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Modal control using real-time identification for time-varying structures

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Abstract – For a controlled structure, the changes of physical characteristics like natural frequencies, modal damping coefficients, and what's more on mode shapes will destabilize the controlled system. To keep the control performance and guarantee its robustness, a new modal control method using real-time identification is proposed to overcome the instability of controlled structure, especially induced by the inversion of mode shape. In this paper, a modal model of time-varying mechanical structure is supplied by an identifier. The choice of the kind of control permits to use this time-varying model for optimizing and self-adapting the controller. In order to supply the controller, a modal observer is used and also updated. For illustrative purpose, this proposed methodology is carried out on a simple but representative time-varying mechanical structure. On this example, an inertia modification leads not only to low modal frequency shifts but also to inversion of a mode shape. This mode shape inversion is shown to destabilize the controlled structure when the control system is not updated. The overall procedure will be described through simulations and performed for different operating conditions, which will prove that mode shapes have to be precisely determined and updated in the controller and observer to guarantee a robust modal control with high performances in spite of the changes of structure.

Key words: Self-adaptive control / real-time identification / modal control / mode shape / stability / structure / robustness

1 Introduction

Active modal control is widely used in many industrial fields and has demonstrated its high performance. But when the structural variation induces the changes of physical characteristics, the robustness of active modal control can not be guaranteed. So considering the robustness, robust control has been extensively studied in the last decades [1], but reduces control performances. Moreover, the robust control is not well adapted to modal formulation which is widely used in the mechanical field. For example, mode shape inversion can not be adapted to robust control. Another solution consists in using adaptive control. For example, a set of controllers can be designed according to the limits of variations of structure such as Multiple Model Adaptive Control (MMAC) [2]. In the case of modal description of the structure, adaptive modal control offers a trade-off between high performances and robustness [3]. But by this approach, some relevant information should be known a priori or must be measured for designing the controller. In this purpose, identification algorithms have been introduced in the closed-loop system for getting an updated model [4,5].

Following this general idea, different methods of identification are developed as direct, indirect, joint input-output [6]. For indirect approaches, in the case of strongly damped structures thanks to the controller, identification methods will not present enough accuracy. However, Gevers [5] has proposed a global approach by changing controller parameters in order to follow the system, but the controlled system is required to be stable. In such Black Box approaches, the causes of instability cannot be known due to the lack of modelling. On the contrary, direct identification approaches aims at finding a model of the structure which is lightly damped. Therefore, identification results can be accurate and this model reconstruction also permits to identify changes which can induce instability for the controlled structure.

A particularly performing model in the case of vibration control of structures is based on modal description which limits the number of sensors and actuators and the model size. Instabilities can be described in modal

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form. For example, when an inversion of mode shape occurs on a changing structure, modal characteristics like natural frequencies, modal damping ratio, and especially mode shapes need to be known to update modal controller with keeping robustness and performance. This modal model cannot be directly obtained by classical identification methods. Recently, Luş [7] provided a unified framework to reconstruct model by some methods considering a full or a non-full set of sensors or actuators. By his approach, the modal characteristics can be deduced based on the complex modes issued from an identified state space model.

Based on these mentioned principles, modal control using real-time identification for time-varying structures is proposed in this paper. Combining online real-time identification and adaptive modal control, this method can be used in order to design an effective controller for a time-varying structure.

This paper is organized as follows. First of all, the modal description of a structure is presented in Section 2, and then a modal control combining a real-time identification is proposed. Section 3 is devoted to the description of the proposed method through a simple mechanical structure. The chosen example is a 3 degrees of freedom (d.o.f.) mechanical structure where inertia of the third d.o.f. can be changed with time, inducing frequency shifts and more specifically mode shape inversion. Results are presented and a conclusion is summarized in the last section.

2 Modal control using real-time identifier

2.1 Principle

In this approach, the time-varying structure is controlled by the help of a feedback control loop combining an identifier. Online modal parameter identification is carried out to identify a modal model of this structure by I/O data, which is directly connected to this structure, at constant time steps. With the same I/O data, a controller is applied to the structure in parallel with identifier. This modal controller is updated and optimized with the help of identified modal model. The updating period of the controller must be linked to the rate of the structure changes and to the first eigen frequency for good identification performances [8]. The optimization algorithm of the control is chosen in order to allow the use of the identified model. The optimization control parameters are defined from the initial state of the structure and are kept for the following updates of the controller. This principle is described in Figure 1: the structure (1) is identified with the help of online identifier (2), which permits to complete the characteristics of modal model (3). This model is used to optimize and update the observer and controller (4) based on a modal state space model. The updating time is defined from the lowest frequency of this structure at initial state and the control time is defined from its highest frequency.

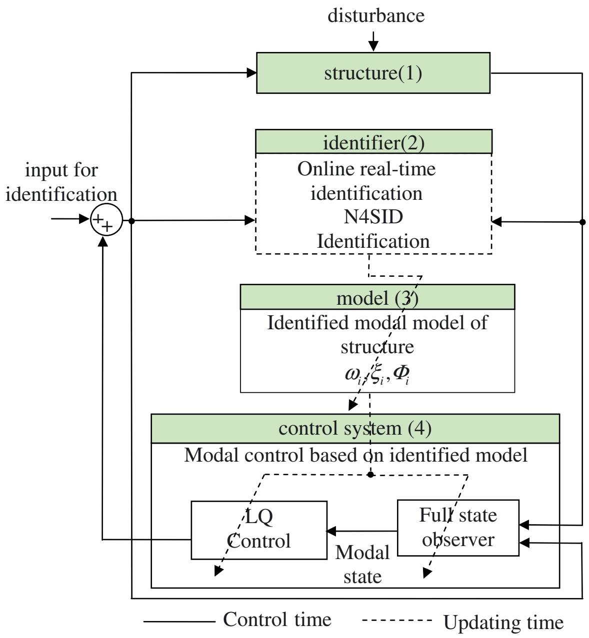

Fig. 1. Principle of modal control using real-time identification.

2.2 Identification of modal model

Actual structure can be modelled by N degrees of freedom (d.o.f.), so second-order differential equation is presented to describe this structure as:

$$\boldsymbol{M}\ddot{\boldsymbol{\delta}} + \boldsymbol{D}\dot{\boldsymbol{\delta}} + \boldsymbol{K}\boldsymbol{\delta} = \boldsymbol{F} \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$ are mass, damping and stiffness matrices, respectively; $\boldsymbol{\delta}$ and $\boldsymbol{F}$, displacement of d.o.f. and external force vectors, respectively. After the change of variable:

$$\boldsymbol{\delta} = \boldsymbol{\Phi}\boldsymbol{q} \tag{2}$$

where $\boldsymbol{q}$ is the modal displacement vector and $\boldsymbol{\Phi}$ is the mode shape matrix reduced to n modes. Under the hypothesis that modes are sufficiently decoupled and the damping is proportional and weak, Equation (1) can be transformed to a state space model in modal basis:

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ &= \begin{bmatrix} 0 & \boldsymbol{I} \\ -\mathrm{diag}\left(\omega_i^2\right) & -2\mathrm{diag}\left(\xi_i\omega_i\right) \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{Bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ \boldsymbol{\Phi}^{\mathrm{T}} \end{bmatrix} \boldsymbol{F} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \tag{3}$$

where ω_i are undamped eigen frequencies and ξ_i modal damping ratio. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are dynamical system, input, output and feedthrough matrix separately. $\boldsymbol{u}$ is input vector. $\boldsymbol{\Phi}$ is normalized as:

$$\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\Phi} = \boldsymbol{I} \tag{4}$$

where $\boldsymbol{I}$ is identity matrix.

In order to finally estimate $\boldsymbol{A}$, $\boldsymbol{B}$, subspace method is chosen for getting a general state space model from input-output data. This identification method can be extended to closed-loop system as direct approaches [6], where an open-loop identification method is directly applied to identify the model of structure by the measured input signal which is necessary for the identification, the measured control signal, and response signal of the controlled structure. So, state space of structure can be identified easily and rapidly, owing to weak damping and decoupled mode shapes. The N4SID identification algorithm [9] is chosen, for its convergence (non-iterative) and numerically stability, regardless of zero and non-zero initial states. As this algorithm is used in real-time, it is necessary to adapt the sampling frequency and the length of the data vectors. Therefore, a discrete time description must be used, leading to the formulation given by Van Overschee and De Moor in [9]. Discrete state space matrices for time k can be estimated by the least-squares method [6], with measured I/O data u and y

$$\begin{bmatrix} \boldsymbol{A}_{\rm d} & \boldsymbol{B}_{\rm d} \\ \boldsymbol{C}' & \boldsymbol{D}' \end{bmatrix} = \left(\begin{bmatrix} \bar{\boldsymbol{X}}_{k+1} \\ \bar{\boldsymbol{Y}}_{k|k} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{X}}_{k} \\ \bar{\boldsymbol{U}}_{k|k} \end{bmatrix}^{\rm T} \right) \times \left(\begin{bmatrix} \bar{\boldsymbol{X}}_{k} \\ \bar{\boldsymbol{U}}_{k|k} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{X}}_{k} \\ \bar{\boldsymbol{U}}_{k|k} \end{bmatrix}^{\rm T} \right)^{-1} \tag{5}$$

where $\bar{\boldsymbol{X}}_{k+1}$, $\bar{\boldsymbol{X}}_{k}$, $\bar{\boldsymbol{U}}_{k|k}$, $\bar{\boldsymbol{Y}}_{k|k}$ are the estimates state vectors, measured input and output vectors after time k. $\boldsymbol{A}_{\rm d}$, $\boldsymbol{B}_{\rm d}$, $\boldsymbol{C}'$, $\boldsymbol{D}'$ are the state space discrete identified matrices.

Then classical method like matrix logarithm is used to transform this identified discrete state space matrices $\boldsymbol{A}_{\rm d}$, $\boldsymbol{B}_{\rm d}$ into continuous state space matrices $\boldsymbol{A}_{\rm c}$, $\boldsymbol{B}_{\rm c}$ and get the eigen value to deduce $\omega_{\rm i}$ and ξ_i. But since the state space basis is unspecified, $\boldsymbol{A}_{\rm c}$, $\boldsymbol{B}_{\rm c}$ can not be directly used to reconstruct complex mode of structure, which is needed to get mode shape in Equation (3).

So, Luş and De Angelis [7, 10] propose a solution to obtain the complex modes of the structure. According to the different positions of actuator and sensor with at least one collocated pair, a transformation matrix $\boldsymbol{\tau}$ can be got from eigen vectors $\boldsymbol{\varphi}$ of $\boldsymbol{A}_{\rm c}$ by [7]:

$$\boldsymbol{C}^{\rm E}(\boldsymbol{l},:)\,\boldsymbol{\varphi}\boldsymbol{\tau}^2 = \left(\boldsymbol{\varphi}^{-1}\boldsymbol{B}_{\rm c}^{\rm E}(:,\boldsymbol{l})\right)^{\rm T} \tag{6}$$

where l is the node of co-located actuator and sensor pair. $\boldsymbol{B}_{\rm c}^{\rm E}$ and $\boldsymbol{C}^{\rm E}$ are the expanded versions of $\boldsymbol{B}_{\rm c}$ and $\boldsymbol{C}'$.

Based on $\boldsymbol{\tau}$, complex modes of the structure $\boldsymbol{\psi}$ can be calculated from $\boldsymbol{\varphi}$. If mode shapes are supposed to be sufficiently decoupled and damping is weak, a mode shape $\boldsymbol{\Phi}_j$ can be estimated by [11]:

$$\boldsymbol{\Phi}_j = \boldsymbol{\psi}_{2j}\sqrt{2i\omega_j\sqrt{1-\xi_j^2}},\ j = 1\cdots N \tag{7}$$

where $\boldsymbol{\psi}_{2j}$ is the complex mode corresponding to eigen value $\lambda_{2j} = -\xi_j\omega_j + i\sqrt{1-\xi_j^2}\,\omega_j$ which is obtained from the identified continuous state space model. These general conditions on uncoupled modes j and h can be evaluated with the following equation (see [11]):

$$2\xi_j\omega_j/|\omega_j-\omega_h| \ll 1 \tag{8}$$

It must be noticed that Equation (7) implies that $\boldsymbol{\Phi}_j$ is normalized as in Equation (4).

In this way, a state space model in modal basis (3) can be reconstructed from an identified discrete state space (5). Then, the control system can be updated with actual information about identified modal behaviour of the changed structure.

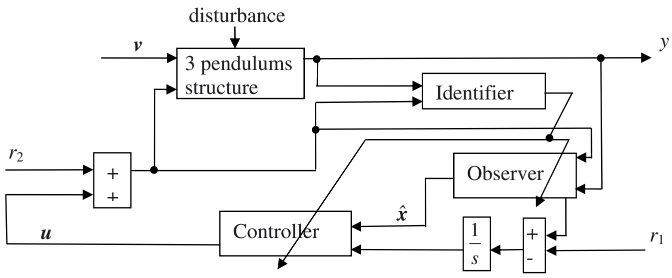

Fig. 2. The scheme of modal control using real-time identification.

2.3 Control and observation design

In this paper, the control algorithm is chosen to optimize the control gains from the identified results ($\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$). So, the classical linear quadratic (LQ) algorithm is chosen for realizing an updated modal control. In order to obtain a good precision of control, an integrator can be introduced. So, in the case of tracking problem, a tracking signal r_1 can be introduced as in the control system presented Figure 2.

The optimization is obtained thanks to Riccati's equations. The linear matrix gain $\boldsymbol{G}$ is got by the minimization of quadratic cost functional:

$$\boldsymbol{J}_{\min} = \int_0^{\infty} \left(\boldsymbol{x}^{\rm T}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^{\rm T}\boldsymbol{R}\boldsymbol{u}\right) {\rm d}t \tag{9}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are weighting matrices. In this paper, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are chosen to be constant and defined from the initial state of the structure. Then the optimal control is got as:

$$\boldsymbol{u} = -\boldsymbol{G}\boldsymbol{x} \tag{10}$$

where $\boldsymbol{G}$ is the optimized matrix gain.

The state vector $\boldsymbol{x}$ can't be obtained or measured directly. So an estimated state vector $\hat{\boldsymbol{x}}$ is reconstructed by an observer. Here, a full state observer, also called Luenberger observer is used:

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}} &= \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}\left(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}} - \boldsymbol{D}\boldsymbol{u}\right) \\ \hat{\boldsymbol{x}}(0) &= \boldsymbol{0} \end{aligned} \tag{11}$$

where $\boldsymbol{L}$ is the optimized observer gain.

Table 1. Mechanical and modal characteristics of the structure (initial state).

Mass of pendulums (kg)	Stiffness of spring ($N.m^{-1}$)	Length of stems (m)	Mass of stems (kg)
$m_1 = 2.61$	$k_1 = 13182$	$Lt_1 = 0.414$	$Mt_1 = 0.305$
$m_2 = 2.61$	$k_2 = 13182$	$Lt_2 = 0.414$	$Mt_2 = 0.305$
$m_3 = 0.875$	$k_3 = 13182$	$Lt_3 = 0.431$	$Mt_3 = 1.205$
Modal damping ratios		Eigen frequencies (3rd pendulum mass at the bottom location)	
$\xi_{10} = 2.61\text{e}{-3}$		$f_1 = 6.44$ Hz	
$\xi_{20} = 2.61\text{e}{-3}$		$f_2 = 17.47$ Hz	
$\xi_{30} = 2.61\text{e}{-3}$		$f_3 = 23.94$ Hz	

2.4 Real-time identification

In this kind of identification chosen for a modal updating, the identifier is supplied with an I/O data directly connected to the structure. These input and output data are correlated due to the feedback control. In order to reduce this correlation, a white noise signal r_2 is added to the input signal $\boldsymbol{u}$ as depicted in Figure 2. The level of this white noise is chosen in order to disturb the structure sufficiently with uncorrelated response, and then to obtain an efficient identification. This excitation must also be sufficiently low in order to limit the control action. This level is also chosen to be greater than the disturbance noise $\boldsymbol{v}$ as in Figure 2.

The disturbances considered in this paper are assumed to not modify the operating dynamic behaviour in a persistent manner. When the collection is large enough to describe the modal response of the lowest frequency mode (two or three periods, see [8]) used in the state space model, the identification is carried out to update modal model and consecutively the controller and observer. This iterative procedure is then performed on new I/O data without overlapping points in the identification windows. This method is well adapted in the case of slow and smooth changes of the structure behaviour.

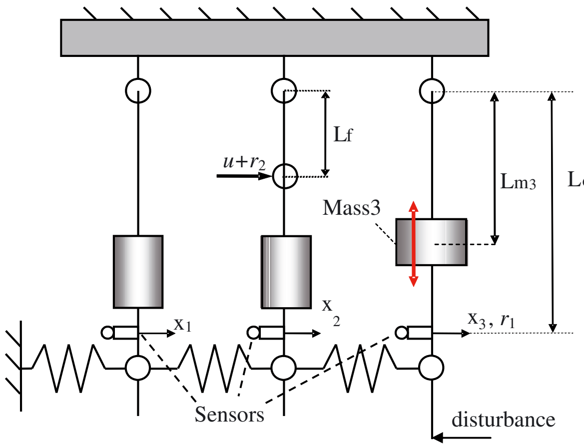

Fig. 3. The 3 pendulums' structure.

3 Application of control system

3.1 Model of a mechanical structure with 3 pendulums

A 3 d.o.f. structure is used to illustrate the previous control/identification process. This 3 pendulums' mechanical structure is presented in Figure 3, where the 3rd pendulum mass can be moved over the vertical direction. Therefore the inertia of this structure is variable and induces mode shape inversion. The characteristics of this structure are summarized in Table 1.

3.2 The instability induced by mode shape inversion

In this example, a regulation problem is considered, where the controller is used to reject disturbances on the structure. When the position of 3rd pendulum mass changes, frequency shifts will occur and the 2nd mode shape will be inverted, i.e. change of the sign of the 2nd component of the 2nd mode shape.

From Equation (3), the poles of the controlled and observed structure can be expressed in modal coordinates with a state space form:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{Z}\boldsymbol{\eta} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} + \boldsymbol{H}\boldsymbol{\eta} \end{aligned} \tag{12}$$

where $\boldsymbol{\eta}$ represents the unknown external disturbance and unknown environmental noise, $\boldsymbol{Z}$ and $\boldsymbol{H}$ are disturbance matrices. Combining Equations (10), (11) and (12), the state space form of the controlled/observed structure is:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}} - \dot{\hat{\boldsymbol{x}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{BG} & \boldsymbol{BG} \\ 0 & \boldsymbol{A} - \boldsymbol{LC} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x} - \hat{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z} \\ \boldsymbol{Z} - \boldsymbol{LH} \end{bmatrix} \boldsymbol{\eta}$$

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{C} - \boldsymbol{DG} & \boldsymbol{DG} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x} - \hat{\boldsymbol{x}} \end{bmatrix} + \boldsymbol{H}\boldsymbol{\eta} \tag{13}$$

Here, the accelerations are used as output for this structure. If the force of control is applied to the 2nd stem, matrices $\boldsymbol{B}$ and $\boldsymbol{D}$ are written as:

$$\boldsymbol{B} = [0_{1\times 3} \ \ \Phi_{21} \ \ \Phi_{22} \ \ \Phi_{23}]' L_c L_f \tag{14}$$

$$\boldsymbol{D} = \boldsymbol{\Phi}[0_{1\times 3} \ \ \Phi_{21} \ \ \Phi_{22} \ \ \Phi_{23}]' L_c L_f \tag{15}$$

where $\Phi_{r,s}$ is the rth component of sth mode shape. Matrix $\boldsymbol{C}$ is:

$$\boldsymbol{C} = \boldsymbol{\Phi}\left[-\mathrm{diag}\left(\omega_i^2\right) \ -2\mathrm{diag}\left(\xi_i\omega_i\right)\right] \tag{16}$$

The poles of the observed and controlled structure are governed by $\boldsymbol{A}-\boldsymbol{LC}$, $\boldsymbol{A}-\boldsymbol{BG}$. For controller, when the force of control is applied to the 2nd stem, the inversion of mode shape (the change of sign of $\Phi_{2,2}$ in this structure) is involved in matrix $\boldsymbol{B}$ and then affects the poles of the controller. For the observer, the inversion of mode shape is always involved with its poles location. The effect of mode shape inversion on the changes of poles is studied in the following.

The control gains and the observer gains are adjusted ($\boldsymbol{G}_0$, $\boldsymbol{L}_0$) by LQG algorithm with weighting matrices Q and R chosen in order to get a high performance when 3rd pendulum mass is located at the bottom location. A pulse force as unknown disturbance is applied at the end of 3rd stem of the controlled structure at time 3.22 s during 20 ms. Modal performances are shown in Figure 4a for the mass of the 3rd pendulum staying at the bottom location. When the 3rd pendulum is moved to the top and the changed structure is controlled by the previous and non-updated $\boldsymbol{G}_0$ and $\boldsymbol{L}_0$, the stability will be lost as shown in Figure 4b. The change of poles is depicted in Figure 5 with constant modal damping ratios. Thanks to some robust and damping of controlled structure, the structure is stable before $Lm_3 = 0.28$ m. But due to mode shape inversion, after this location, the controller $\boldsymbol{G}_0$ is no more stable as detailed in Figure 5b. The 2nd mode shape will be inverted at the position $Lm_3 = 0.294$ m. Of course, if a controller designed for weaker performances is used, the damping of the controlled structure will be decreased and the instability which is induced by the mode shape inversion will occur in a higher location. It is possible to find a controller weak enough to get the controlled structure always stable. These characteristics are obtained via Riccati's equation using constant matrices Q, R, and updated matrices A and B (see Sect. 2.3).

If the force of control is applied to the 1st stem with the same conditions as the above mentioned example, the inversion of mode shape isn't involved in matrix B. So it will not affect stability of the constant initial controller but only affect performances. However in the case of location constraints of actuator (industrial cases), the instabilities can be avoided with the proposed method also maintaining a high performance.

So, the real-time identification is necessary for updating the control.

3.3 Validation of real-time identification for controlled system

To verify the performances of identification, firstly external disturbance $\boldsymbol{v}$ is not applied to the structure and

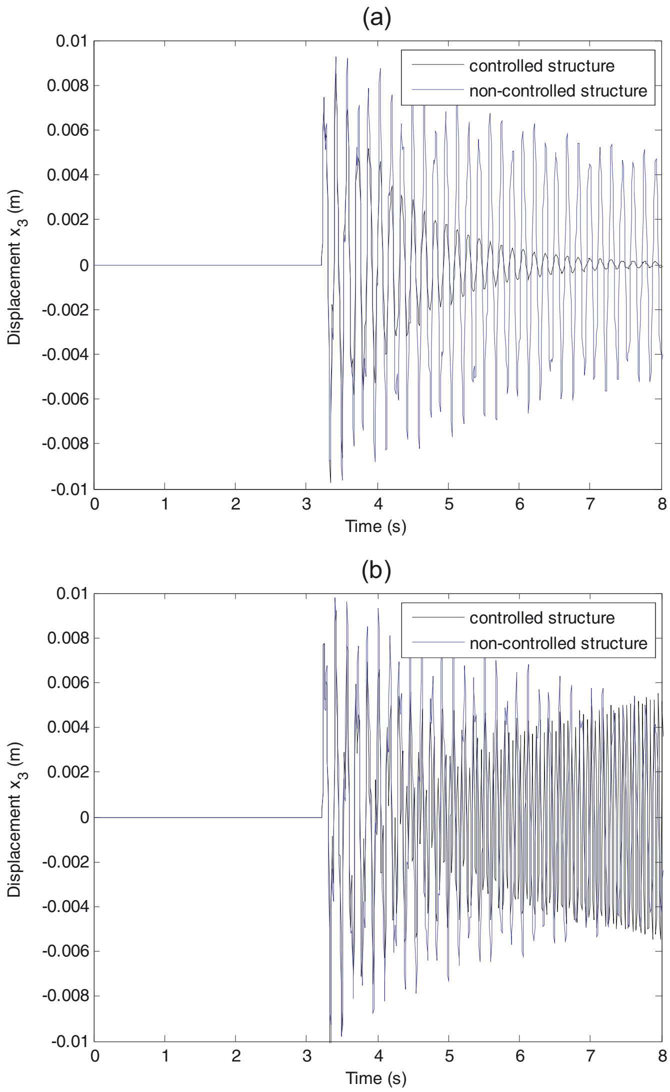

Fig. 4. Displacement x_3, 3rd pendulum mass at bottom location (a) and at high location (b).

the 3rd mass moves from $Lm_3 = 0.342$ m to $Lm_3 = 0.234$ m with the velocity of 3.6 mm.s^{-1}. The same weighting matrices as in Section 3.2 are used by LQG to optimize controller and observer. By contrast with Section 3.2, when the 3rd mass moves, the controller $\boldsymbol{G}$ and observer $\boldsymbol{L}$ are successively updated according to the identified modal model ($\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$). By this way, the control system is adapted to the changed structure.

Acceleration and force signals are sampled at 1000 Hz and filtered, focusing the frequency bandwidth of interest [2 Hz−50 Hz]. The length of the identification window is set to 500 points (0.5 s) [8]. With these operating conditions, $\Phi_{r,s}/\Phi_{1,s}$ and frequencies are identified as modal characteristics and compared to their theoretical values. Some identified results as $\Phi_{2,2}/\Phi_{1,2}$, $\Phi_{3,2}/\Phi_{1,2}$ and frequency ω_2 are shown in Figures 6a–c. For the 1st and 3rd mode, the difference between theoretical and identified mode shape is less than 2% like the 3rd component of 2nd mode shown in Figure 6b. But there is a

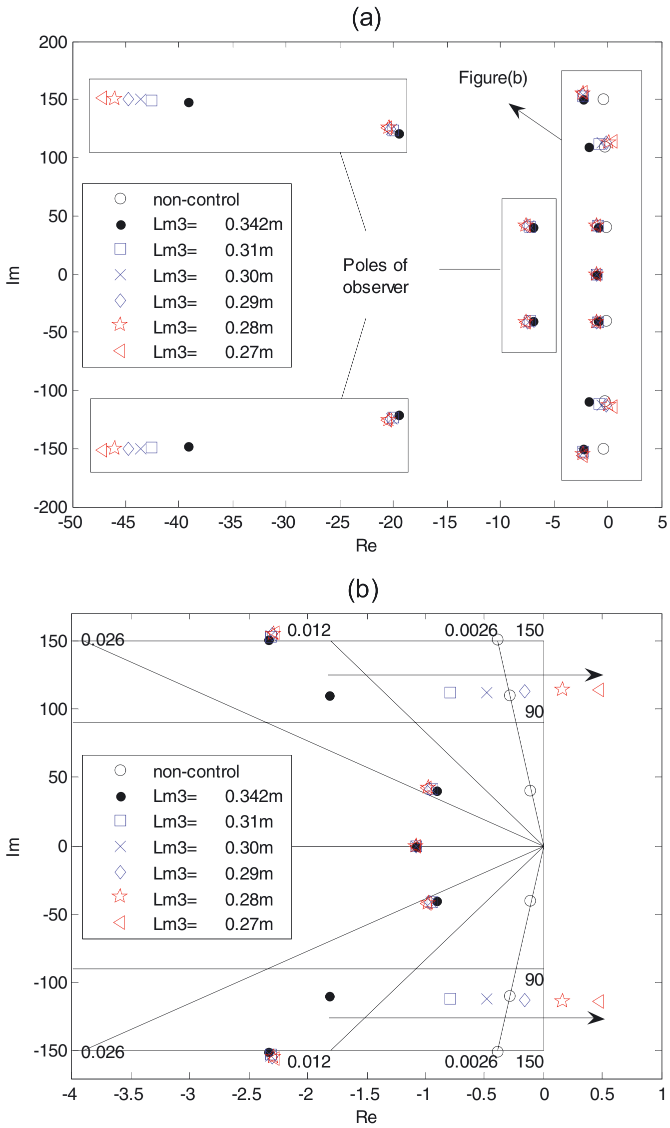

Fig. 5. Pole location in the case of non-updated controller $\boldsymbol{G}_0$ and observer $\boldsymbol{L}_0$(a) and details (b).

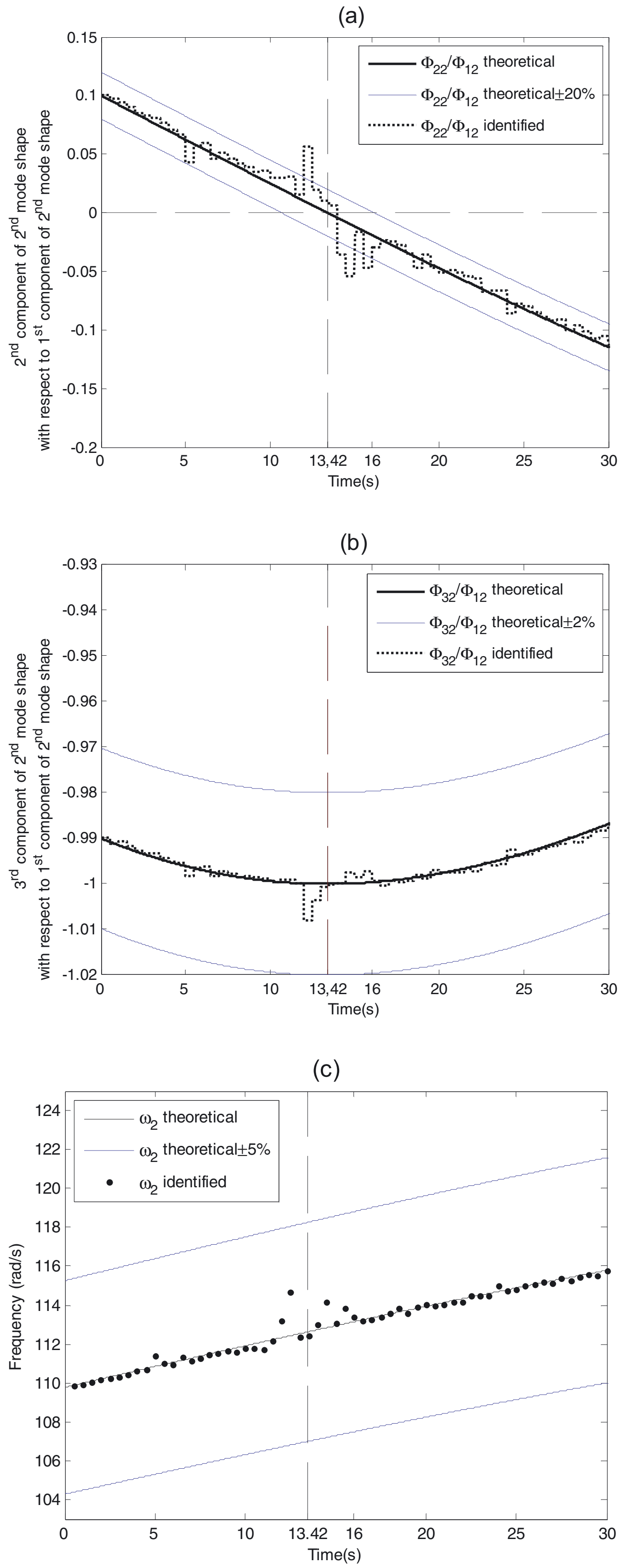

Fig. 6. Results of modal control with real-time identification without disturbance $\boldsymbol{v}$.

greater difference between theoretical and identified value for the 2nd component of 2nd mode shape $\Phi_{2,2}/\Phi_{1,2}$ as in Figure 6a, especially near to the time 13.42 s where the 2nd mode shape is inverted. For the 1st frequency as well as for the 3rd frequency, the identified frequencies are always close to the theoretical values. But for the 2nd frequency, the difference between theoretical and identified values is increased when the mode shape is inverted, but always less than 5% as in Figure 6c. Indeed, when the 2nd mode shape is near to be inverted, i.e. $\Phi_{2,2} \approx 0$, the acceleration of 2nd pendulum mass is near to zero. Then the identifier can not get enough measured outputs, and identified results are not exact.

When the mass moves, the stability of the control can be checked from the displacement response x_3 when the necessary noise for the identification is introduced on the structure (Fig. 7) due to a continuing update of the controller and observer.

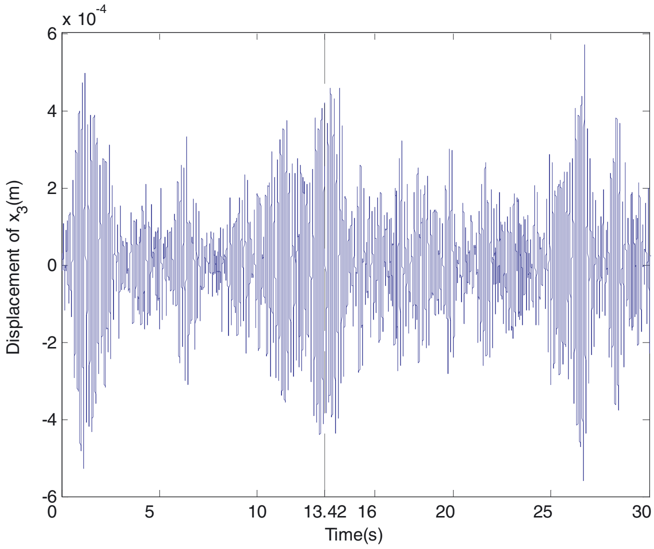

Fig. 7. Displacement x_3 without disturbance $\boldsymbol{v}$.

3.4 Results of modal control using real-time identification

Since the identification works properly, an updated control system should be realized to reject the disturbance on the structure. As the disturbance does not affect the dynamic behaviour, the result of the identification is not seriously affected by this disturbance. For getting changes of identified results induced by disturbance, a pulse force (50 N) as an example of the unknown disturbance is applied at the end of 3rd stem at time 20.22 s and during 20 ms. This time location is chosen to clearly distinguish effects of disturbance from changes induced by mode shape inversion. The environmental white noise $\boldsymbol{v}$ is also introduced in the system in order to simulate external noise. Other operating conditions are the same as in Section 3.3. Some results as identified $\Phi_{2,2}/\Phi_{1,2}$, $\Phi_{3,2}/\Phi_{1,2}$ and frequency ω_2 are shown in Figures 8a–c.

Before the disturbance occurs, almost the same results as in Section 3.3 can be got for identified mode shapes and frequencies, in spite of the environmental white noise. Obviously, near the time 20.22 s where the disturbance occurs, there is a greater difference between theoretical and identified value of mode shapes and frequencies. Even if the identified modal model is not exact during the time interval [20.5 s−21.0 s] and thanks to the updated controller and observer, the controlled system remains stable. A high performance can be obtained to reject the effect of disturbance as shown in Figures 9a−b.

From time [12.0 s–12.5 s], the identified model will be deteriorated by the emergence of mode shape inversion as in Figure 8, so there is a peak value of the control force in the time interval [12.0 s−12.5 s] to overcome this effect, as shown in Figure 9b. But this peak is eliminated due to the recovered exactness of the identified model in the control system. The global robustness of the adaptive controlled system seems to be improved.

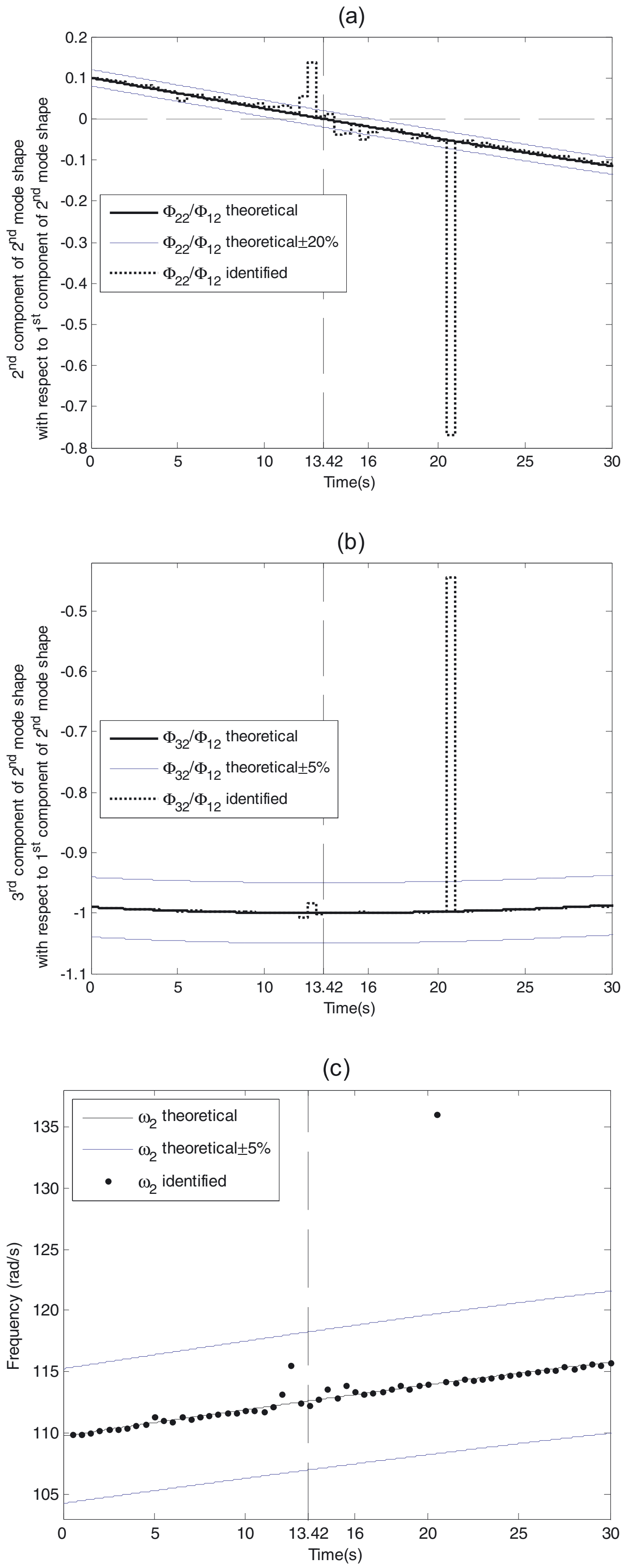

Fig. 8. Results of modal control with real-time identification with disturbance $\boldsymbol{v}$.

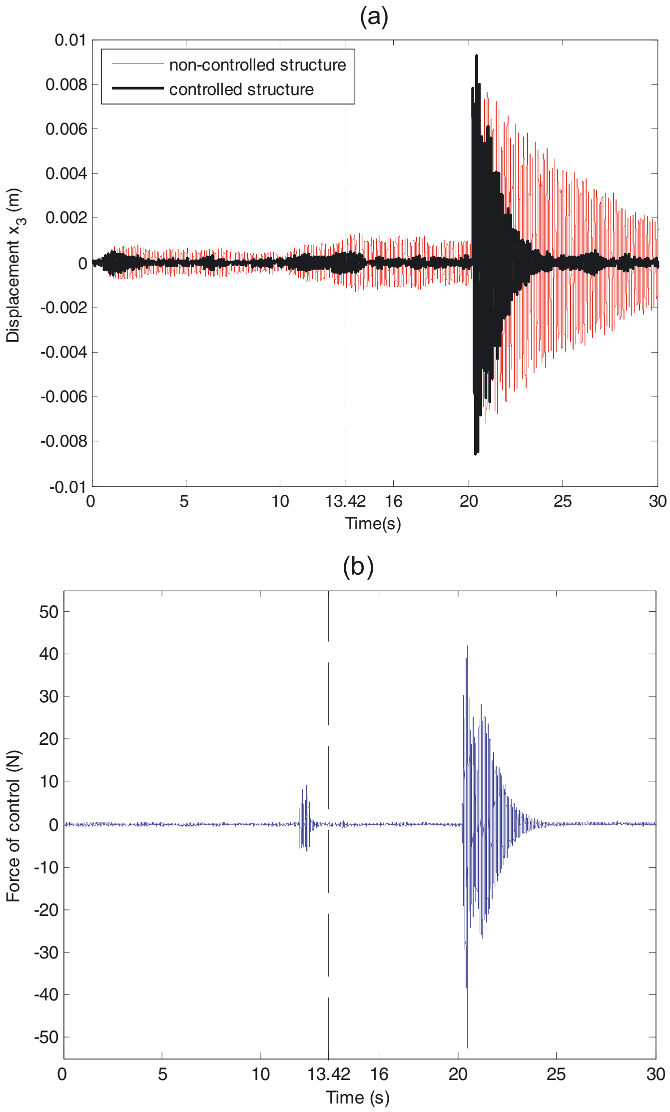

Fig. 9. Displacement x_3 (a) and force of control (b) with disturbance $\boldsymbol{v}$.

4 Conclusion

The aim of this paper is to propose a modal control using a real-time identification which is automatically adapted and intended to time-varying structures. This method can overcome instabilities which are induced by changes of modal characteristics, especially the inversion of mode shape. At the same time, this modal control keeps its high performances.

To get a model of time-varying structure, N4SID is used as a direct approach of identification of the structure under control to get a modal state space model through a pertinent change of modal basis. According to this reconstructed model, the control system can be updated at each step of identification considering the lowest eigen frequency. This modal self-adaptive control can be used for discrete or continuous time-varying structures.

An application has been carried out concerning a 3-d.o.f. discrete structure. The method can exactly identify almost all modal characteristics with ±5% precision at least. Evidently when the inversion and the disturbance occur, the exactness of identified mode shapes is reduced. But due to the updated control system, the stability is guaranteed. Moreover, the application shows that the modal self-adaptive control is globally very robust and permits to keep the same high performance in spite of the changes of the dynamic behaviour of structure. Lastly, this method applied to industrial structures permits to avoid the problems of dispersion and to use an imprecise modelling of the structure.

In the future, this kind of control will be applied to continuous time-varying structures.

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