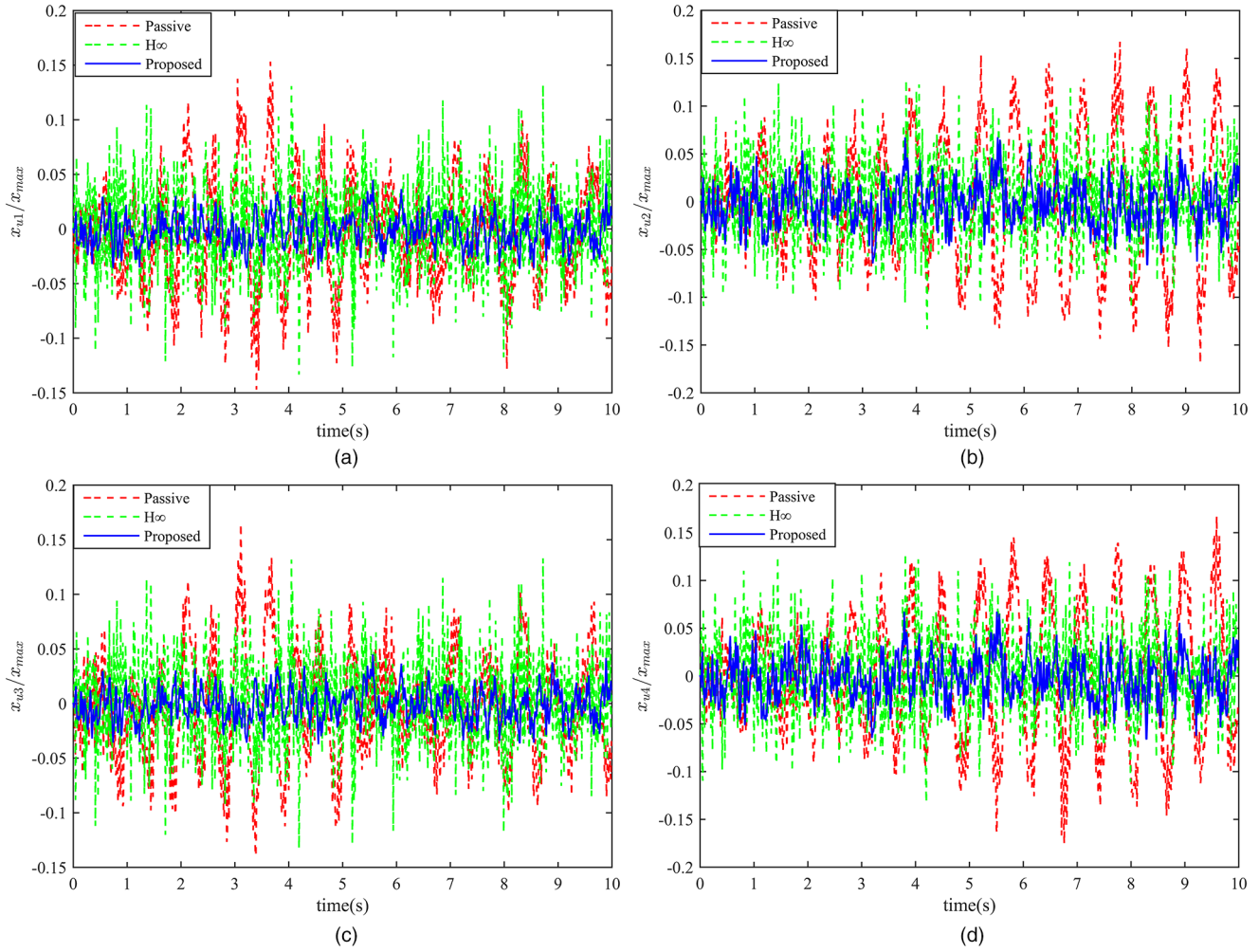
- Based on the model, the state space equation of elevator car system considering the uncertainty of guide shoe parameters is established by LFT method, and the mixed  $H_2/H_\infty$  optimal guaranteed cost state feedback control strategy is proposed. In the process of controller design, free matrix is introduced to reduce the conservatism of uncertainty matrix and further improve the robust stability of the system.



**Fig. 9.** Time response of vibration displacement constraint of each guide shoe (case 2). (a) Vibration displacement constraint of upper left guide shoe. (b) Vibration displacement constraint of left lower guide shoe. (c) Vibration displacement constraint of upper right guide shoe. (d) Vibration displacement constraint of right lower guide shoe.



**Fig. 10.** Time response of horizontal vibration acceleration at the center of car floor (case 3).



**Fig. 11.** Time response of vibration displacement constraint of each guide shoe (case 3). (a) Vibration displacement constraint of upper left guide shoe. (b) Vibration displacement constraint of left lower guide shoe. (c) Vibration displacement constraint of upper right guide shoe. (d) Vibration displacement constraint of right lower guide shoe.

- The simulation results of the 4m/s high-speed elevator car under the conditions of guide rail irregularity disturbance and guide shoe parameter uncertainty show that: compared with passive and  $H_{\infty}$  control, the mixed  $H_2/H_{\infty}$  optimal guaranteed cost control makes the elevator car system have smaller peak value of car horizontal acceleration, and ensures that the vibration displacement of guide shoe meets the limit of displacement constraint, which proves that the proposed method can effectively suppress the horizontal vibration of elevator car and has good robustness to parameter uncertainty.
- In this paper, a state feedback control strategy is proposed, which requires online measurement of all state variables of the system. When the elevator is running at high speed under the actual working condition, some state variables in the system can't be accurately measured, and there will be some measurement errors. However, these limitations don't affect the controlled performance of the system. The proposed control strategy can still fully describe the advantages of the internal dynamic characteristics of the system, has better

robustness to parameter uncertainty, and ensures the elevator car to obtain superior ride comfort.

## Nomenclature

|                      |                                                                                                           |
|----------------------|-----------------------------------------------------------------------------------------------------------|
| $\mathbf{A}$         | Nominal part of the coefficient matrix                                                                    |
| $\Delta\mathbf{A}$   | Uncertainty part of the coefficient matrix                                                                |
| $\mathbf{B}_1$       | Control input matrix                                                                                      |
| $\Delta\mathbf{B}_1$ | Uncertainty part of the control input matrix                                                              |
| $\mathbf{B}_2$       | Excitation input matrix                                                                                   |
| $\mathbf{C}_X$       | Generalized damping matrix of the system                                                                  |
| $\mathbf{C}_1$       | Output matrix of the $H_{\infty}$ norm                                                                    |
| $\mathbf{C}_2$       | Output matrix of the $H_2$ norm                                                                           |
| $c_{sf}$             | Equivalent damping coefficient between the guide shoes and the car ( $\text{N.s.m}^{-1}$ )                |
| $c_{tf}$             | The normal contact damping coefficient between the guide shoes and the guide rail ( $\text{N.s.m}^{-1}$ ) |
| $\bar{c}$            | Nominal values of damping ( $\text{N.s.m}^{-1}$ )                                                         |
| $\mathbf{D}_2$       | Output matrix of the $H_2$ norm                                                                           |

|                                     |                                                                                                       |
|-------------------------------------|-------------------------------------------------------------------------------------------------------|
| $d_c, d_k$                          | Perturbation values of parameters                                                                     |
| $\mathbf{E}_1, \mathbf{E}_2$        | Known matrices to represent the structure of uncertain matrices                                       |
| $\Delta(t)$                         | Norm-bounded time-varying uncertain matrix function                                                   |
| $\mathbf{H}$                        | Known matrices to represent the structure of uncertain matrices                                       |
| $I_c$                               | The moment of inertia of elevator car centroid ( $\text{kg.m}^{-2}$ )                                 |
| $J(K)$                              | Upper bound of $H_2$ performance index                                                                |
| $\mathbf{K}_x$                      | Generalized stiffness matrix of the system                                                            |
| $\mathbf{K}$                        | State-feedback gain matrix to be designed                                                             |
| $k_{sf}$                            | Equivalent stiffen coefficient between the guide shoes and the car ( $\text{N.m}^{-1}$ )              |
| $k_{tf}$                            | Normal contact stiffness coefficient between the guide shoes and the guide rail ( $\text{N.m}^{-1}$ ) |
| $\bar{k}$                           | Nominal values of stiffness and damping of guide shoes ( $\text{N.m}^{-1}$ )                          |
| $L_2(0 \rightarrow \infty)$         | Space of square-integrable vector function over $[0 \rightarrow \infty)$                              |
| $l_1, l_2$                          | Vertical distances from the active shoe centroid to the car centroid (m)                              |
| $\mathbf{M}$                        | Generalized mass matrix of the system                                                                 |
| Max                                 | Maximum value of horizontal acceleration at the center of car floor ( $\text{m.s}^{-1}$ )             |
| $m_c, m_{ui}$                       | Mass of the car and four GSs (kg)                                                                     |
| $\ T_{z_\infty} \varpi(s)\ _\infty$ | Transfer function of $H_\infty$ norm                                                                  |
| $\mathbf{F}_x(t)$                   | Generalized force of the system (N)                                                                   |
| $u_i(t)$                            | Control force exerted by active guide shoe (N)                                                        |
| $\mathbf{x}(t)$                     | The state vector of the system                                                                        |
| $\mathbf{Y}$                        | Degree of freedom of the horizontal vibration of the system                                           |
| $\mathbf{Z}_2$                      | Controlled performance of $H_2$ norm                                                                  |
| $\mathbf{Z}_\infty$                 | Performance index of $H_\infty$ norm                                                                  |
| $x_{ui}$                            | Vibration displacement of the guide shoe system (m)                                                   |
| $x_{ri}$                            | Guide rail excitation of the four guide shoes                                                         |
| $x_{\max}$                          | Maximum allowable overlap size of the guide rail (m)                                                  |
| $R^{m \times n}$                    | Represents the real number matrix of $m \times n$ dimension                                           |
| RMS                                 | Root mean square value of horizontal acceleration at the center of car floor ( $\text{m.s}^{-1}$ )    |
| $\theta$                            | The deflection angles of elevator car centroid (rad)                                                  |
| $\varpi(t)$                         | Guide rail excitation                                                                                 |
| $\gamma$                            | Disturbance suppression degree                                                                        |
| $\alpha$                            | Free matrix                                                                                           |
| *                                   | Term which is induced by symmetry                                                                     |
| $\Gamma_u$                          | LFT transformation matrix                                                                             |
| $\delta(t)$                         | Time-varying functions used to describe the perturbation range of parameters                          |
| $\psi$                              | Nominal model of guide shoe without parameter uncertainty                                             |
| $\ \cdot\ _2$                       | Usual $L_2(0 \rightarrow \infty)$ norm                                                                |
| $\ \cdot\ _\infty$                  | $H_\infty$ norm                                                                                       |
| $\text{tr}(\Lambda)$                | Trace function of matrix $\Lambda$                                                                    |
| $\Lambda^{-1}$                      | Inverse of matrix $\Lambda$                                                                           |
| $\Lambda^T$                         | Transpose of the matrix $\Lambda$                                                                     |

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## Appendix A

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{B}_{11} \\ \mathbf{B}_{12} \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} \mathbf{B}_{21} \\ \mathbf{B}_{22} \end{bmatrix}, \mathbf{C}_2 = [\lambda] \begin{bmatrix} \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}, \mathbf{C}_1 = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{C}_{13} \end{bmatrix}, \mathbf{A}_{21} = \mathbf{I}_{6 \times 6},$$

$$\mathbf{A}_{22} = \text{zeros}(6) \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \quad \mathbf{B}_{12} = \text{zeros}(6 \times 4) \quad \mathbf{B}_{22} = \text{zeros}(6 \times 8)$$

$$\mathbf{C}_{11} = \text{zeros}(4), \mathbf{C}_{12} = \text{zeros}(4), \mathbf{E}_1 = \text{diag}(\underbrace{\bar{c}, \bar{c} \dots, \bar{c}}_6, \underbrace{\bar{k}, \bar{k} \dots, \bar{k}}_6), \mathbf{E}_2 = \text{zeros}(12, 4)$$

$$\mathbf{C}_{21} = \begin{bmatrix} \frac{-4\bar{c}}{m_c} + \frac{2l_2\bar{c}\Delta l}{I_c} & \frac{2\bar{c}\Delta l}{m_c} - \frac{2l_2\bar{c}\Delta l'}{I_c} & \frac{\bar{c}}{m_c} + \frac{\bar{c}l_1l_2}{I_c} & \frac{\bar{c}}{m_c} - \frac{\bar{c}l_2^2}{I_c} & \frac{\bar{c}}{m_c} + \frac{\bar{c}l_1l_2}{I_c} & \frac{\bar{c}}{m_c} - \frac{\bar{c}l_2^2}{I_c} \end{bmatrix}$$

$$\mathbf{C}_{22} = \begin{bmatrix} \frac{-4\bar{k}}{m_c} + \frac{2l_2\bar{k}\Delta l}{I_c} & \frac{2\bar{k}\Delta l}{m_c} - \frac{2l_2\bar{k}\Delta l'}{I_c} & \frac{\bar{k}}{m_c} + \frac{\bar{k}l_1l_2}{I_c} & \frac{\bar{k}}{m_c} - \frac{\bar{k}l_2^2}{I_c} & \frac{\bar{k}}{m_c} + \frac{\bar{k}l_1l_2}{I_c} & \frac{\bar{k}}{m_c} - \frac{\bar{k}l_2^2}{I_c} \end{bmatrix}$$

$$\mathbf{D}_2 = [\lambda] \begin{bmatrix} \frac{1}{m_c} - \frac{l_1l_2}{I_c} & \frac{1}{m_c} + \frac{l_2^2}{I_c} & \frac{1}{m_c} - \frac{l_1l_2}{I_c} & \frac{1}{m_c} + \frac{l_2^2}{I_c} \end{bmatrix},$$

$$\mathbf{A}_{11} = \begin{bmatrix} \frac{-4\bar{c}}{m_c} & \frac{2\bar{c}\Delta l}{m_c} & \frac{\bar{c}}{m_c} & \frac{\bar{c}}{m_c} & \frac{\bar{c}}{m_c} & \frac{\bar{c}}{m_c} \\ \frac{2\bar{c}\Delta l}{I_c} & \frac{-2\bar{c}\Delta l'}{I_c} & \frac{\bar{c}l_1}{I_c} & -\frac{\bar{c}l_2}{I_c} & \frac{\bar{c}l_1}{I_c} & -\frac{\bar{c}l_2}{I_c} \\ \frac{\bar{c}}{m_{u1}} & \frac{\bar{c}l_1}{m_{u1}} & \frac{-(\bar{c} + c_{tf})}{m_{u1}} & 0 & 0 & 0 \\ \frac{\bar{c}}{m_{u2}} & -\frac{\bar{c}l_2}{m_{u2}} & 0 & \frac{-(\bar{c} + c_{tf})}{m_{u2}} & 0 & 0 \\ \frac{\bar{c}}{m_{u3}} & \frac{\bar{c}l_1}{m_{u3}} & 0 & 0 & \frac{-(\bar{c} + c_{tf})}{m_{u3}} & 0 \\ \frac{\bar{c}}{m_{u4}} & -\frac{\bar{c}l_2}{m_{u4}} & 0 & 0 & 0 & \frac{-(\bar{c} + c_{tf})}{m_{u4}} \end{bmatrix},$$

$$\mathbf{A}_{12} = \begin{bmatrix} \frac{-4\bar{k}}{m_c} & \frac{2\bar{k}\Delta l}{m_c} & \frac{\bar{k}}{m_c} & \frac{\bar{k}}{m_c} & \frac{\bar{k}}{m_c} & \frac{\bar{k}}{m_c} \\ \frac{2\bar{k}\Delta l}{I_c} & \frac{-2\bar{k}\Delta l'}{I_c} & \frac{\bar{k}l_1}{I_c} & -\frac{\bar{k}l_2}{I_c} & \frac{\bar{k}l_1}{I_c} & -\frac{\bar{k}l_2}{I_c} \\ \frac{\bar{k}}{m_{u1}} & \frac{\bar{k}l_1}{m_{u1}} & \frac{-(\bar{k} + k_{tf})}{m_{u1}} & 0 & 0 & 0 \\ \frac{\bar{k}}{m_{u2}} & -\frac{\bar{k}l_2}{m_{u2}} & 0 & \frac{-(\bar{k} + k_{tf})}{m_{u2}} & 0 & 0 \\ \frac{\bar{k}}{m_{u3}} & \frac{\bar{k}l_1}{m_{u3}} & 0 & 0 & \frac{-(\bar{k} + k_{tf})}{m_{u3}} & 0 \\ \frac{\bar{k}}{m_{u4}} & -\frac{\bar{k}l_2}{m_{u4}} & 0 & 0 & 0 & \frac{-(\bar{k} + k_{tf})}{m_{u4}} \end{bmatrix}$$

$$\mathbf{B}_{11} = \begin{bmatrix} \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} \\ -\frac{l_1}{I_c} & \frac{l_2}{I_c} & -\frac{l_1}{I_c} & \frac{l_2}{I_c} \\ \frac{-1}{m_{u1}} & 0 & 0 & 0 \\ 0 & \frac{-1}{m_{u2}} & 0 & 0 \\ 0 & 0 & \frac{-1}{m_{u3}} & 0 \\ 0 & 0 & 0 & \frac{-1}{m_{u4}} \end{bmatrix}, \mathbf{C}_{13} = \begin{bmatrix} \frac{1}{x_{\max}} & 0 & 0 & 0 \\ 0 & \frac{1}{x_{\max}} & 0 & 0 \\ 0 & 0 & \frac{1}{x_{\max}} & 0 \\ 0 & 0 & 0 & \frac{1}{x_{\max}} \end{bmatrix},$$

$$\mathbf{B}_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_{tf}}{m_{u1}} & 0 & 0 & 0 & \frac{c_{tf}}{m_{u1}} & 0 & 0 & 0 \\ 0 & \frac{k_{tf}}{m_{u2}} & 0 & 0 & 0 & \frac{c_{tf}}{m_{u2}} & 0 & 0 \\ 0 & 0 & \frac{k_{tf}}{m_{u3}} & 0 & 0 & 0 & \frac{c_{tf}}{m_{u3}} & 0 \\ 0 & 0 & 0 & \frac{k_{tf}}{m_{u4}} & 0 & 0 & 0 & \frac{c_{tf}}{m_{u4}} \end{bmatrix}.$$