

# Vibrations of a dual-rotor system in aero-engine induced by the support misalignment

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**Abstract.** In this paper, an analytical model of a rotor-bearing-rotor coupling system is established for aero-engine from the Lagrange equation, based on the dynamics theory of the dual-rotor system and combined with its stiffness, damping, and misalignment characteristics. In the model, the misalignment fault simulation method is introduced. The coupling effect between the low-pressure rotor and high-pressure rotor and the influence of misalignment of the support on low-pressure coupling are considered. The vibration characteristics of the dual-rotor system with the misalignment fault of the low-pressure turbine rear support are studied. The time-domain responses, the frequency spectra, and the shaft-center trajectory of the dual-rotor system with different misalignments are obtained. The influence of unbalance on the vibration characteristics of the low-pressure rotor and the high-pressure rotor with misalignment fault is analyzed. Finally, test verification is carried out by the designed dual-rotor test rig. The experimental results are consistent with the analytical results, confirming the established model's feasibility and simulation method in this paper. The results may provide a theoretical basis for research on the misalignment fault in aero-engine dual-rotor systems.

**Keywords:** Dual-rotor system / misalignment faults / vibration characteristics / theoretical analysis / experiment

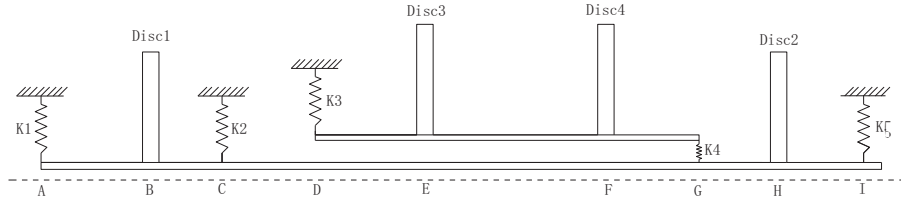
## 1 Introduction

The vibration fault forms of aero-engine rotor systems are various, such as unbalance, rub-impact, and misalignment [1]. The misalignment fault is one of the most common faults of the aero-engine rotor system. In actual production, the support misalignment fault of the rotor system is caused by the manufacturing error, installation error thermal expansion of the bearing base, etc. Misalignment fault will lead to excessive stress of the rotor shaft and instability of system motion. It can also cause other faults in the rotor system. The research on the influence of misalignment faults on dual-rotor system vibration response characteristics occupies an important role in the diagnosis and identification of aero-engine misalignment faults.

In recent years, research on the dynamic characteristics of the rotor system with misalignment has been conducted by relevant scholars. Wang and Jiang [2], Qingkai et al. [3],

Hongxian and Xuejun [4], Zhansheng et al. [5] and Pachpor et al. [6] summarized the misalignment of the rotor system. The rotor misalignment fault is mainly divided into coupling misalignment and support misalignment. As early as 1976, Gibbon [7] deduced and obtained the calculation formula of the force and torque generated at the connection point between the coupling and the rotating shaft. Sekhar and Prabhu [8] derived the calculation formula of angular misalignment force based on the Gibbons formula. Lee and Lee [9] derived the formula of excitation force decomposition when the coupling on the rotor system is misalignment, and the vibration response of the rotor system was analyzed. Al-Hussain and Redmond [10] derived the kinematics differential equation of the rotor system with parallel misalignment of rigid coupling using the basic kinematic principle, based on the Lagrange equation. Guo and Xingyan [11] studied the vibration characteristics of rotor systems under sleeve coupling misalignment faults. Some researchers [12] studied the misalignment excitation when the bearing of the rotor is misalignment, and further analyzed the influence of the misalignment of the bearing on the dynamics of the rotor

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**Fig. 1.** Structural sketch of a dual-rotor system.

system. Guoquan et al. [13] took the double rotor system of an aero-engine as the research object and considered the misalignment of outer rotor bearing, established the dynamic model of the dual-rotor system by using the finite element method, and studied the vibration response characteristics of the rotor under different rotating speeds and misalignment through numerical simulation. Xiao [14] deduces the dynamic models of misalignment of supporting bearings and intermediate bearings in the dual-rotor system using the finite element method. Meipeng et al. [15] equated the eccentricity of the high-pressure rotor support with the additional misalignment moment at the support of the high-pressure rotor and deduced the equation of the dual-rotor system considering the eccentricity of the high-pressure rotor support by using the Lagrange equation method.

Research on rotor misalignment mainly focuses on coupling misalignment in the rotor system [5,16–19]. The team [20,21] established a simplified model of a dual rotor system with four supports and four disks. The proper orthogonal decomposition method is used to reduce the dimension of the dual-rotor system for the first time. With the help of the finite element method, the nonlinear response caused by the misalignment of the coupling between the rotor and the motor was studied and verified by the test method. Research on the misalignment of the support is aimed at the misalignment of bearing, but misalignment of bearing is only a part of the misalignment of the support, and lack of research on a misalignment of rolling bearing. The research on a misalignment of support has theoretical and practical engineering needs.

Although many scholars have done a lot of research on the vibration of rotor systems with misalignment faults, in the current research, the modeling of coupling misalignment and bearing misalignment is relatively mature, and the impact on rotor system vibration is mainly concentrated on a single rotor. For the dual-rotor system, the vibration coupling problem of the dual-rotor system considering the support misalignment fault and the influence of the fault on the coupling is seldom involved. In this paper, a typical aero-engine model tester is taken as the research object, considering the load torque force of the low-pressure caused by the misalignment of the rear support of the low-pressure turbine, the excitation mechanism of support misalignment is analyzed. The motion differential equation of the dual-rotor system with misalignment faults is established, and the effects of misalignment, unbalanced mass, and load torque on the vibration response of the dual-rotor are studied. Finally, test verification is carried out by the designed dual-rotor test rig.

## 2 The dynamic model for a dual-rotor system

With the increase of the thrust-weight ratio of the aero-engine, the structure of the rotor system changes greatly, and the high speed of the aero-engine makes the vibration problem of the aero-engine more prominent. The coupling characteristics of aero-engine rotor dynamics and the transfer law of rotor misalignment vibration between the high-pressure rotor and the low-pressure rotor are still unclear and need to be studied urgently. In this paper, an advanced aero-engine rotor in reference [22] was simplified as a rigid rotor with elastic supports, as shown in Figure 1. The inner rotor is also called a low-pressure rotor, and the outer rotor is also called the high-pressure rotor. The low-pressure rotor consists of one fan (referred to as LPC) and one turbine (referred to as LPT), supported by bearings 1, 2, 4, and 5. The high-pressure rotor is composed of one compressor disc (referred to as HPC) and a high-pressure turbine (referred to as HPT), which is supported by bearings 3 and 4.

The parameters of the dual-rotor system are shown in Tables 1–4, among which the parameters of four equivalent discs are shown in Table 1, the equivalent parameters of the rotating shaft are shown in Table 2, and the distance parameters are shown in Table 3. According to the principle of dynamic similarity, the first two natural frequencies and vibration modes of the simplified dual-rotor system are kept consistent with the prototype by adjusting the support stiffness. Thus achieving the goal of dynamic similarity between the simplified model and the prototype's dual rotor system [22]. The designed stiffness parameters are shown in Table 4.

## 3 Motion equations of the dual-rotor system

The motion equations of the dual-rotor system are as follows.

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_j(t), \quad j = 1, 2, 3 \quad (1)$$

where the kinetic energy of the system can be recorded as follows.

$$T = T_{t1} + T_{r1} + T_{t2} + T_{r2} + T_{t3} + T_{r3} + T_{t4} + T_{r4}. \quad (2)$$

$T_{ti}$  and  $T_{ri}$  are the translational kinetic energy and rotational kinetic energy of disc respectively.

**Table 1.** Disc parameters of the dynamic model.

| Disc | Thickness | External diameter | Internal diameter | Mass    | Moment of inertia (kg · m <sup>2</sup> ) |
|------|-----------|-------------------|-------------------|---------|------------------------------------------|
| LPC  | 18 mm     | 330 mm            | 35 mm             | 14.0 kg | $J_p = 0.164 \ J_d = 0.101$              |
| LPT  | 10 mm     | 316 mm            | 48 mm             | 11.3 kg | $J_p = 0.059 \ J_d = 0.060$              |
| HPC  | 35 mm     | 305 mm            | 73 mm             | 20 kg   | $J_p = 0.230 \ J_d = 0.120$              |
| HPT  | 14 mm     | 304 mm            | 62 mm             | 10 kg   | $J_p = 0.102 \ J_d = 0.050$              |

**Table 2.** Shaft parameters of the dual-rotor system.

| Shaft                      | External diameter | Internal diameter | Length |
|----------------------------|-------------------|-------------------|--------|
| Fan-rotor                  | 30 mm             | 0 mm              | 381 mm |
| Low-pressure turbine shaft | 30 mm             | 0 mm              | 923 mm |
| High-pressure shaft        | 65 mm             | 40 mm             | 680 mm |

**Table 3.** Distance parameters of the dual-rotor system.

| L <sub>AB</sub> | L <sub>AC</sub> | L <sub>AD</sub> | L <sub>AE</sub> | L <sub>AF</sub> | L <sub>AG</sub> | L <sub>AH</sub> | L <sub>AI</sub> |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 127mm           | 372 mm          | 488 mm          | 610 mm          | 1000 mm         | 1069 mm         | 1109 mm         | 1186 mm         |

**Table 4.** Equivalent stiffness of five supports.

| $k_{b1}$              | $k_{b2}$              | $k_{b3}$              | $k_{b4}$              | $k_{b5}$              |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $0.9 \times 10^7$ N/m | $2.5 \times 10^8$ N/m | $0.3 \times 10^7$ N/m | $2.5 \times 10^8$ N/m | $0.3 \times 10^7$ N/m |

The total potential energy of the system can be recorded as follows.

$$U = U_1 + U_2 + U_3 + U_4 + U_c + U_5. \quad (3)$$

It includes the elastic potential energy of five supports and the potential energy of coupling.

$$U_i = \frac{1}{2} \mathbf{q}_{B_i}^T \mathbf{K} \mathbf{q}_B = \frac{1}{2} \{\mathbf{B}_i \mathbf{q}_i\}^T \mathbf{K} \{\mathbf{B}_i \mathbf{q}_i\}, i = 1 - 4. \quad (4)$$

The elastic potential energy of the intermediate bearing is as follows.

$$U_5 = \frac{1}{2} (\mathbf{q}_{C_H} - \mathbf{q}_{C_L})^T \mathbf{K}_C (\mathbf{q}_{C_H} - \mathbf{q}_{C_L}). \quad (5)$$

The elastic potential energy at the coupling is as follows.

$$\begin{aligned} U_C &= \frac{1}{2} (\mathbf{q}_{C_2} - \mathbf{q}_{C_1})^T \mathbf{K}_C (\mathbf{q}_{C_2} - \mathbf{q}_{C_1}) \\ &= \frac{1}{2} \{\mathbf{B}_5 \mathbf{q}_2 - \mathbf{B}_4 \mathbf{q}_1\}^T \mathbf{K}_C \{\mathbf{B}_5 \mathbf{q}_2 - \mathbf{B}_4 \mathbf{q}_1\}. \end{aligned} \quad (6)$$

The analytic model of the dual-rotor system is established by using the Lagrange equation as shown in

equation (7). Please refer to reference [22] for the specific modeling process.

$$\begin{aligned} m_1 \ddot{y}_1 - (k_{b1y} \theta_{z1} a_1 - k_{b1y} y_1) + \frac{12E_C I_{yc}}{L_c^3} \left( \left( \frac{\theta_{z1}}{2} + \frac{\theta_{z2}}{2} \right) \right. \\ \left. L_c + \theta_{z1} c_1 + \theta_{z2} c_2 + y_1 - y_2 \right) = 0 \end{aligned} \quad (7.a)$$

$$\begin{aligned} m_1 \ddot{z}_1 + k_{b1z} \theta_{y1} a_1 + k_{b1z} z_1 + \frac{-12E_C I_{yc}}{L_c^3} \left( \left( \frac{\theta_{y1}}{2} + \frac{\theta_{y2}}{2} \right) \right. \\ \left. L_c + \theta_{y1} c_1 + \theta_{y2} c_2 - z_1 + z_2 \right) = 0 \end{aligned} \quad (7.b)$$

$$\begin{aligned} J_{d1} \ddot{\theta}_{y1} + J_{p1} \Omega_1 \dot{\theta}_{z1} + \frac{4E_C I_{yc} (\theta_{y1} + \theta_{y2}/2)}{L_c} \\ + \frac{12E_C I_{yc} c_1 (\theta_{y1} c_1 + \theta_{y2} c_2 - z_1 + z_2)}{L_c^3} \\ + \frac{12E_C ((\theta_{y1} + \theta_{y2}/2) c_1 + 1/2 \theta_{y2} c_2 - z_1/2 + z_2/2) I_{yc} L_c}{L_c^3} \\ + k_{b1z} \theta_{y1} a_1 + k_{b1z} z_1 = 0 \end{aligned} \quad (7.c)$$

$$\begin{aligned}
& J_{d1}\dot{\theta}_{z1} - J_{p1}\Omega_1\dot{\theta}_{y1} \\
& + \frac{4(\theta_{z1} + \theta_{z2}/2)E_C I_{yc}}{L_c} + \frac{12E_C I_{yc}c_1(\theta_{z1}c_1 + \theta_{z2}c_2 + y_1 - y_2)}{L_c^3} \\
& + \frac{12E_C I_{yc}L_c((\theta_{z1} + \theta_{z2}/2)c_1 + 1/2\theta_{z2}c_2 + y_1/2 - y_2/2)}{L_c^3} \\
& + k_{b1y}\theta_{z1}a_1^2 - k_{b1y}y_1a_1 = 0
\end{aligned} \quad (7.d)$$

$$\begin{aligned}
& m_2\dot{y}_2 + (-k_{b2y}a_2 - k_{b4y}a_{41} + k_{b5y}a_5)\theta_{z2} \\
& + (y_2 + (-a_4 + a_{34})\theta_{z4} - y_4)k_{b4y}y_2 \\
& - \frac{6E_C I_{yc}(\theta_{z1} + \theta_{z2})L_c + 12E_C I_{yc}(\theta_{z1}c_1 + \theta_{z2}c_2 + y_1 - y_2)}{L_c^3} + (k_{b5y} + k_{b2y})y = 0
\end{aligned} \quad (7.e)$$

$$\begin{aligned}
& m_2\dot{z}_2 + (k_{b2z}a_2 + k_{b4z}a_{41} - k_{b5z}a_5)\theta_{y2} \\
& + \frac{(\dot{z}_2 + (\frac{a_4}{\theta_{y1}} + \frac{a_{34}}{\theta_{y2}})L_c + \frac{12E_C I_{yc}(\theta_{y1}c_1 + \theta_{y2}c_2 - z_1 + z_2)}{L_c^3} + (k_{b5z} + k_{b2z})z_2)}{L_c^3} = 0
\end{aligned} \quad (7.f)$$

$$\begin{aligned}
& J_{d2}\dot{\theta}_{y2} + J_{p2}\Omega_1\dot{\theta}_{z2} + (k_{b2z}a_2^2 + k_{b4z}a_{41}^2 + k_{b5z}a_5^2 + k_{b4\theta y})\theta_{y2} \\
& + (z_2 + (a_4 - a_{34})\theta_{y4} - z_4)k_{b4z}a_{41} \\
& + (k_{b2z}a_2 - k_{b5z}a_5)z_2 - k_{b4\theta y}\theta_{y4} \\
& + \frac{6E_C I_{yc}(\theta_{y1}c_1 + \theta_{y1}c_2 + 2\theta_{y2}c_2 - z_1 + z_2)L_c}{L_c^3} \\
& + \frac{2E_C I_{yc}(\theta_{y1} + 2\theta_{y2})L_c^2 + 12E_C c_2 I_{yc}}{L_c^3} \\
& + \frac{(\theta_{y1}c_1 + \theta_{y2}c_2 - z_1 + z_2)}{L_c^3} = 0
\end{aligned} \quad (7.g)$$

$$\begin{aligned}
& J_{d2}\dot{\theta}_{z2} - J_{p2}\Omega_1\dot{\theta}_{y2} \\
& + (a_2^2 k_{b2y} + a_5^2 k_{b5y} + a_{41}^2 k_{b4y} + k_{b4\theta z})\theta_{z2} \\
& + a_{41}(-y_2 + (a_4 - a_{34})\theta_{z4} + y_4)k_{b4y} \\
& + (-a_2 k_{b2y} + a_5 k_{b5y})y_2 - k_{b4\theta z}\theta_{z4} \\
& + \frac{2E_C I_{yc}((\theta_{z1} + 2\theta_{z2})L_c^2 + (3c_1\theta_{z1} + 3c_2\theta_{z1} + 6c_2\theta_{z2} + 3y_1 - 3y_2)L_c)}{L_c^3} \\
& + \frac{12E_C I_{yc}c_2(c_1\theta_{z1} + c_2\theta_{z2} + y_1 - y_2)}{L_c^3} = 0
\end{aligned} \quad (7.h)$$

$$\begin{aligned}
& m_3\dot{y}_3 - a_3\theta_{z3}k_{b3y} + y_3k_{b3y} \\
& + \frac{6E_g I_y((\theta_{z3} + \theta_{z4})L + 2y_3 - 2y_4)}{L^3} = 0
\end{aligned} \quad (7.i)$$

$$\begin{aligned}
& m_3\dot{z}_3 + a_3\theta_{y3}k_{b3z} + z_3k_{b3z} \\
& - \frac{6E_g I_y((\theta_{y3} + \theta_{y4})L - 2z_3 + 2z_4)}{L^3} = 0
\end{aligned} \quad (7.j)$$

$$\begin{aligned}
& J_{d3}\dot{\theta}_{y3} + J_{p3}\Omega_3\dot{\theta}_{z3} + a_3^2 k_{b3z}\theta_{y3} + a_3 k_{b3z}z_3 \\
& + \frac{4E_g I_y((\theta_{y3} + 1/2\theta_{y4})L - 3/2z_3 + 3/2z_4)}{L^2} = 0
\end{aligned} \quad (7.k)$$

$$\begin{aligned}
& J_{d3}\dot{\theta}_{z3} - J_{p3}\Omega_3\dot{\theta}_{y3} + a_3^2 k_{b3y}\theta_{z3} - a_3 k_{b3y}y_3 \\
& + \frac{4E_g I_y((\theta_{z3} + 1/2\theta_{z4})L + 3/2y_3 - 3/2y_4)}{L^2} = 0
\end{aligned} \quad (7.l)$$

$$\begin{aligned}
& m_4\dot{y}_4 + (y_4 + (a_4 - a_{34})\theta_{z4} + \theta_{z2}a_{41} - y_2)k_{b4y} \\
& - \frac{6E_g I_y((\theta_{z3} + \theta_{z4})L + 2y_3 - 2y_4)}{L^3} = 0
\end{aligned} \quad (7.m)$$

$$\begin{aligned}
& m_4\dot{z}_4 + \frac{(a_4 - a_{34})\theta_{y4} + \theta_{y2}a_{41} + z_2 - z_4}{L^3} k_{b4z} \\
& + \frac{6E_g I_y((\theta_{y3} + \theta_{y4})L - 2z_3 + 2z_4)}{L^3} = 0
\end{aligned} \quad (7.n)$$

$$\begin{aligned}
& J_{d4}\dot{\theta}_{y4} + J_{p4}\Omega_3\dot{\theta}_{z4} \\
& + (a_4 - a_{34})((a_4 - a_{34})\theta_{y4} + \theta_{y2}a_{41} + z_2 - z_4)k_{b4z} \\
& - k_{b4\theta y}\theta_{y2} + k_{b4\theta y}\theta_{y4} \\
& + \frac{2E_g I_y((\theta_{y3} + 2\theta_{y4})L - 3z_3 + 3z_4)}{L^2} = 0
\end{aligned} \quad (7.o)$$

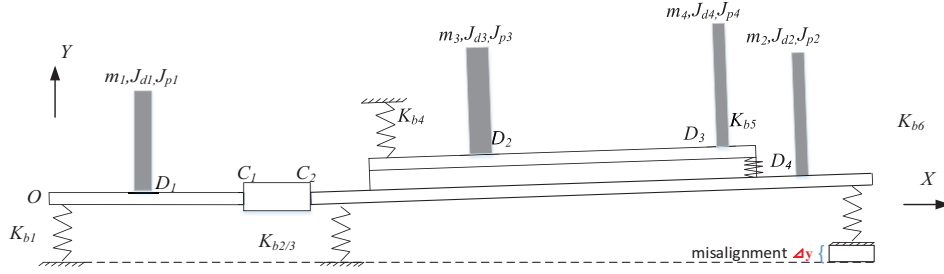
$$\begin{aligned}
& J_{d4}\dot{\theta}_{z4} - J_{p4}\Omega_3\dot{\theta}_{y4} \\
& + (a_4 - a_{34})(y_4 + (a_4 - a_{34})\theta_{z4} + \theta_{z2}a_{41} - y_2)k_{b4y} \\
& - k_{b4\theta z}\theta_{z2} + k_{b4\theta z}\theta_{z4} \\
& + \frac{2E_g I_y((\theta_{z3} + 2\theta_{z4})L + 3y_3 - 3y_4)}{L^2} = 0.
\end{aligned} \quad (7.p)$$

## 4 The excitation mechanism of the support misalignment

The characteristics of the slender shaft of the low-pressure rotor make the assembly more difficult, which increases the possibility of support misalignment. The misalignment of the rear support of the low-pressure turbine rotor in the dual-rotor system will not only change the potential energy of the rotor system but also affect the coupling that connects the fan rotor and the low-pressure turbine rotor. The structure diagram of the low-pressure turbine rear support misalignment is shown in [Figure 2](#).

When the support has a misalignment of  $\Delta y$  in the  $y$ -direction and a misalignment of  $\Delta z$  in the  $z$ -direction, the support has a deformation in the equilibrium state, the expression of the elastic potential energy of the support becomes:

$$U_6 = \frac{1}{2}(\mathbf{q}_{B_6}^T - \Delta^T)\mathbf{K}_{B_6}(\mathbf{q}_{B_6} - \Delta) \quad (8)$$



**Fig. 2.** Schematic diagram of rear support misalignment of the low-pressure turbine.

where  $\Delta = (0 \ \Delta y \ \Delta z \ 0 \ 0)^T$ .  $K_{B_6}$  is the stiffness matrix of misalignment support.  $K_{B_6}$  is the generalized displacement of the rear support of the low-pressure turbine disc.

When the low-pressure rotor is a misalignment, the sleeve coupling on the low-pressure rotor is bent, and there is a misalignment angle  $\alpha$  between the fan rotor and the low-pressure turbine rotor. The rotational angular velocities of the fan rotor and the low-pressure turbine rotor are  $\Omega_1$  and  $\Omega_2$  respectively. Fan rotor drives the low-pressure turbine rotor through the coupling when the angular displacement of the fan rotor is  $\phi_1$ , the angle of rotation of the coupling gear sleeve is  $\phi_2$ , and the relationship between  $\phi_1$  and  $\phi_2$  is as follows.

$$\tan \phi_1 = \tan \phi_2 \cos \alpha. \quad (9)$$

Considering the small misalignment angle  $\alpha$ , i.e.  $\tan \alpha \cong \alpha$ . Derivatives on both sides of the above equation (8).

$$\frac{\Omega_2}{\Omega_1} = \frac{C}{1 + D \cos 2\phi_1} \quad (10)$$

where  $C = \frac{4 \cos \alpha}{3 + \cos 2\alpha} D = \frac{\cos 2\alpha - 1}{3 + \cos 2\alpha}$ . The calculation method refers to Wang Meiling [23]. The angular acceleration of the low-pressure turbine rotor can be obtained by derivation of this formula:

$$\dot{\Omega}_2 = \frac{2CD\Omega_1^2 \sin 2\phi_1}{(1 + D \cos 2\phi_1)^2}. \quad (11)$$

The torque transmitted from the fan rotor shaft to the low-pressure turbine rotor shaft through the coupling is mainly used for two parts, one part is the accelerated motion of the low-pressure turbine rotor, and the other part is used to balance the resistance suffered by the low-pressure turbine rotor, namely:

$$T_x = T_a + T_f \quad (12)$$

where  $T_a = I\dot{\Omega}_2$ ,  $T_f = c\Omega_2$ .  $I$  is the moment of inertia of the low-pressure turbine rotor.  $c$  is the damping of the rotor, and the torque of the fan rotor is in the horizontal direction. When torque passes through the coupling, it divides into two parts:

$$T_x = T \cos \alpha, T_s = T \sin \alpha. \quad (13)$$

Combining equation (11) and equation (12), it can be concluded that:

$$T \cos \alpha = I\dot{\Omega}_2 + c\Omega_2 \quad (14)$$

where  $T_s$  is in the axial direction of the low-pressure turbine rotor. Further, decomposes into  $y$  and  $z$  directions.

$$T_y = T \sin \alpha \sin \beta, T_z = T \sin \alpha \cos \beta \quad (15)$$

where  $\beta$  is the intersection angle between the misalignment vector and the  $z$ -axis. The torque forces of the low-pressure turbine rotor in the  $y$  and  $z$  directions at the coupling can be obtained.

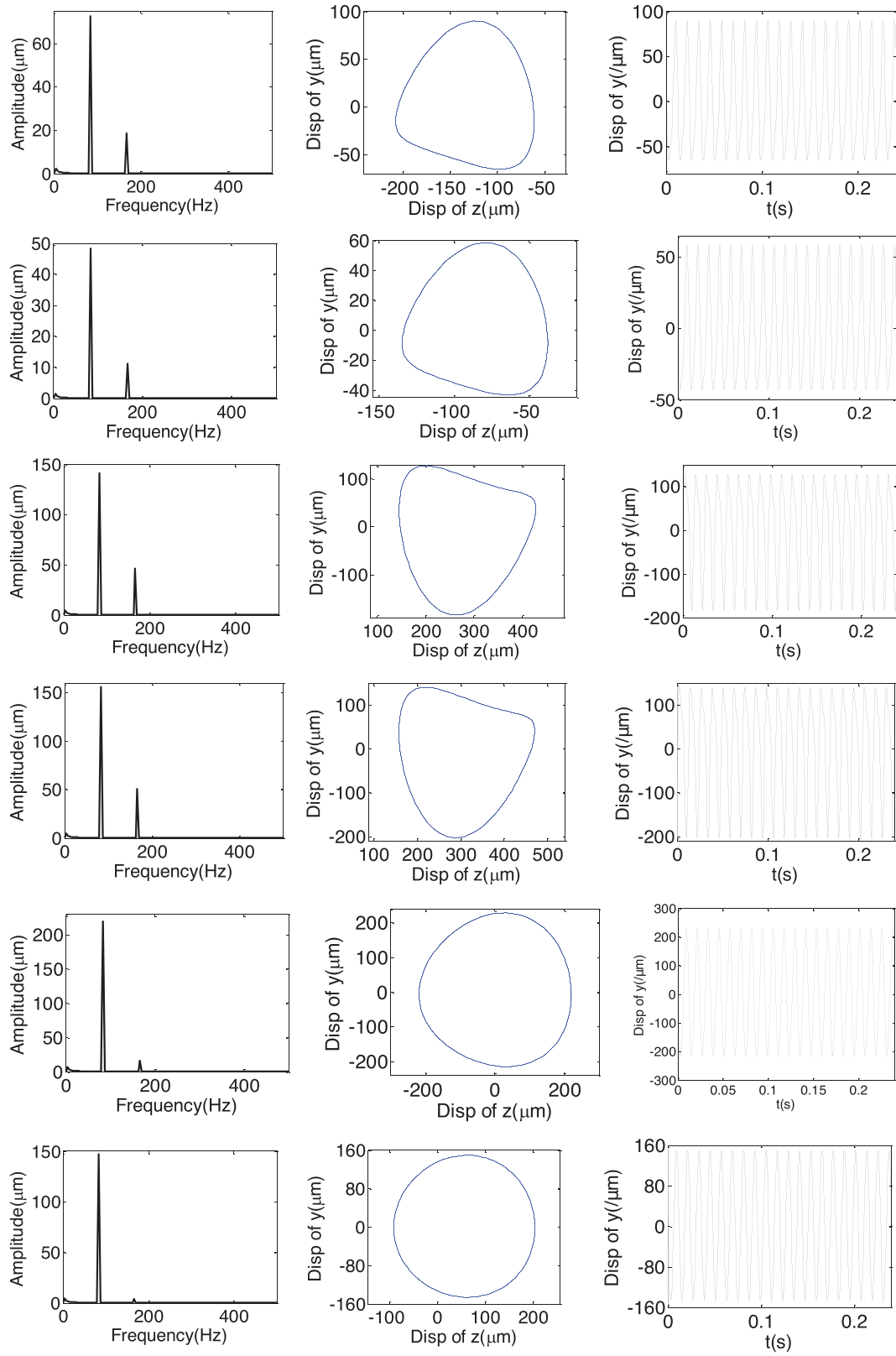
## 5 Misalignment excitation force

In the case of misalignment, the speed of the low-pressure turbine rotor has changed, and its excitation force also changes accordingly. The expression of the misalignment excitation force as follows

$$\mathcal{Q}_{2u} =$$

$$\begin{pmatrix} -\alpha m_2 e_2 \left[ \frac{C\Omega_1}{1 + D \cos(2\Omega_1 t)} \right]^2 \cos\left(\frac{C\Omega_1 t}{1 + D \cos(2\Omega_1 t)} + \varphi_{20}\right) \\ m_2 e_2 \left[ \frac{C\Omega_1}{1 + D \cos(2\Omega_1 t)} \right]^2 \cos\left(\frac{C\Omega_1 t}{1 + D \cos(2\Omega_1 t)} + \varphi_{20}\right) \\ m_2 e_2 \left[ \frac{C\Omega_1}{1 + D \cos(2\Omega_1 t)} \right]^2 \sin\left(\frac{C\Omega_1 t}{1 + D \cos(2\Omega_1 t)} + \varphi_{20}\right) \\ 0 \\ 0 \end{pmatrix}. \quad (16)$$

From equation (15), it can be concluded that the excitation frequency of the low-pressure turbine disc is  $\frac{C}{1 + D \cos(2\Omega_1 t)}$  times of rotating frequency of the fan rotor. The amplitude of excitation force is directly proportional to the sum of unbalance  $m_2 e_2$  of the low-pressure turbine disc are  $\left[ \frac{C\Omega_1}{1 + D \cos(2\Omega_1 t)} \right]^2$ .



**Fig. 3.** Vibration response of measuring points 1–8 with z-direction misalignment.

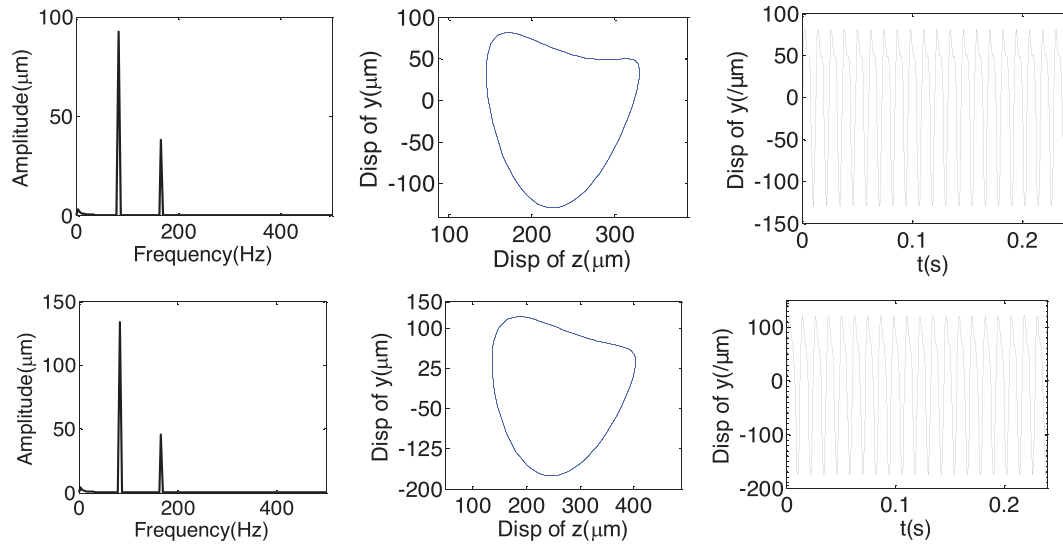


Fig. 3. (Continued).

## 6 The vibration of the dual-rotor excited by both unbalanced and support misalignment

### 6.1 Misalignment in Z-direction

The misalignment of the rear support of the low-pressure turbine included the  $y$ -direction misalignment,  $z$ -direction misalignment, and comprehensive misalignment in the two directions. Four measuring points are selected on the low-pressure and the high-pressure rotor respectively, including 1# support (No. 1), the centroid of LPC (No. 2), the centroid of LPT (No. 3), 6# support (No. 4), 4# support (No. 5), the centroid of HPC (No. 6), the centroid of HPT (No. 7) and the intermediate bearing (No. 8). The rotational speed of the low-pressure rotor and the high-pressure rotor are 5000 r/min (N1) and 6000r/min (N2) respectively, and the initial unbalance of the fan disc is  $3.0 \times 10^{-3}$  kg m.

The situation that misalignment of 0.5 mm exists in the  $z$ -direction at 6# supporting was analyzed, and the time-domain responses, the frequency spectra, and the shaft-center trajectories of each measuring point for the dual-rotor system are obtained, as shown in Figure 3.

It can be seen from the frequency spectra that the vibration in the  $y$ -direction at 1#, 2#, 3#, 4#, 7#, and 8# measuring points has two obvious frequencies, one is the rotational frequency of the low-pressure rotor (N1), the other one is 2 times of rotational frequency of the low-pressure rotor (2N1). The orbit of the axis at the center is not a regular circle, but an approximate 'concave' shape, which indicates that the vibration response in the  $y$ -direction and the  $z$ -direction is coupled. The measuring points 5# and 6# on the high-pressure rotor are less affected by the misalignment, and the amplitude of the 2 times rotational frequency of the low-pressure rotor (2N1) in the spectrum diagram is very small, which can be ignored, and the orbit of the axis center is a regular circle.

### 6.2 Y-direction and Z-direction misalignment

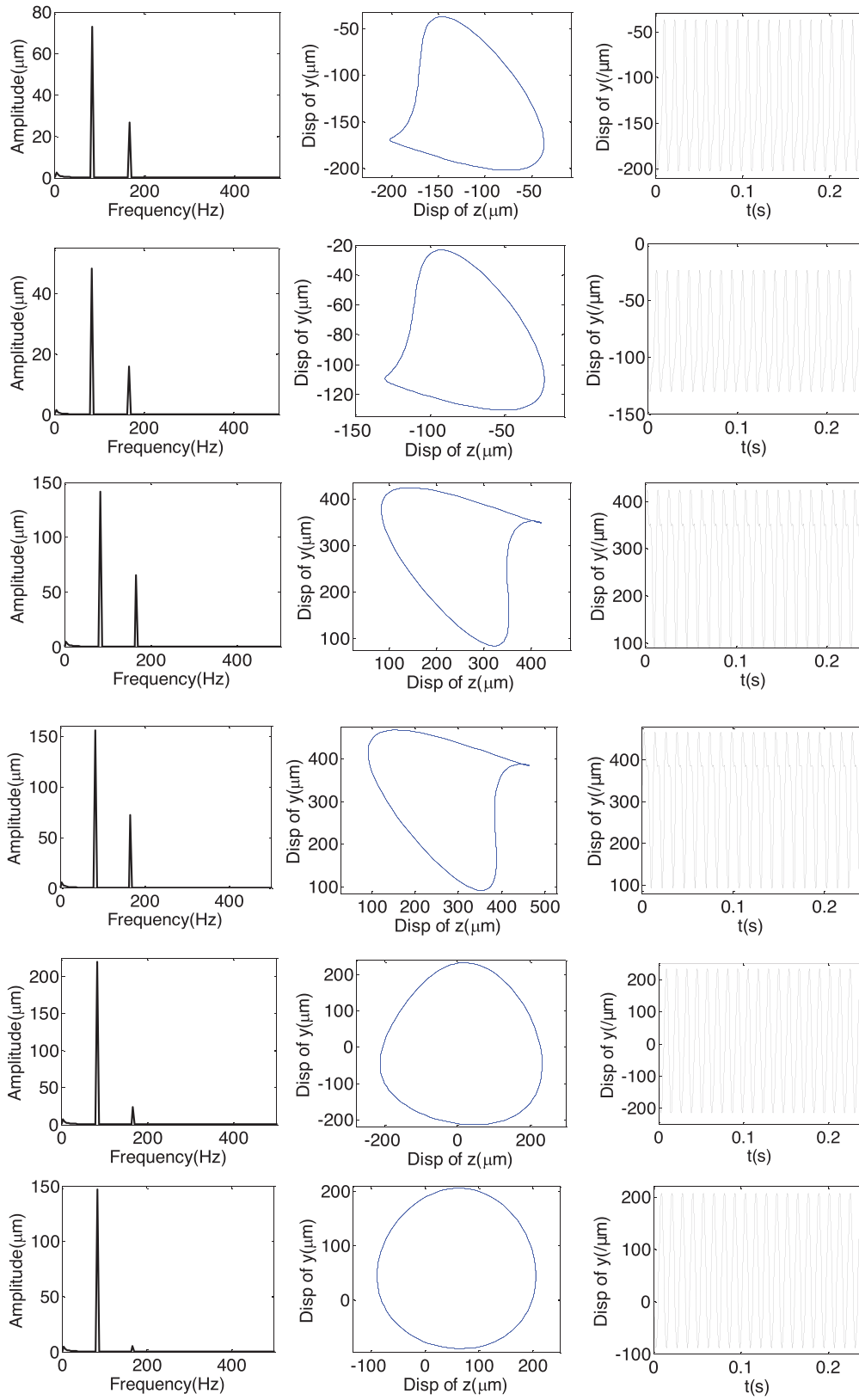
The situation that the misalignment of 0.5mm exists in the  $z$ -direction and  $y$ -direction at 6# supporting was analyzed, and the time-domain responses, the frequency spectra, and the shaft-center trajectories of each measuring point for the dual-rotor system are obtained, as shown in Figure 4.

According to the vibration response of 8 measuring points, the following conclusions can be drawn:

- It can be seen from the spectrum diagram that the misalignment of the rear support of the low-pressure turbine will cause the 2 times rotational frequency of the low-pressure rotor.
- It can be seen from the axle center trajectory in the figure that 'concave' axle center trajectory appears at measuring point 1 and point 2; The axial trajectories of measuring point 3, point 4, point 7 and point 8 are an external '8' shape. And axial trajectories of measuring points 5 and 6 are close to circular. This is because measuring points 3, 4, 7 and 8 are close to the rear support of the misalignment low-pressure turbine while measuring points 5 and 6 are coupled with the low-pressure rotor to generate frequency 2N1 and are far away from the rear support of the misalignment low-pressure turbine, so they are least affected by misalignment.
- It can be seen from the frequency spectra that the magnitude of frequency N1 of 5# and 6# is larger, which indicates that the two measuring points are greatly affected by unbalanced excitation.

### 6.3 Influence of misalignment on the vibration response of the dual-rotor system

In this section, the variation law of the dual-rotor system vibration with misalignment of the support is studied and the fitting curves of vibration amplitude of each measuring



**Fig. 4.** Vibration response of measuring points 1–8 with  $z$ -direction and  $y$ -direction misalignment.

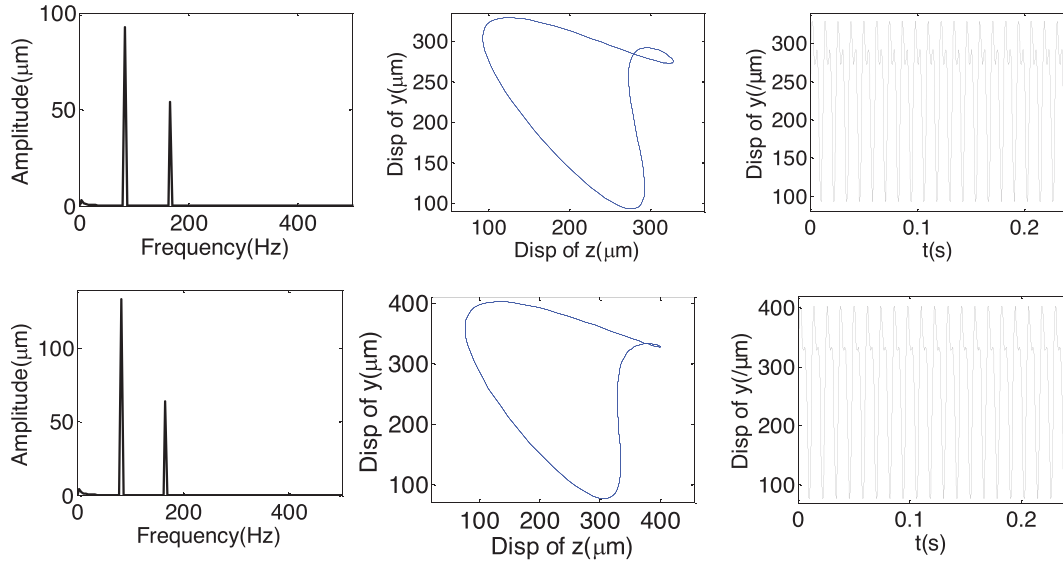


Fig. 4. (Continued).

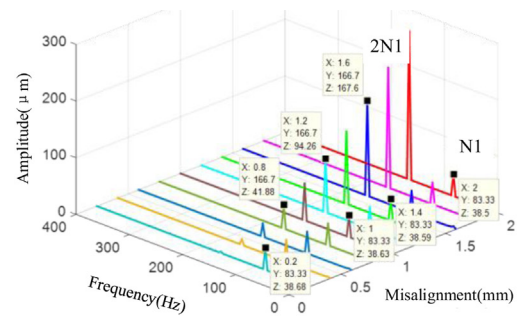
point of the dual-rotor system vary with misalignment are obtained, as showed in Figure 5 and Table 5. Among them, Figure 5a is a three-dimensional spectrum diagram of the dual-rotor system vibration with different misalignment.

Figure 5b is a fitting curve of the vibration amplitude of frequency 2N1 varying with the size of misalignment. The fitting curve equation of the vibration amplitude of frequency 2N1 changing with misalignment of the dual-rotor system is shown in Table 5.

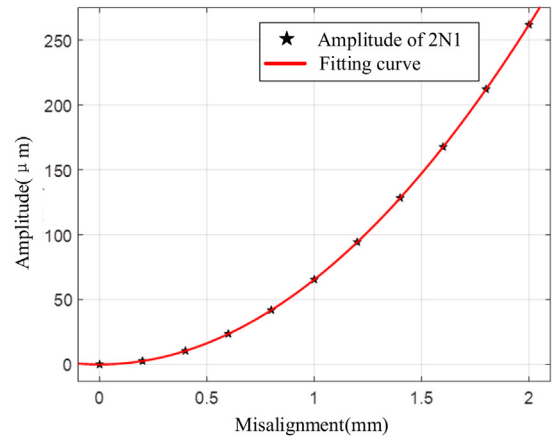
It can be seen from Figure 5a that with the increase of the misalignment of the support, the amplitude of frequency N1 has no obvious change, while the amplitude of frequency 2N1 increases. From Figure 5b, it can be seen that the vibration amplitude of frequency 2N1 is approximately quadratic with the misalignment. And the fitting curve equation and fitting error are obtained by fitting the quadratic curve, as showed in Table 5. Through analysis, it is found that the fitting error is very small. It can be considered that the amplitude of the rotor vibration frequency 2N1 caused by the misalignment of the rear support of the low-pressure turbine has a quadratic relationship with the misalignment.

#### 6.4 Influence of rotor load torque on the vibration response of the dual-rotor system

The influence of load torque on the response of the dual-rotor support misalignment is studied in this section. The three-dimensional spectrum of vibration amplitude for every test point varying with load torque are obtained, as showed in Figure 6a. The relationship between the vibration response amplitude of the dual-rotor system and the load torque of the rotor is analyzed, and the fitting curve is obtained, as showed in Figure 6b. And the fitting equation and error are shown in Table 6.



(a) The 3D spectrum of vibration amplitude with misalignment

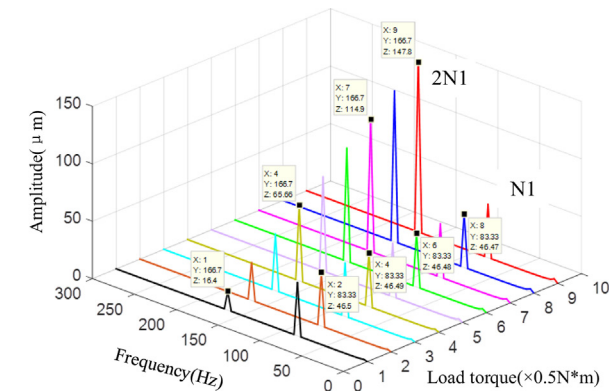


(b) Fitting curve of vibration amplitude with misalignment

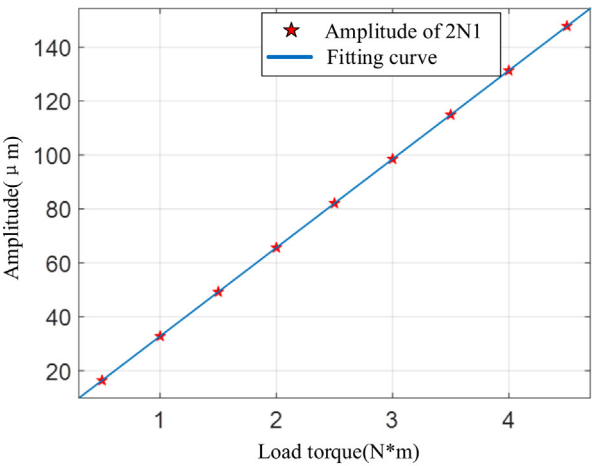
Fig. 5. Influence of misalignment on vibration response of the dual-rotor system.

**Table 5.** Fitting curve equation of vibration amplitude with misalignment.

| Misalignment<br>(mm) | 2N1( $\mu\text{m}$ ) | Fitting function value( $\mu\text{m}$ )<br>$y = 65.5x^2 - 0.041x - 0.01$ | Error |
|----------------------|----------------------|--------------------------------------------------------------------------|-------|
| 0                    | 0                    | -0.01                                                                    | —     |
| 0.2                  | 2.59                 | 2.60                                                                     | 0.45% |
| 0.4                  | 10.45                | 10.45                                                                    | 0.03% |
| 0.6                  | 23.54                | 23.55                                                                    | 0.02% |
| 0.8                  | 41.88                | 41.88                                                                    | 0.01% |
| 1.0                  | 65.45                | 65.45                                                                    | 0.01% |
| 1.2                  | 94.26                | 94.26                                                                    | 0.01% |
| 1.4                  | 128.3                | 128.3                                                                    | 0.01% |
| 1.6                  | 167.6                | 167.6                                                                    | 0.01% |
| 1.8                  | 212.1                | 212.1                                                                    | 0.02% |
| 2.0                  | 261.9                | 261.9                                                                    | 0.01% |



(a) The 3D spectrum of vibration amplitude with load torque



(b) Fitting curve of vibration amplitude variation with load torque

**Fig. 6.** Influence of rotor load torque on vibration response of the dual-rotor system.

As can be seen from Figure 6a, the amplitude of vibration frequency N1 of the rotor system does not change with the increase of rotor load torque. And the amplitude of the vibration frequency 2N1 is approximately linearly

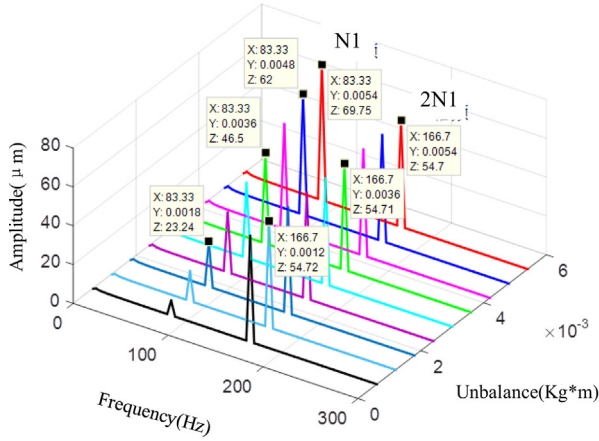
**Table 6.** Fitting curve equation of vibration amplitude with load torque.

| Load torque<br>(N·m) | 2N1( $\mu\text{m}$ ) | Fitting function<br>value( $\mu\text{m}$ )<br>$y = 32.84x - 0.03$ | Error  |
|----------------------|----------------------|-------------------------------------------------------------------|--------|
| 0.5                  | 16.41                | 16.39                                                             | 0.061% |
| 1.0                  | 32.80                | 32.81                                                             | 0.030% |
| 1.5                  | 49.24                | 49.23                                                             | 0.020% |
| 2.0                  | 65.66                | 65.65                                                             | 0.015% |
| 2.5                  | 82.08                | 82.07                                                             | 0.012% |
| 3.0                  | 98.51                | 98.49                                                             | 0.010% |
| 3.5                  | 114.90               | 114.91                                                            | 0.008% |
| 4.0                  | 131.30               | 131.33                                                            | 0.023% |
| 4.5                  | 147.81               | 147.75                                                            | 0.034% |

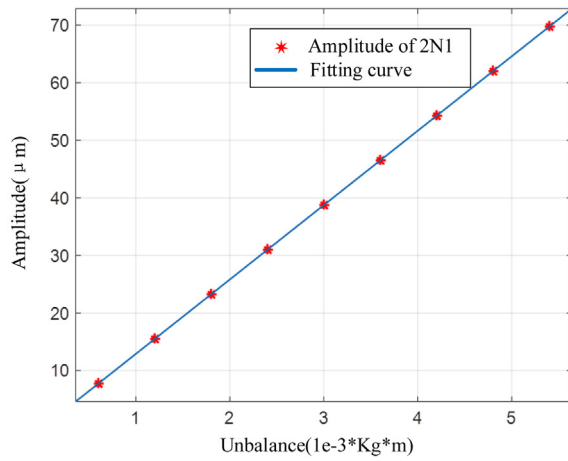
related to the load torque as showed in Figure 6b. The linear fitting results between the amplitude of frequency 2N1 and the load torque are shown in Table 6. It can be seen that the error of the fitting equation is within 0.1%. It can be considered that the amplitude of the rotor vibration frequency 2N1 caused by the misalignment of the rear support of the low-pressure turbine is linearly related to the load torque.

**6.5 Influence of unbalance on vibration response of the dual-rotor system**

In this section, the influence of unbalanced mass on the vibration response of the dual-rotor system with support misalignment faults is studied. The three-dimensional spectrum of the dual-rotor vibration response varying with the unbalance of the fan disc is plotted, as showed in Figure 7a. The fitting curve of the amplitude of frequency N1 with unbalance in the vibration response of the dual-rotor system is obtained, as showed in Figure 7b. And the fitting curve equation and fitting error between the



(a) The 3D spectrum of vibration amplitude with unbalance



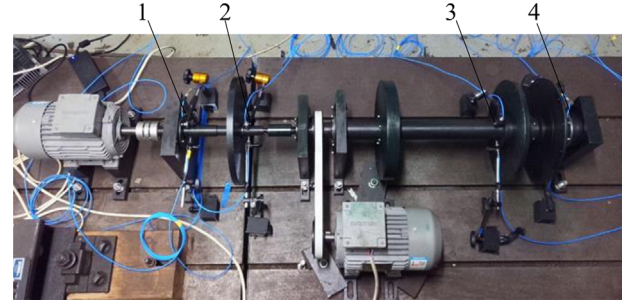
(b) Fitting curve of vibration amplitude variation with unbalance

**Fig. 7.** Influence of unbalance on vibration response of the dual-rotor system.**Table 7.** Fitting curve equation of vibration amplitude with unbalance.

| x   | N1(μm) | Fitting function(μm)<br>$y = 12.92x - 0.18$ | Error |
|-----|--------|---------------------------------------------|-------|
| 0.6 | 7.73   | 7.57                                        | 2.04% |
| 1.2 | 15.49  | 15.32                                       | 1.07% |
| 1.8 | 23.24  | 23.08                                       | 0.71% |
| 2.4 | 30.99  | 30.83                                       | 0.52% |
| 3.0 | 38.74  | 38.58                                       | 0.41% |
| 3.6 | 46.50  | 46.33                                       | 0.36% |
| 4.2 | 54.25  | 54.08                                       | 0.31% |
| 4.8 | 62.02  | 61.84                                       | 0.26% |
| 5.4 | 69.75  | 69.59                                       | 0.23% |

amplitude of the vibration frequency N1 and the magnitude of the unbalance are obtained by fitting the quadratic curve, as showed in Table 7.

As can be seen from Figure 7a, the amplitude of the vibration response frequency N1 of the dual-rotor system gradually increases with the increase of the unbalanced

**Fig. 8.** Measuring points on the dual-rotor system test-rig.

mass, the amplitude of 2N1 does not change. Furthermore, the fitting curve of the vibration amplitude of the frequency N1 with the unbalanced mass is given in Figure 7b, it is found that the amplitude of the frequency N1 has a linear relationship with the magnitude of the unbalanced mass. From the fitting results in Table 7, it can be seen that the error of the fitting function gradually decreases with the increase of unbalanced mass. It can be considered that the amplitude of the frequency N1 is linearly related to the unbalanced mass.

## 7 Experimentation research

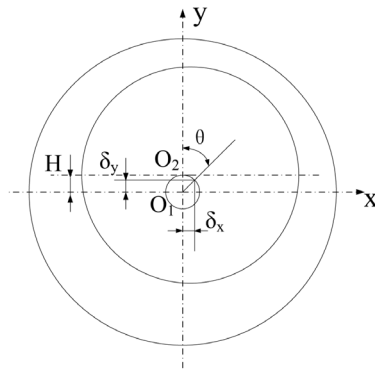
By arranging four measuring points on the dual-rotor system as shown in Figure 8, the vibration characteristics of the dual-rotor are studied. The 6# support of the test rig has the function of adjusting the misalignment of the support, as shown in Figure 9a. The misalignment adjusting device mainly consists of 6# support, outer adjusting ring and inner adjusting ring of support misalignment. The misalignment of the 6# support can be adjusted by adjusting the angles of the two adjusting rings, and the adjustment range is 0–3 mm. Figure 9b shows the adjustment principle of the misalignment adjusting device. There is eccentricity  $H$  between the inner adjustment ring and the outer adjustment ring. When the inner adjustment ring is rotated, the center  $O_2$  of the inner ring rotates through the angle  $\theta$  along the center  $O_1$  of the outer adjustment ring, so that the inner adjustment ring is non-concentric concerning the outer adjustment ring, so that the rotor support is out of alignment. An annular groove is arranged on the outer ring of the adjusting device to control and adjust the eccentric angle. The different mean values (horizontal direction and vertical direction) of the rotor support are as follows.

$$\delta_z = H \sin(\theta), \delta_y = H[1 - \sin(\theta)].$$

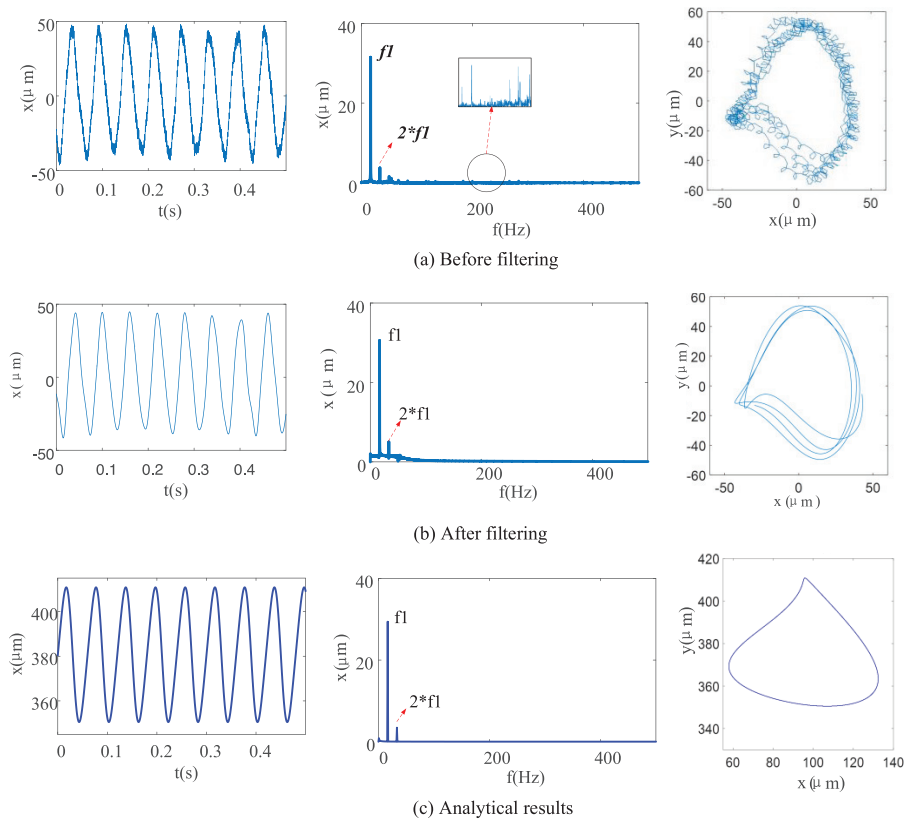
Vibration response of the dual-rotor system with 6# support misalignment was tested on the test rig. Rotational speed of the inner and outer rotors is 1000 r/min and 0 r/min respectively. The time-domain responses, the frequency spectra and the shaft-center trajectories at the rear support of the low-pressure turbine are obtained employing eddy current sensors with the 2mm misalignment, as showed in Figures 10a and 10b. And the analytical

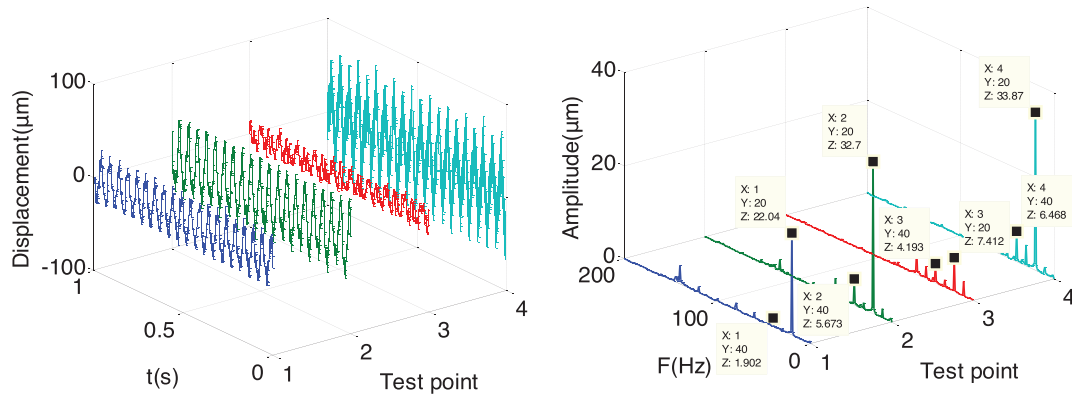


(a) Sketch of test rig of 6#fulcrum

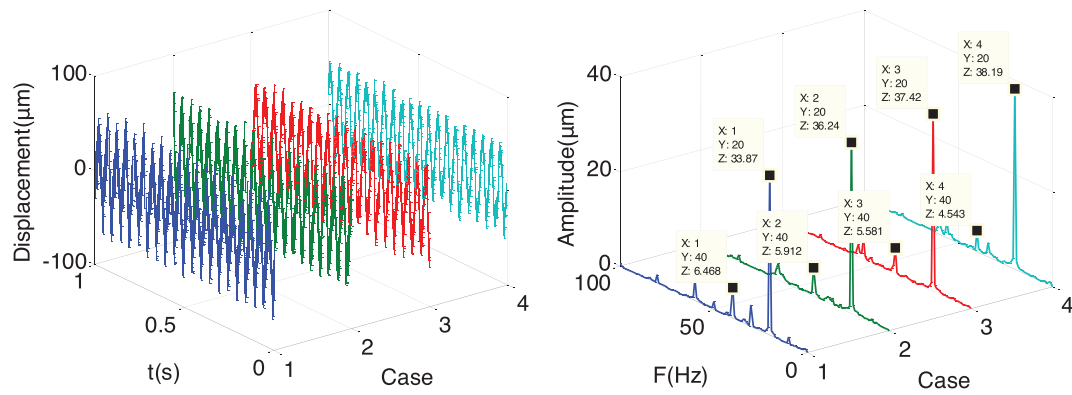


(b) Schematic diagram

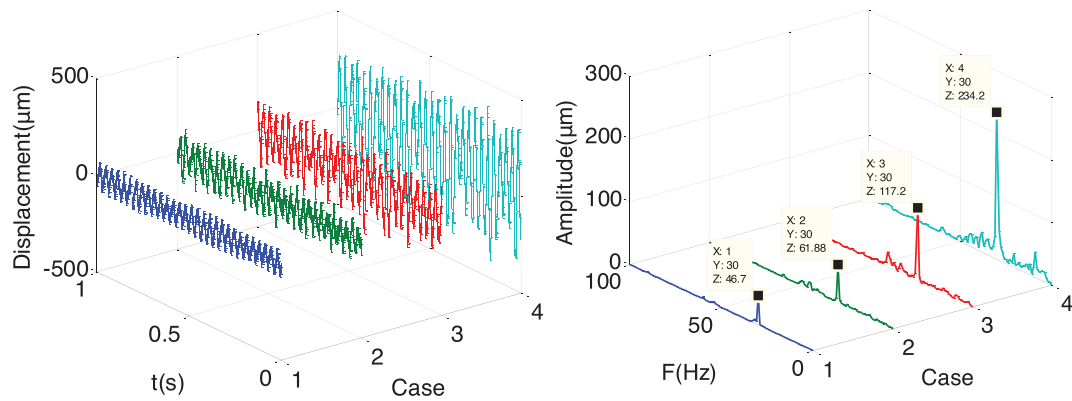
**Fig. 9.** 6# fulcrum misalignment adjusting device.**Fig. 10.** The time-domain responses, the frequency spectra and the shaft-center trajectories of dual-rotor.



(a) The vibration time domain and frequency spectrum of different measuring points with the same misalignment



(b) The vibration time domain and frequency spectrum of test point 4 with different misalignment



(c) The vibration time domain and frequency spectrum of test point 4 with different unbalance

**Fig. 11.** The displacement vibration of the low-pressure turbine shaft under four kinds of misalignment.

results are also given, as showing in Figure 10c. The analytical results also conform to the practical vibration measurement due to support misalignment, which confirms the feasibility of the established model and an analytical method.

As can be seen from Figure 10a, miscellaneous edges appear at the peaks and valleys of the time-domain displacement curve, which are caused by the influence of high-frequency components in the signal. The main components of the spectrum are frequencies N1 and 2N1, the peak value of component frequency is relatively low. And N1 is the rotating frequency of the low-pressure rotor, 2N1 is mainly caused by the misalignment of the support. The displacement time domain curve and spectrum after filtering out high-frequency components are shown in Figure 10b, which is regular waveforms. The measured axis trajectory is relatively messy due to the influence of high-frequency signals caused by gear meshing, bearing defects and other factors. And the axis trajectory after filtering out high-frequency components presents an obvious “water-drop” shape, which is consistent with the simulation results of analytical calculation.

The vibration time domain and frequency spectrum of different measuring points with the same misalignment is obtained are shown in Figure 11a. By adjusting the angle  $\theta$ , the displacement vibration time domain and frequency domain of the low-pressure turbine shaft under four kinds of misalignment are obtained as shown in Figure 11b. The vibration time domain and frequency domain of the low-pressure turbine shaft under four different unbalances are obtained as shown in Figure 11c by changing the unbalance of the fan disc. The magnitude of the four imbalances from case 1–4 are 20 g, 26 g, 42 g, and 63 g respectively.

As can be seen from Figure 11a, the magnitude order of the four vibration measuring points is  $3 < 1 < 2 < 4$ . It shows that the vibration decreases on the low-pressure rotor transmission path: LPT-LPC-Fan shaft. The vibration of the low-pressure rotor causes the vibration of N1 as the main frequency at the measuring point 3 of the high-pressure rotor, which indicates that there is coupling between vibration of the high-pressure rotor and the low-pressure rotor. It can be seen from Figure 11b that during the change of  $\theta$  from  $90^\circ$  to  $0^\circ$ , the amplitude of frequency N1 basically has no change, and the amplitude of frequency 2N1 gradually decreases. It can be seen from Figure 11c that the amplitude of frequency N1 is linear with the magnitude of unbalance.

## 8 Conclusions

In this paper, the vibration characteristics of the dual-rotor system with support misalignment faults are studied by analytical method. Based on the self-designed test-rig of the dual-rotor system, the vibration characteristics of the dual-rotor system under the misalignment fault of support are studied. After calculation and analysis, the following conclusions are obtained:

- The 2 times of rotational frequency of the low-pressure rotor (2N1) will occur in the vibration of the dual-rotor when the support is misalignment. And coupling will occur in the misalignment of supports in horizontal and vertical directions.
- The amplitude of the frequency 2N1 in the vibration response of the dual-rotor system is quadratic with the misalignment.
- The amplitude of the frequency 2N1 in the vibration response of the dual-rotor system is linearly related to the load torque.
- The amplitude of the rotor frequency in the vibration response of the dual-rotor system has a linear relationship with the unbalanced mass, while the frequency 2N1 does not correlate with the unbalanced mass.

The simplified model of the dual-rotor system eliminates the interference of many factors on the vibration characteristics of the dual-rotor system, and clarifies the feasibility of using the analytical dynamic model of the dual-rotor system to study the vibration problem under the rotor misalignment fault. The establishment of the dual rotor system test-rig with a misalignment adjustment device can simulate the misalignment of support. The theoretical results are verified by experimental tests. The conclusion of this paper has important value for vibration prediction, evaluation and control of aero-engine internal and external dual-rotor systems unbalance and misaligned support faults.

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## Conflicts of interest

The authors declare no conflict of interest in preparing this article.

## Data availability statement

All data, models, and code generated or used during the study appear in the submitted article.

## Author contribution statement

Pingping Ma's contribution to the work can be summarized in the following roles: Writing – Review and Editing, Writing-Original Draft Preparation Software, Conceptualization, and Methodology. Muge Liu's contribution to the work can be summarized in the following roles: Visualization and formal Analysis. Junkai Guo's contribution to the work can be summarized in the following roles: Data Curation and investigation. Haoyu Wang's contribution to the work can be summarized in the following roles: Data Curation, Formal Analysis, and Resources. Yuehui Dong's contribution to the work can be summarized in the following roles: Visualization, Data Curation, and Investigation.

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