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APTA: advanced probability-based tolerance analysis of products

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Abstract – In mass production, the customer defines the constraints of assembled products by functional and quality requirements. The functional requirements are expressed by the designer through the chosen dimensions, which are linked by linear equations in the case of a simple stack-up or non-linear equations in a more complex case. The customer quality requirements are defined by the maximum allowable number of out-of-tolerance assemblies. The aim of this paper is to prove that quality requirements can be accurately predicted in the design stage thanks to a better knowledge of the statistical characteristics of the process. The authors propose an approach named Advanced Probability based Tolerance Analysis (APTA), assessing the defect probability (called P_{D}) that the assembled product has of not conforming to the functional requirements. This probability depends on the requirements (nominal value, tolerance, capability levels) set by the designer for each part of the product and on the knowledge of production devices that will produce batches with variable statistical characteristics (mean value, standard deviation). The interest of the proposed methodology is shown for linear and non-linear equations related to industrial products manufactured by the RADIALL SA Company.

Key words: APTA / tolerance analysis / defect probability / capability levels / FORM approximations

1 Introduction

In industry, the customer's technical requirements regarding an assembled product are translated into specifications. These specifications list functional requirements to guarantee the performance of the delivered assembled product. These functional requirements are justified by using mathematical equations that can be linear for a linear stack-up, non-linear, or even originating in CAD software (see [1]). In mass production, quality requirements are added in order to guarantee to the customer that the delivered assembled product is robust with respect to manufacturing variability. For each functional requirement, an allowable defect probability, expressed in parts per million (ppm), is required. In the mechanical industry, the link between part tolerances and the associated defect probability of the mechanical product has not previously been analysed in depth. Experience feedback enables the quality level of the design to be validated.

In a more and more competitive world, industrial companies feel the need to manage defect probability P_{D} in the design stage for economic and environmental reasons, reducing warranty returns and wastage in production. The calculation of such a probability would enable design tolerancing to be managed or optimized – see for example [1–3] – by proposing the most economic design (target value and/or tolerance of component dimensions) with respect to the allowable defect probability. However, tolerance optimization results are highly dependent on the accuracy of the predicted defect probability. It is easier to obtain a precise prediction of defect probability, which would also enable the defect probability sensitivities to be examined for various design parameters, in order to argue in favor of the choice of a particular dimensional specification with a supplier.

This paper deals with the issue of defect probability P_{D} assessment in the design stage. From a scientific point of view, the calculation of P_{D} is not simple and concerns the assessment of a very low probability (a few ppm). Some approaches exist under various assumptions concerning the statistical distribution of dimensions. After Section 2, which illustrates the problem using a simple two-part reference example and introduces the necessity of a new approach, Section 3 comprises a brief bibliography review regarding the probabilistic modeling of dimensions and defect probability computation. Section 4 details the proposed APTA methodology. The applicability of the proposed methodology is demonstrated in Section 5 for the simple two-part reference example mentioned above. The methodology is then applied to two

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Notations

Y	Functional variable of the assembled product;
T_Y	Target value for Y;
LSL_Y, USL_Y	Functional variable bounds;
t_Y	Tolerance on Y;
f	Mathematical relationship enabling the computation of Y;
P_{D}	Defect probability evaluated from the proposed APTA method;
$P_{\mathrm{D}}^{\mathrm{U}}$	Upper bound of the defect probability;
X_i	Part dimension;
n	Number of dimensions in f;
T_i	Target value of the X_i dimension;
LSL_i, USL_i;	X_i bounds;
t_i	X_i tolerance;
$C_{pi}^{(r)}, C_{pki}^{(r)}$	X_i capability requirements;
C_{pi}, C_{pki}	Capability levels of a X_i manufactured batch;
σ_i	Standard deviation of a X_i manufactured batch;
μ_i	Mean value of a X_i manufactured batch;
δ_i	Mean shift of a X_i manufactured batch;
$\delta_i^{(\max)}$	Maximum allowable mean shift of a X_i manufactured batch;
$P_{\mathrm{D}\vert\delta,\sigma}$	Conditional defect probability, knowing δ_i, σ_i;
$h_{\delta,\sigma}(\delta_i, \sigma_i)$	Joint probability density function of δ_i, σ_i;
Φ	Cumulative density function of the standard Gaussian distribution.

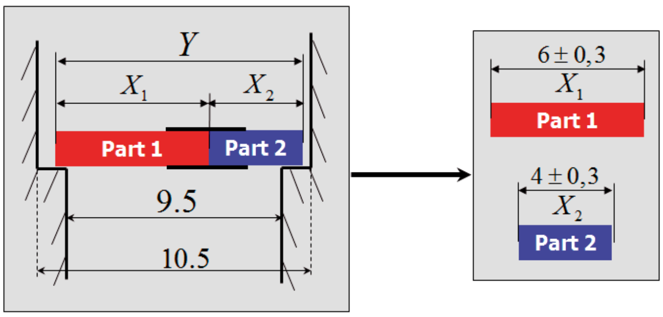

Fig. 1. Illustration of the issue of P_{D} computation in the design stage on a simple two-part example.

industrial applications in Sections 6 and 7 to assess assembly non-conformance probability for electrical connectors produced by the RADIALL company. The first industrial application deals with a linear functional requirement and the second problem deals with a non-linear assembly requirement requiring advanced probabilistic computations.

2 Illustration using a reference example

To illustrate the subject, let us consider a very simple mechanical assembly of two parts, 1 and 2 (Fig. 1). The mechanical requirement of such an assembly is $Y = X_1 + X_2 \in [9.5; 10.5]$ and the customer specifies a quality requirement of 5 ppm i.e. $P_{\mathrm{D}} = \mathrm{Prob}\{Y \notin [9.5; 10.5]\} \leq 5$ ppm. Parts 1 and 2 of the product are specified as follows:

- The target dimensions are $T_1 = 6$ for part 1 and $T_2 = 4$ for part 2.
- The tolerances on parts 1 and 2 are set to $t_1 = t_2 = 1/(1.2\sqrt{2}) = 0.59$, corresponding to the modified root sum of squares tolerancing method [4]. A different choice could have been made.
- The capability requirements are $C_{pi}^{(r)} = 1$ and $C_{pki}^{(r)} = 1$ for each part.

Each production batch of X_i must verify that $C_{pi} \leq C_{pi}^{(r)}$ and $C_{pki} \leq C_{pki}^{(r)}$. C_{pi}, C_{pki} are defined with the well-known equations:

$$C_{pi} = \frac{t_i}{6\sigma_i}, \quad C_{pki} = \frac{t_i/2 - |\delta_i|}{3\sigma_i} \tag{1}$$

where σ_i and δ_i are the statistical parameters (standard deviation and mean shift) of the X_i production batch. δ_i is defined by the difference between the mean value of the X_i production batch μ_i and the target value T_i ($\delta_i = \mu_i - T_i$). The choice of the $C_{pi}^{(r)}, C_{pki}^{(r)}$ capability indexes comes from the industrial requirements of our partner RADIALL S.A. All the following is based on this kind of capability. Nevertheless, the applicability of the proposed methodology in the case of the Taguchi capability index [5] or inertial capability index [6] is discussed in Section 4.6.

The question which now arises is: what is the product defect probability associated with the specifications of parts 1 and 2, and consequently, is it suitable as regards the requirement of 5 ppm or, if not, must the part specifications or the manufacturing process be changed? The answer to this question requires the statistical parameters (δ_i, σ_i) of production batch X_i to be known, but due to tool wear, settings and material variability, different processes for the same part, multi-cavity tools, etc., they are variable over time, as underlined by most authors [7, 8]. On one day, the manufacturing process may work in ideal

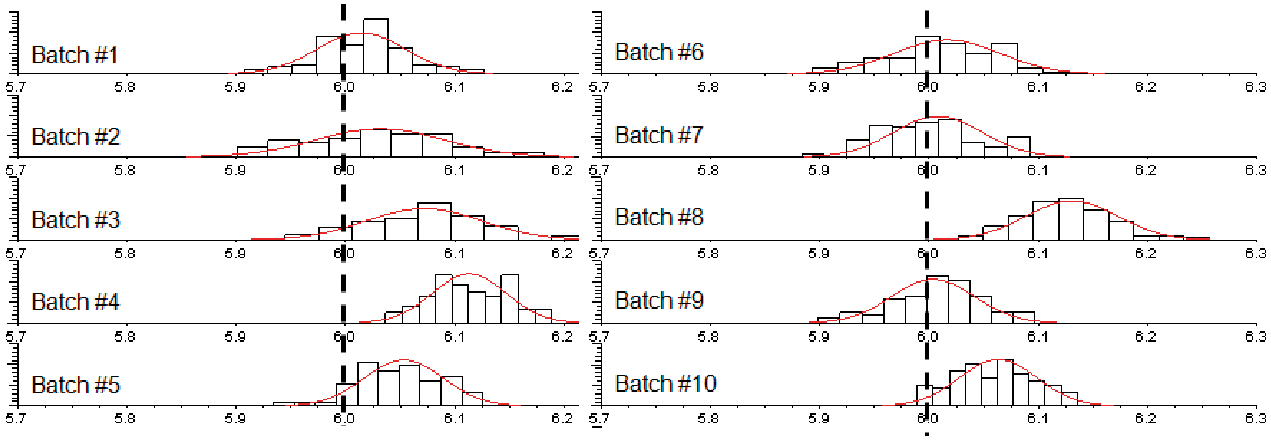

Fig. 2. Variability of the statistical properties of 10 different production batches of X_1. All are in conformance with the specifications $T_1 = 6$, $t_1 = 0.6$, $C_{p1}^{(r)} = 1$, $C_{pk1}^{(r)} = 1$.

Table 1. Capability levels of 10 X_1 production batches, X_1 defined by: $T_1 = 6$, $t_1 = 0.59$, $C_{p1}^{(r)} = 1$, $C_{pk1}^{(r)} = 1$.

Batch #	C_{p1}	C_{pk1}	$\lvert\delta_1\rvert$	σ_1
(1)	2.47	2.35	0.014	0.040
(2)	1.69	1.51	0.031	0.058
(3)	1.88	1.43	0.071	0.052
(4)	2.96	1.84	0.112	0.033
(5)	2.78	2.28	0.053	0.035
(6)	2.05	1.94	0.016	0.048
(7)	2.46	2.39	0.008	0.040
(8)	2.36	1.33	0.130	0.042
(9)	2.63	2.60	0.003	0.037
(10)	2.81	2.22	0.062	0.035

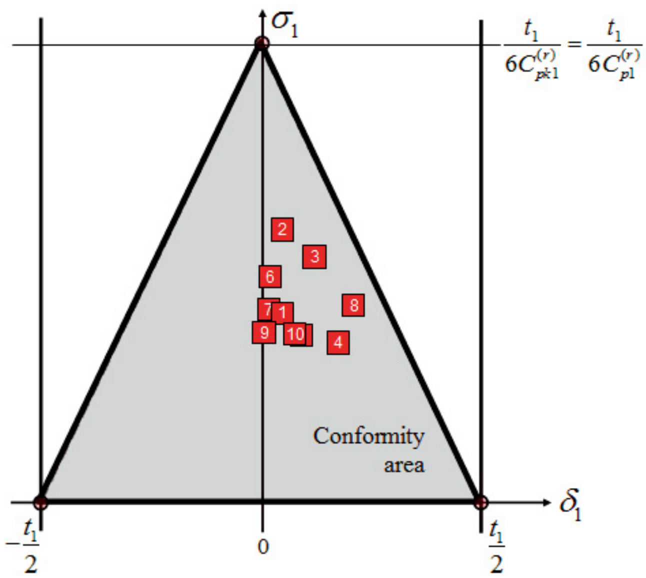

Fig. 3. Conformity area in grey in a mean shift (δ_1), standard deviation (σ_1) diagram. The mean shift is bounded by $\pm t_1/2$ when the standard deviation is zero. The standard deviation is bounded between 0 and $t_1/(6C_{p1}^{(r)})$ corresponding to $C_{p1} = C_{p1}^{(r)}$. The 10 batches of Section 2 are indicated by numbered squares.

manufacturing conditions (high quality material, maintenance has just been carried out, new tools have been installed . . .), and on another day, the manufactured batch may have suitable characteristics but with large deviations due to poor material quality, worn tools, and so on. To illustrate this issue, Figure 2 shows 10 synthetic part batch characteristics, all compliant with the specifications. Table 1 shows the 10 capability levels, standard deviation and mean shift of each production batch. The mean shift $\delta_1 = \mu_1 - T_1$ and the standard deviation σ_1 of each distribution fluctuate according to the production conditions. The 10 compliant batches are indicated by squares in a standard δ, σ diagram (Fig. 3) where the conformity area is represented in grey. In the context of the $C_p^{(r)}, C_{pk}^{(r)}$ requirements, the conformity area is bounded by the equations $C_{pi}(\sigma_i) = C_{pi}^{(r)}, C_{pki}(\sigma_i, \delta_i) = C_{pki}^{(r)}$ (Fig. 3 illustrates $C_{pi}^{(r)} = C_{pki}^{(r)}$ since this is the case for the reference example). According to most authors [4, 9–12], these allowable statistical variations have a non-negligible impact on defect probability and must be taken into account in its calculation. The present study proposes to tackle the issue by suggesting a probabilistic model of mean shifts δ_i and standard deviations σ_i based on the knowledge of each manufacturing process.

3 Short literature review

Very fortunately for the production department, there exists an infinite number of different production batches respecting design conformity requirements. The larger the grey area in Figure 3 (large tolerance and small capability requirements), the easier the production is. To compute defect probability, assumptions on statistical models must be made, and several assumptions exist. A very interesting overview is proposed by [13]. Some of the possible models that could be considered are:

– ***Uniformly distributed statistical model*** [4, 7] in the range of the lower and upper bounds

$[LSL_i–USL_i]$. It is a very conservative model from a quality point of view. It supposes that there is the same likelihood of producing parts close to the tolerance bounds as there is close to the target, an unrealistic assumption in mass production and very pessimistic to assess defect probability.

– ***Non-shifted distribution*** – Gaussian statistical model centered on the target, that is to say $\delta_i = 0$: the standard deviation is proportional to the tolerance using the k parameter $\sigma_i = t_i/k$ proposed by [14] and used for various applications in [3, 15]. k is in general set to 6. This model can seem to be optimistic because various phenomena (tool wear, tool settings ...) tend to generate a mean shift δ_i between μ_i and T_i. Some authors suggest integrating a correlation factor between the dimensions coming from the same part [16, 17]. Others wonder about the probabilistic distributions of the part variables: truncated, multimodal, non-Gaussian [7, 14].
– ***Shifted distribution*** – non-centered Gaussian distributions [4,7,9,12]: the mean shifts δ_i have a crucial importance in the evaluation of P_{D}, as shown in [11, 18]. The various authors agree that the standard deviation of X_i is linked to process capability and that the mean value μ_i can be moved from a maximum value in either one direction or the other. The mean shift is defined by Scholtz [7] using $|\delta_i| = \eta/2$, η_i being an a priori fixed value chosen between 0 and 1; $\eta_i = 0.2$ was suggested by Scholtz. The standard deviation σ_i can be deduced considering that $C_{pki} = 1$ by the relation:

$$\sigma_i^0 = \frac{t_i}{6}(1 - \eta) \quad (2)$$

– ***Dynamic shifted distribution*** [8]: this proposed model is based on two observations. The first is that, as with shifted distribution, the standard deviation of a batch is linked to process capability. The second is that tools tend to wear down, gradually modifying the mean value of the manufactured batches until the production tool is repaired. The batch mean value μ_i is thus itself a random variable which can vary between a minimum and a maximum bound. In [8] this kind of model is only used to predict T_Y with a certain confidence level. The computation of defect probability based on this model does not seem to have been performed.

These models have been widely used for defect probability evaluation and in some cases for the robust optimization of specifications [1–3]. The first drawback is that it remains difficult to know which model to use – which ones are conservative regarding defect probability. The second drawback is that the statistical properties of each dimension (standard deviation for non-shifted distribution, mean shift for shifted distribution ...) are not directly linked to the capability requirements. The third very important drawback is that the knowledge of the production device is not considered in all these assumptions. Only parts with independent Gaussian models will be considered in the following, even if other assumptions could be taken into account.

For all the bibliography references, P_{D} is evaluated using deterministic assumptions about δ_i, σ_i as we saw before. The obtained probability is only a conditioned probability, knowing the value of δ_i, σ_i; it is called $P_{\mathrm{D}|\delta,\sigma}$ in the following. If $Y = f(X_i)$ takes a linear form (i.e. $Y = a_0 + \sum_{i=1}^{n} a_i X_i$), the defect probability is a function of δ_i, σ_i and can be computed using the following common relation:

$$P_{\mathrm{D}|\delta,\sigma}(\delta_i, \sigma_i) = \Phi\left(-\frac{\mu_Y - LSL_Y}{\sigma_Y}\right) + \Phi\left(-\frac{USL_Y - \mu_Y}{\sigma_Y}\right) \quad (3)$$

Φ is the cumulative density function of the standard Gaussian distribution (zero mean and unit standard deviation). μ_Y is the mean value of Y, depending on the mean shifts δ_i of each dimension with the equation $\mu_Y(\delta_i) = a_0 + \sum_{i=1}^{n} a_i(T_i + \delta_i)$. σ_Y is the standard deviation of Y, depending on the standard deviation of each dimension σ_i with the equation $[\sigma_Y(\sigma_i)]^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2$ without correlation between dimensions. The first term in Φ (Eq. (3)) represents the defect probability toward the lower bound of Y and the second term in Φ toward the upper bound of Y. If the functional requirement is defined by a one-sided requirement on Y, the appropriate term in Equation (3) must be removed. When f takes a non-linear form, Equation (3) cannot be applied as is. Whatever the expression of f and the statistical distribution of X_i, the computation of $P_{\mathrm{D}|\delta,\sigma}$ can always be carried out by Monte Carlo simulations, but this calculation may quickly become very time-consuming. To reduce calculation time, other authors [3, 12] propose to use a FORM approximation to evaluate $P_{\mathrm{D}|\delta,\sigma}$. This consists of a linearization of the limit state functions $G_1(X_i) = f(X_i) - LSL_Y$ and $G_2(X_i) = USL_Y - f(X_i)$ around the most central failure point [19, 20]. Another interesting method, named the SOTA method [21], is based on a second-order Taylor expansion of Y and an approximation of the Y distribution using the generalized Lambda distribution involving 4 parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. Following this, the probability computation of $P_{\mathrm{D}|\delta,\sigma}$ knowing (δ_i, σ_i) is performed using the cumulative density function of the generalized Lambda distribution.

This short bibliography review underlines the limits of the existing methods that do not take into account the variability of parameters (δ_i, σ_i), characterizing the manufacturing process, for the prediction of P_{D}. Based on probability approximation tools, we propose an innovative approach to the computation of P_{D} that leads to a more precise prediction of defect probability, subject to investment in the knowledge of manufacturing process variability. We call this approach APTA for "Advanced Probability based Tolerance Analysis" based on the integration of (δ_i, σ_i) variability into the computation.

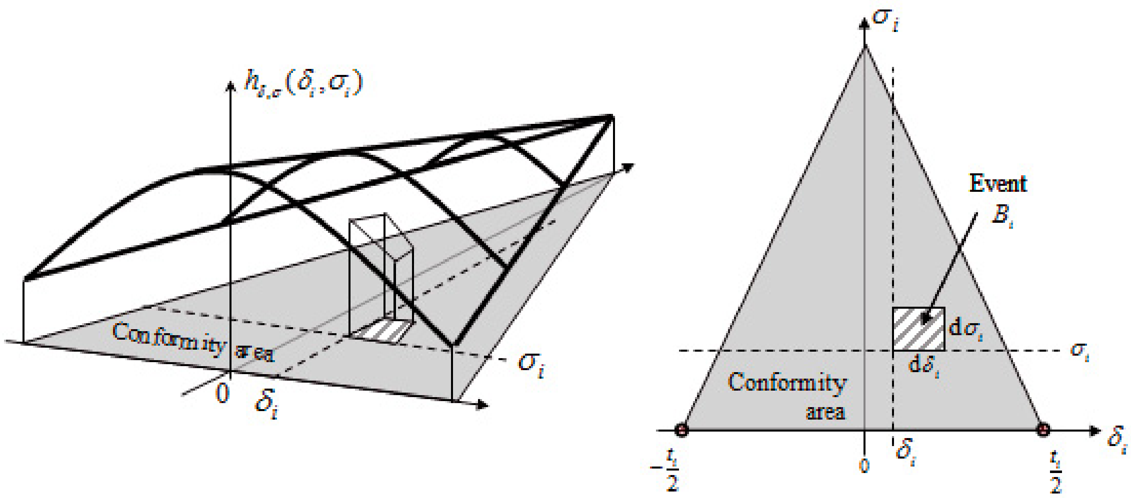

Fig. 4. Illustration of the joint probability density function $h_{\delta,\sigma}(\delta_i,\sigma_i)$ on the left and event B_i on the right.

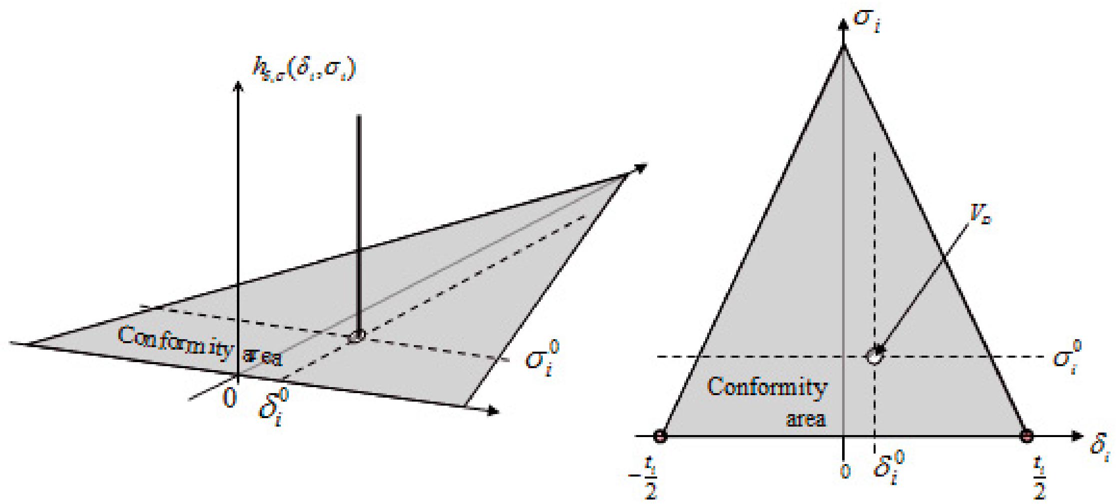

Fig. 5. Density function $h_{\delta,\sigma}(\delta_i,\sigma_i)$ for a fixed known value of $\delta_i = \delta_i^0, \sigma_i = \sigma_i^0$. The variability domain V_{D} is characterized here by only one point $\in$ conformity area.

4 Proposed APTA methodology for P_{D} assessment

4.1 Mathematical formulation of the APTA method

The objective of the proposed APTA methodology is to take into account variable mean shifts and standard deviations in the evaluation of defect probability. In other words, the aim is to compute P_{D} rather than only computing the conditioned probability $P_{\mathrm{D}|\delta,\sigma}$. In the following, each dimension X_i is considered to have an independent Gaussian distribution within the production batch, with a mean shift δ_i and a standard deviation σ_i. In any case, the proposed methodology could be usable and spread to non-Gaussian and/or dependent variables. The two quantities δ_i, σ_i are considered as random variables defined by a joint probability density function called $h_{\delta,\sigma}(\delta_i,\sigma_i)$ which depends on the production device. This density function is bounded by the conformity area. This function is equal to zero outside the conformity area (Fig. 4) because out-of-tolerance batches are considered as being excluded. It can be defined over the whole conformity area or over a reduced domain named variability domain: $V_{\mathrm{D}} \subset$ Conformity Area (Figs. 5–8). Let us consider the following three events:

- A: the functional requirement is not satisfied (Y is outside the tolerance);
- B_i: the mean shift of the X_i batch is in the range $[\delta_i; \delta_i + \mathrm{d}\delta_i]$ and its standard deviation is in the range $[\sigma_i; \sigma_i + \mathrm{d}\sigma_i]$, see Figure 4 for an illustration of B_i over the conformity area;
- B: event B consists of the intersection of the B_i events $\cap_{i=1}^{n} B_i \in [\delta_i; \delta_i + \mathrm{d}\delta_i] \times [\sigma_i\ ;\ \sigma_i + \mathrm{d}\sigma_i]$, that is to say that each dimension is in the specified ranges.

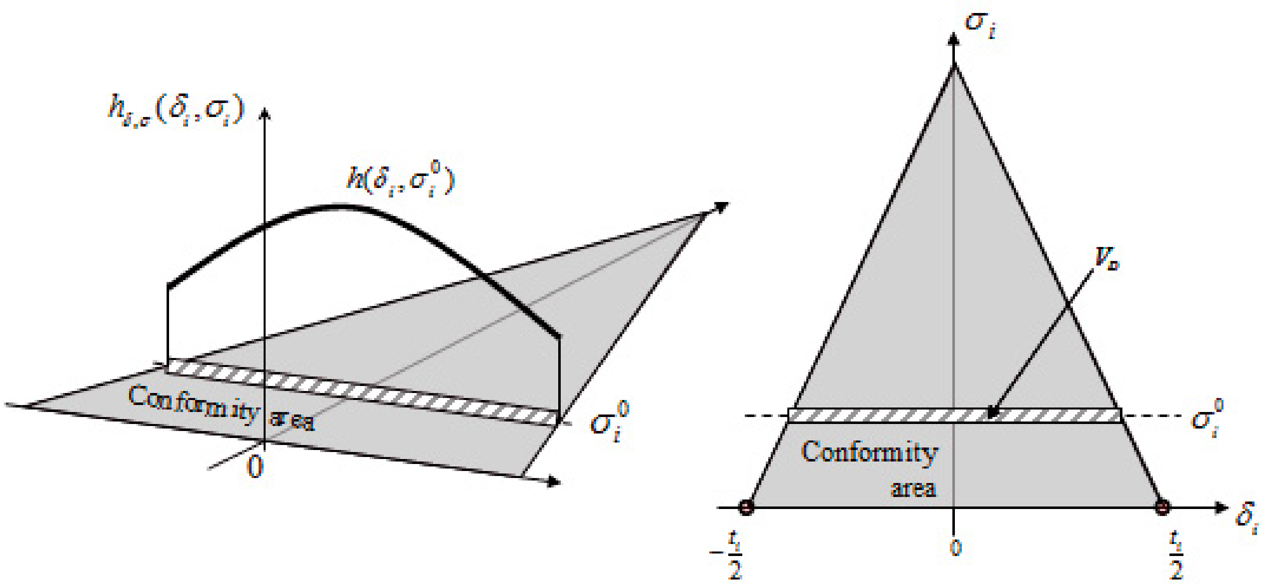

Fig. 6. Joint density function $h_{\delta,\sigma}(\delta_i,\sigma_i) = h_{\delta,\sigma}(\delta_i,\sigma_i^0)$, illustration for a fixed known value of $\sigma_i = \sigma_i^0$. The variability domain V_{D} is here characterized by the hatched line $\subset$ conformity area.

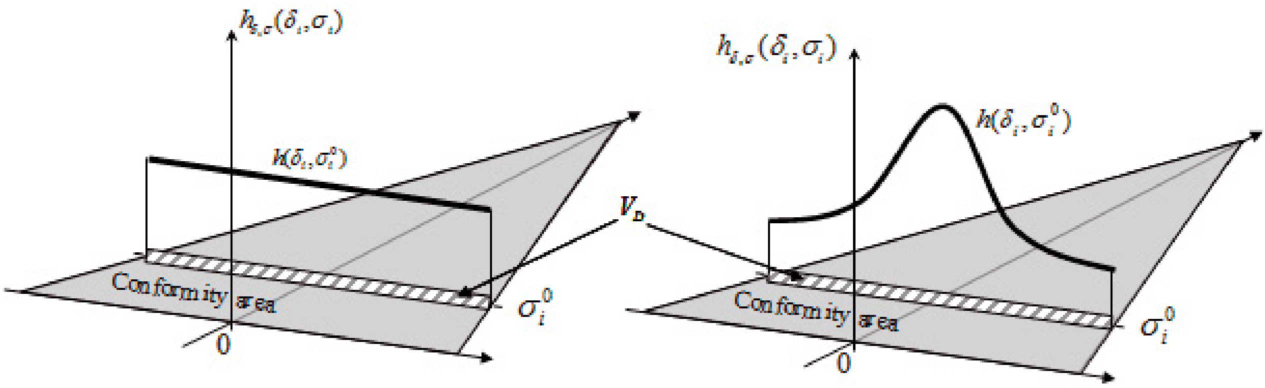

Fig. 7. Joint density function $h_{\delta,\sigma}(\delta_i,\sigma_i) = h_{\delta,\sigma}(\delta_i,\sigma_i^0)$ for a fixed value of $\sigma_i = \sigma_i^0$. Uniform distribution (left), truncated Gaussian distribution (right).

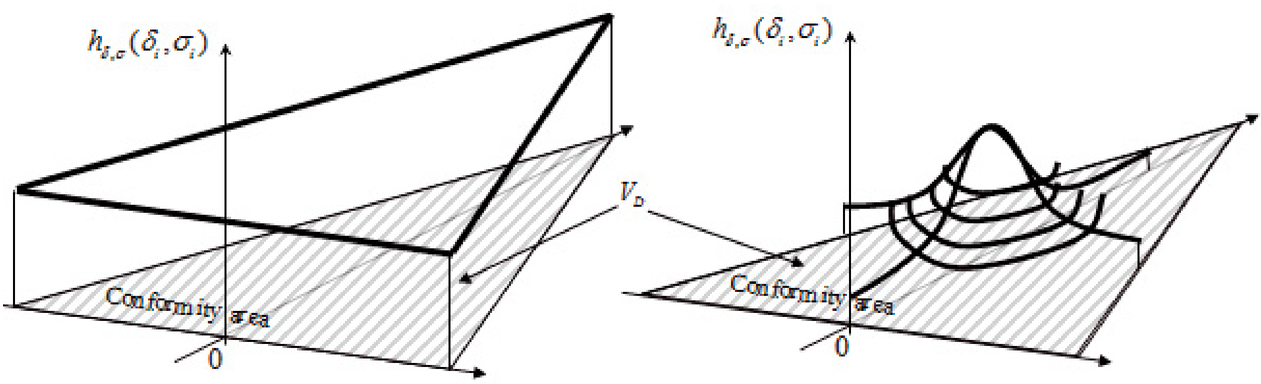

Fig. 8. Configuration where δ_i, σ_i are random. Assumption of a uniform distribution (left) over the whole conformity area, Gaussian distribution (right). The variability domain V_{D} is here the whole conformity area.

The probability measurement of event B_i is then:

$$\text{Prob}(B_i) = h_{\delta,\sigma}(\delta_i, \sigma_i)\,\mathrm{d}\delta_i\,\mathrm{d}\sigma_i \tag{4}$$

and:

$$\text{Prob}(B) = \prod_{i=1}^{n} h_{\delta,\sigma}(\delta_i, \sigma_i)\,\mathrm{d}\delta_i\,\mathrm{d}\sigma_i \tag{5}$$

with an assumption of batch parameter independence for two different dimensions. Then:

$$\text{Prob}(A|B) = P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i) \tag{6}$$

Consequently, using Bayes' theorem:

$$\text{Prob}(A \cap B) = \text{Prob}(A|B)\text{Prob}(B) \tag{7}$$

$$\text{Prob}(A \cap B) = P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i) \prod_{i=1}^{n} h_{\delta,\sigma}(\delta_i, \sigma_i)\,\mathrm{d}\delta_i\,\mathrm{d}\sigma_i \tag{8}$$

And finally, by extension to the whole domain V_{D}:

$$P_{\mathrm{D}} = \int_{V_{\mathrm{D}}} P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i) \prod_{i=1}^{n} h_{\delta,\sigma}(\delta_i, \sigma_i)\,\mathrm{d}\delta_i\,\mathrm{d}\sigma_i \tag{9}$$

The dimension of this integral is $2n$ (n being the number of dimensions in f). P_{D} is the expectation of $P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i)$ weighted by the $\prod h_{\delta,\sigma}(\delta_i, \sigma_i)$ product. It is the defect probability evaluated with the APTA method considering all the possible mean shifts and standard deviations with a joint density function defined by $h_{\delta,\sigma}(\delta_i, \sigma_i)$. All the manufactured batches are assumed to be in the conformity area and consequently out-of-tolerance batches are not considered in the formulation. Equation (9) is the basis of the proposed methodology. In addition, according to Equation (9), P_{D} can be bounded by the upper value of $P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i)$ when (δ_i, σ_i) vary inside the variability domain V_{D}:

$$P_{\mathrm{D}} \leq \max_{\delta,\sigma \in V_{\mathrm{D}}} P_{\mathrm{D}|\delta,\sigma}\,(\delta_i, \sigma_i) \tag{10}$$

This upper value, called $P_{\mathrm{D}}^{\mathrm{U}}$, is easier to compute than the whole integral defined in (9) and does not require any knowledge of $h_{\delta,\sigma}(\delta_i, \sigma_i)$.

To take advantage of the proposed methodology, preliminary statistical analyses must be performed in order to determine a suitable expression for $h_{\delta,\sigma}(\delta_i, \sigma_i)$. To do so, capability monitoring is necessary and knowledge must be accumulated to characterize $h_{\delta,\sigma}(\delta_i, \sigma_i)$ for each kind of process used in manufacturing. The use of this methodology requires an effort concerning production monitoring analysis. In this way, for a new product using parts manufactured with a well-known process, it is possible to use the knowledge of the old production process to validate assumptions about $h_{\delta,\sigma}(\delta_i, \sigma_i)$. Several expressions of $h_{\delta,\sigma}(\delta_i, \sigma_i)$ can be given, depending on the statistical analysis of a particular production device. The aim of the following sub-sections is to present some of the possibilities.

4.2 Application to a production device with fixed known values of $\delta_i = \delta_i^0, \sigma_i = \sigma_i^0$ (static assumptions)

For a production device with fixed known static values of $\delta_i = \delta_i^0, \sigma_i = \sigma_i^0$ (this is the case of all the proposals exposed in the bibliography review), that is to say that the characteristics of the manufactured batch are deterministic and do not change with time, the density function $h_{\delta,\sigma}(\delta_i, \sigma_i)$ takes the following form (Fig. 5 for a graphic illustration):

$$\begin{aligned} h_{\sigma,\delta}(\sigma_i, \delta_i) &= +\infty \quad \text{if} \quad \sigma_i = \sigma_i^0 \quad \text{et} \quad \delta_i = \delta_i^0 \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{11}$$

Equation (9) becomes:

$$P_{\mathrm{D}} = P_{\mathrm{D}|\delta,\sigma}\left(\delta_i^0, \sigma_i^0\right) \tag{12}$$

corresponding to the usual static approach. Depending on the values of (δ_i^0, σ_i^0), it can correspond to various assumptions found exposed in Section 3:

- non shifted Gaussian distribution: $\delta_i^0 = 0$ and $\sigma_i^0 = \frac{t_i}{6}$;
- shifted Gaussian distribution: $\delta_i^0 = \eta_i t_i/2$ and $\sigma_i^0 = \frac{t_i}{6}(1-\eta)$.

4.3 Application to a production device with variable δ_i and fixed standard deviation $\sigma_i = \sigma_i^0$

For a production device with a fixed known standard deviation $\sigma_i = \sigma_i^0$ and variable mean shift δ_i, the joint density function (reduced to a simple density function) can take the following expression (Fig. 6 for a graphic illustration):

$$\begin{aligned} h_{\sigma,\delta}(\sigma_i, \delta_i) &= h(\delta_i, \sigma_i^0) \quad \text{if} \quad \sigma_i = \sigma_i^0 \quad \text{and} \\ &\qquad \delta_i \in \left[-\delta_i^{(\max)}(\sigma_i^0);\ \delta_i^{(\max)}(\sigma_i^0)\right] \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{13}$$

$\delta_i^{(\max)}$ is the maximum allowable mean shift depending on σ_i^0. If the mean shifts are assumed to be uniformly distributed within $\left[-\delta_i^{(\max)}; \delta_i^{(\max)}\right]$, the density function becomes (Fig. 8 left):

$$\begin{aligned} h_{\sigma,\delta}(\sigma_i, \delta_i) &= \frac{1}{2\delta_i^{(\max)}(\sigma_i^0)} \quad \text{if} \quad \sigma_i = \sigma_i^0 \quad \text{and} \\ &\qquad \delta_i \in \left[-\delta_i^{(\max)}(\sigma_i^0); \delta_i^{(\max)}(\sigma_i^0)\right] \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{14}$$

If the mean shifts are assumed to have a truncated Gaussian distribution, the density function takes the following expression (Fig. 7, right):

$$\begin{aligned} h_{\sigma,\delta}(\sigma_i, \delta_i) &= \frac{K}{\sqrt{2\pi}\sigma_\delta} \exp\left(-\frac{\delta_i^2}{2\sigma_\delta^2}\right) \quad \text{if} \quad \sigma_i = \sigma_i^0 \quad \text{and} \\ &\qquad \delta_i \in \left[-\delta_i^{(\max)}(\sigma_i^0); \delta_i^{(\max)}(\sigma_i^0)\right] \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{15}$$

σ_δ is the standard deviation of δ_i; it has to be set from analyses of the production device that will be used for the production of X_i. The K parameter is here to norm the integral of $h_{\sigma,\delta}$ over $\mathbb{R}$ to 1 since $h_{\sigma,\delta}$ is truncated to V_{D}.

4.4 Application to a production device with random δ_i and σ_i

If the standard deviations and mean shifts can be considered simultaneously random, a joint density function $h_{\delta,\sigma}$ has to be set. Considering the case where the joint (δ_i, σ_i) density function is uniform (the most severe case), then (Fig. 7, left):

$$h_{\sigma,\delta}(\sigma_i,\delta_i) = \frac{1}{A_{\mathrm{V}}} \quad \text{if} \quad \delta_i, \sigma_i \in \text{Variability domain } V_{\mathrm{D}}$$
$$= 0 \quad \text{otherwise} \tag{16}$$

where A_{V} is the surface of the variability domain that can be the whole conformity area or a reduced domain included within the conformity area. If the standard deviations and mean shifts are independent, the joint density function can follow a truncated Gaussian distribution (Fig. 8 right):

$$h_{\sigma,\delta}(\sigma_i,\delta_i) = \frac{K}{2\pi\sigma_\sigma\sigma_\delta}\exp\left(-\frac{1}{2}\left(\frac{\sigma_i - m_\sigma}{\sigma_\sigma}\right)^2 -\frac{1}{2}\left(\frac{\delta_i - m_\delta}{\sigma_\delta}\right)^2\right) \quad \text{if} \quad \delta_i,\sigma_i \in V_{\mathrm{D}}$$
$$= 0 \quad \text{otherwise} \tag{17}$$

where m_σ, σ_σ are the mean value and standard deviation of σ_i and m_δ, σ_δ are the mean value and standard deviation of δ_i to be evaluated from a preliminary statistical analysis of the production device. As previously, the K parameter is here to norm the integral of $h_{\sigma,\delta}$ over $\mathbb{R}^2$ to 1 since $h_{\sigma,\delta}$ is truncated to V_{D}.

4.5 Numerical computation of P_{D}

Equation (9) represents the mathematical expectation of $P_{\mathrm{D}|\delta,\sigma}(\delta_i,\sigma_i)$ with respect to the joint density function of δ_i, σ_i. To reduce computation time, Equation (9) is assessed in two steps.

Computation of $P_{\mathrm{D}}|_{\delta,\sigma}(\delta_i,\sigma_i)$

For a linear f function, the evaluation of $P_{\mathrm{D}|\delta,\sigma}(\delta_i,\sigma_i)$ is performed using the analytical Equation (9). For a non-linear f, the evaluation of $P_{\mathrm{D}|\delta,\sigma}(\delta_i,\sigma_i)$ could be assessed by Monte Carlo simulation. In order to limit computation time (since $P_{D|\delta,\sigma}(\delta_i,\sigma_i)$ must be evaluated several times), the use of approximation algorithms is proposed. The SOTA algorithm requires a numerical time-consuming computation of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, which can be inapplicable when repeated many times. FORM appears more efficient and is therefore chosen here.

Numerical integration of Equation (9)

For the high dimensional numerical integration of $P_{\mathrm{D}|\delta,\sigma}(\delta_i,\sigma_i)$, a Monte Carlo scheme is proposed to compute Equation (9):

$$P_{\mathrm{D}} = \int_{\text{Conformity Area}} P_{\mathrm{D}|\delta,\sigma}(\delta_i,\sigma_i)\prod_{i=1}^{n} h_{\delta,\sigma}(\delta_i,\sigma_i)\,\mathrm{d}\delta_i\,\mathrm{d}\sigma_i$$
$$= E(P_{\mathrm{D}|\delta,\sigma}) \quad \text{Expectation operator}$$
$$\approx \tilde{P}_{\mathrm{D}} = \frac{1}{N}\sum_{k=1}^{N} P_{\mathrm{D}|\delta,\sigma}\left(\delta_i^{(k)},\sigma_i^{(k)}\right) \tag{18}$$

where $\delta_i^{(k)}, \sigma_i^{(k)}$ are random vectors simulated according to $h_{\delta,\sigma}$. The number N must be set according to the 95% confidence interval on P_{D}:

$$\tilde{P}_{\mathrm{D}} - \frac{1.96\sigma_{\mathrm{P}}}{\sqrt{N}} \le P_{\mathrm{D}} \le \tilde{P}_{\mathrm{D}} + \frac{1.96\sigma_{\mathrm{P}}}{\sqrt{N}} \tag{19}$$

where σ_{P} is the standard deviation of the $\tilde{P}_{\mathrm{D}}$ estimation defined by:

$$\sigma_{\mathrm{P}} = \tilde{P}_{\mathrm{D}}\sqrt{\frac{1-\tilde{P}_{\mathrm{D}}}{N\tilde{P}_{\mathrm{D}}}} \tag{20}$$

In the following, and particularly in the applications (Sects. 5, 6 and 7), N is set in order to reach a 95% confidence interval size lower than 1 ppm, that is to say:

$$3.92\frac{\sigma_{\mathrm{P}}}{\sqrt{N}} \le 10^{-6} \tag{21}$$

4.6 Extension of the APTA method in the case of the Taguchi and inertial capability indexes

The APTA method can be extended for other capability indexes based on the Taguchi loss function, such as the Taguchi capability index [5] or the inertial capability index [6]. For these two indexes, the shape of the conformity area in the δ, σ diagram is a circle centered on $\delta = 0$ and $\sigma = 0$. The joint density function $h_{\sigma,\delta}$ can take one of Equations (11) and (14)–(17) but truncation must occur in the circular conformity area.

4.7 Concluding remarks on the APTA methodology

The proposed mathematical expression of P_{D} (Eq. (9)) is the basis of the APTA method. It requires the knowledge of the production device of each dimension X_i in order to set the (δ_i,σ_i) joint probability density function. This can be achieved thanks to a statistical analysis of the evolution of (δ_i,σ_i) over time. This requires a lot of information to be gathered. If this information is not available, the upper bound $P_{\mathrm{D}}^{\mathrm{U}}$ can be computed knowing only the variability domain. All the results are highly dependent on the (δ_i,σ_i) joint density function assumption and this

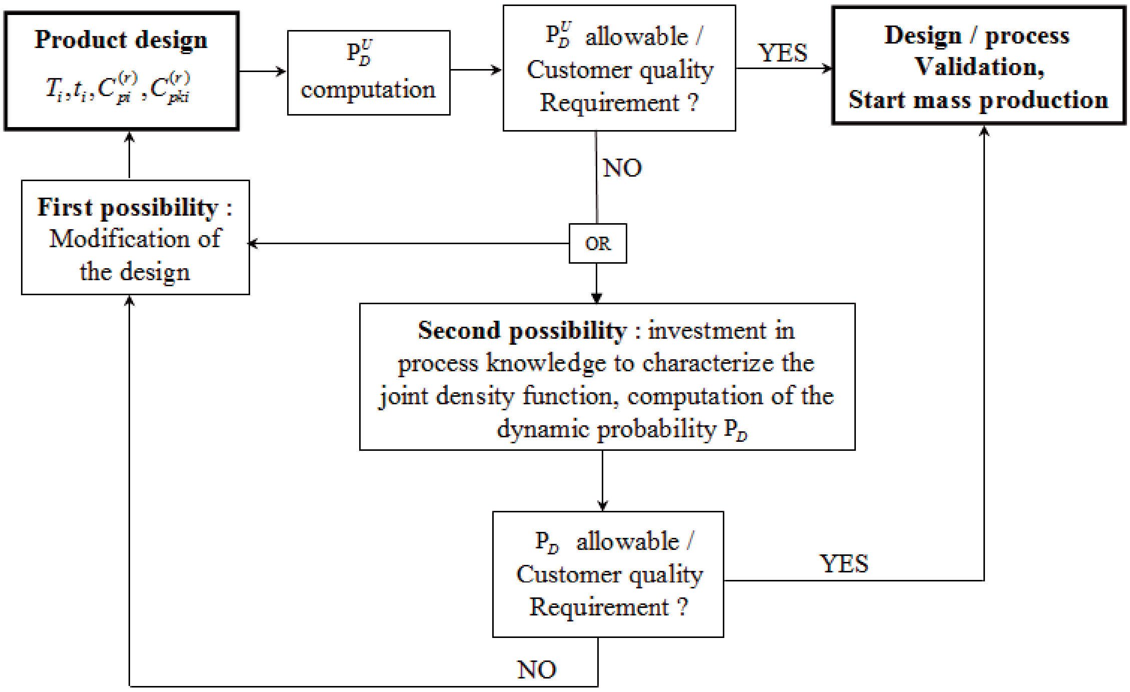

Fig. 9. Position of the APTA methodology in the design stage.

step is fundamental. A general synoptic (Fig. 9) is proposed to show how to use this methodology in the design stage. A preliminary defect probability can be assessed using the upper bound P_D^U. If the obtained value is in conformance with the customer quality requirement, the product/process coupling is validated, but with an uncontrolled margin. If not, or if an increase in part tolerances is necessary to optimize the product cost, knowledge of the joint density function $h_{\delta,\sigma}$ can lead to a better prediction of defect probability, enabling the product/process coupling to be validated or the part tolerances to be increased for the same customer quality requirement. If the customer quality requirement is not met, the choice of a new process with a more favourable $h_{\delta,\sigma}$ function with respect to P_D can also be considered.

5 Reference example application

The reference example defined in Section 2 is used to illustrate the applicability of the proposed methodology (see Fig. 1 and data in Sect. 2). To show the applicability of the APTA method to this reference application, several assumptions about $h_{\delta,\sigma}$ were tested and compared.

5.1 Static assumptions

X_1 and X_2 are considered to have fixed distribution parameters. Thus:

$$h_{\sigma,\delta}(\sigma_i, \delta_i) = +\infty \quad \text{for} \quad \sigma_i = \sigma_i^0 \quad \text{and} \quad \delta_i = \delta_i^0$$
$$= 0 \quad \text{otherwise} \tag{22}$$

σ_i^0 and δ_i^0 are supposed to be the same for each dimension (i.e. $\sigma_i^0 = \sigma_1^0 = \sigma_2^0$ and $\delta_i^0 = \delta_1^0 = \delta_2^0$), they are set here to explore the border of the conformity area. Results are given graphically in Figure 10 (left). Each black mark corresponds to a particular assumption about σ_i^0 and δ_i^0. σ_i^0 is characterized by an associated $C_{pi}^{(0)}$ capability value, that is to say:

$$\sigma_i^0 = \frac{t_i}{6C_{pi}^{(0)}} \tag{23}$$

The results show that static defect probability is more sensitive to mean shift than to standard deviation. Furthermore, the smaller the standard deviation, the higher the possible mean shift and static defect probability.

5.2 APTA method with uniform mean shift and fixed standard deviation

For this reference example, defect probability is computed according to the uniform density assumption:

$$h_{\sigma,\delta}(\sigma_i, \delta_i) = \frac{1}{2\delta_i^{(\max)}(\sigma_i^0)} \quad \text{if} \quad \sigma_i = \sigma_i^0 \quad \text{and}$$
$$\delta_i \in \left[-\delta_i^{(\max)}(\sigma_i^0); \delta_i^{(\max)}(\sigma_i^0)\right]$$
$$= 0 \quad \text{otherwise} \tag{24}$$

As previously, σ_i^0 is characterized by an associated $C_{pi}^{(0)}$ capability value. $\delta_i^{(\max)}$ corresponds to the maximum allowable mean shift depending on σ_i^0. The results are presented graphically in Figure 10 (right) according to the

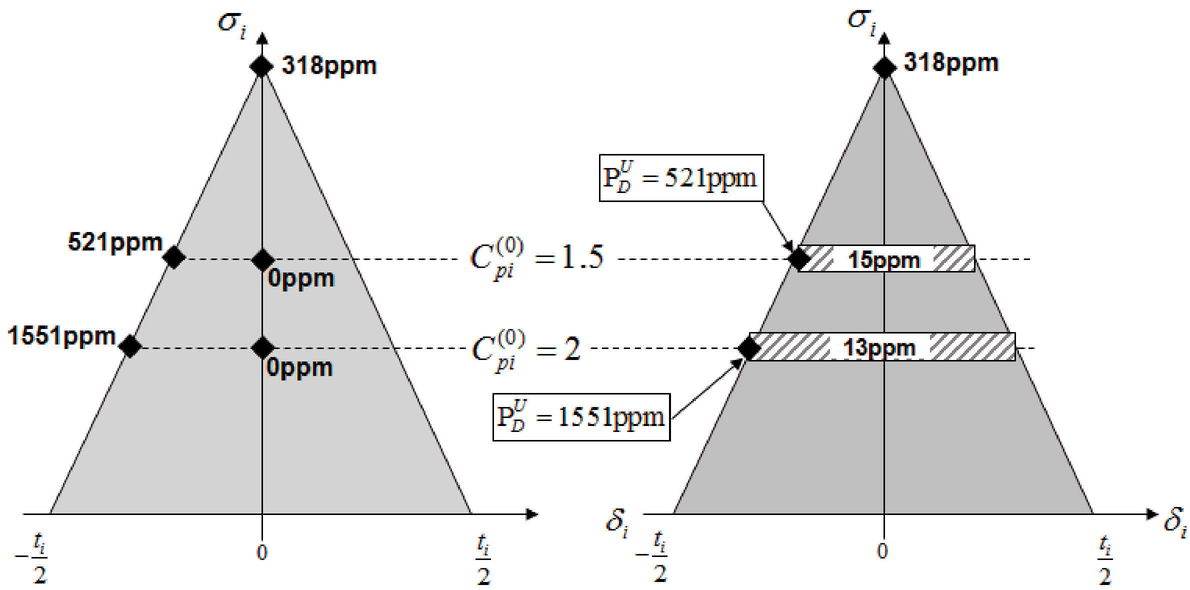

Fig. 10. Reference example – left: static defect probability considering different assumptions about δ_i, σ_i. Right: APTA defect probability considering different constant values of σ_i^0 and random values of δ_i within the hatched conformity area. The black marks on the right locate the δ_i, σ_i corresponding to the $P_{\mathrm{D}}^{\mathrm{U}}$ values.

value of $C_{pi}^{(0)}$. The results show that, contrary to the static assumption, the P_{D} values decrease when the $C_{pi}^{(0)}$ capability level is higher. $P_{\mathrm{D}}^{\mathrm{U}}$ evaluation of the upper bound of P_{D} over V_{D} can be computed and is plotted in Figure 10 (right) by black lozenge marks to locate the corresponding δ_i, σ_i values. This leads to overestimations of P_{D}. The value of $P_{\mathrm{D}}^{\mathrm{U}}$ is obtained for the highest possible mean shift within V_{D}.

5.3 APTA method with uniform mean shift and standard deviation

For this reference application, the density function is assumed to be the following:

$$h_{\sigma,\delta}(\sigma_i, \delta_i) = \frac{1}{A_{\mathrm{V}}} \quad \text{if} \quad \delta_i, \sigma_i \in \text{conformity area}$$
$$= 0 \quad \text{otherwise} \tag{25}$$

The variability domain V_{D} is here considered to be a space included in the conformity area. It is represented here by a triangle where $\sigma_i \in \left[\frac{t_i}{6C_{pi}^{(0)}}; \frac{t_i}{6C_{pi}^{(r)}}\right]$. In this case $C_{pi}^{(0)}$ represents the maximum capability of the variability domain. This domain is represented by the hatched area in Figure 11 for two different values of $C_{pi}^{(0)}$. The results with regard to $C_{pi}^{(0)}$ are presented graphically in Figure 11. $P_{\mathrm{D}}^{\mathrm{U}}$ evaluation of the upper bound of P_{D} can be computed (see the black lozenge marks in Fig. 11 to locate the corresponding δ_i, σ_i values). This leads to overestimations of P_{D}. As previously, the value of $P_{\mathrm{D}}^{\mathrm{U}}$ is obtained for the highest allowable mean shift within V_{D}.

5.4 Comparison with the bibliography

P_{D} computations are compared with literature proposals for the X_i distribution: uniformly distributed statistical model, non-shifted Gaussian distribution with $\sigma_i = t_i/6$, shifted Gaussian distribution with $\sigma_i = t_i/6$ and $\eta_i = 0.2$. The results are presented in Table 2. These results are quite different from those obtained with knowledge about the production device, that is to say when using the joint density function. For a customer quality requirement of 50 ppm, $P_{\mathrm{D}}^{\mathrm{U}}$ and all the literature proposals would lead to the rejection of the design even though a more precise analysis of the (δ, σ) distribution would lead to the acceptance of the part specifications.

5.5 Reference example conclusions

The predicted P_{D} greatly depends on assumptions about the statistical properties of X_i. For the same problem, results can vary from 0 ppm (case of centered distribution with weak standard deviation) to 1551 ppm (case with the most shifted distribution) for Gaussian distribution, to 18 000 ppm for worst case uniform distribution. Consequently, the study of the evolution of the statistical properties of each production batch will enable greater confidence concerning the predicted P_{D} evaluation. Without the knowledge of $h_{\sigma,\delta}(\sigma_i, \delta_i)$, an evaluation of an upper bound of P_{D} can be given knowing only the variability domain V_{D}. The $P_{\mathrm{D}}^{\mathrm{U}}$ probability is obtained from the smallest standard deviation allowing the highest mean shift.

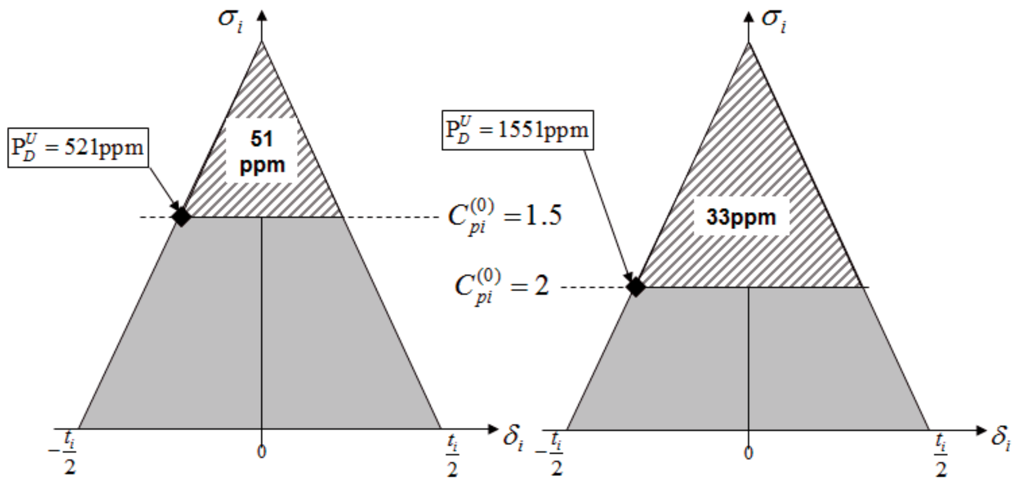

Fig. 11. Reference example – APTA defect probability considering different random values of σ_i and δ_i within the variability domain (hatched). The black marks locate the δ_i, σ_i values corresponding to the P_D^U values.

Table 2. Reference example – comparison of the proposed methodology with literature proposals.

Proposed methodology results		Bibliography results		
P_D	P_D^U	Uniformly distributed statistical model	Non-shifted Gaussian distribution with $\sigma_i = t_i/6$	Shifted Gaussian distribution with $C_{pki} = 1$ and $\eta_i = 0.2$
33 ppm	1551 ppm	18 000 ppm (95% conf. interval 220 ppm)	318 ppm	291 ppm

6 Linear industrial application: electrical pin contact length

6.1 Presentation of the study

The first industrial application is proposed by the RADIALL Company. The considered f function is composed of 17 dimensions linked together in a linear mathematical equation as follows:

$$Y = f(X_i) = \sum_{i=1}^{17} k_i X_i \quad k_i = \pm 1 \tag{26}$$

The k_i coefficients, target values, tolerances and capability requirements of each dimension are given in Table 3. The functional requirement is $Y \geq S$, $S = 1.75$ being a threshold value to be exceeded. This requirement corresponds to the minimum contact length of a metallic pin in its socket to enable good conductivity in an electrical circuit (Fig. 12).

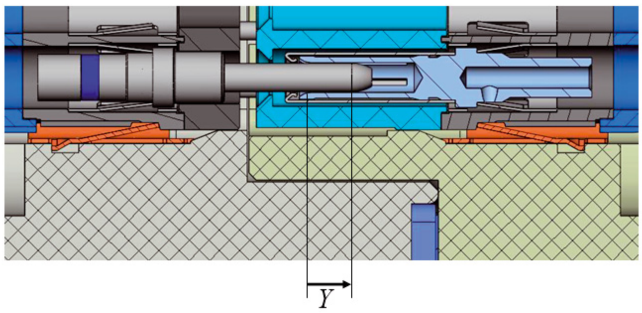

Fig. 12. Linear industrial application – contact length of an electrical pin.

6.2 (δ, σ) joint density function

The joint density functions are proposed by RADIALL from statistical analysis on process devices that are used in the manufacture of the electrical system. Of the 17 dimensions, 14 dimensions have uniform variable mean shift and standard deviation in a variability domain defined by a triangle in the δ, σ diagram (as in Fig. 11) and involving a $C_{pi}^{(0)}$ upper capability bound. The other 4 dimensions (#4, #5, #12, #13) are considered to be very difficult to manufacture within tolerance. Consequently each dimension is verified after fabrication and the only requirement is that each dimension fall within tolerance t_i. After statistical analysis, the dimension is variable over the whole tolerance range and a uniform distribution of X_i within t_i appears to be the best choice. In this context, for dimensions #4, #5, #12 and #13, σ_i is constant and set to zero as if each production batch was constituted of only one part and the mean shift can uniformly vary within the whole tolerance range.

Table 3. Linear industrial application – dimension details.

Dim.	k_i	T_i	t_i	$C_{pki}^{(r)} = C_{pi}^{(r)}$	(δ, σ) Density function	$C_{pi}^{(0)}$
1	–1	0.75	0.2	1.8	σ, δ uniformly variable in the variability domain bounded by $C_{pi}^{(0)}$	2.6
2	1	10.71	0.16	1.8		2.6
3	1	3.75	0.1	1.8		2.6
4	–1	5.125	0.05	*No requirement*	$\sigma_i = 0$ and $\delta_i \in [-t_i/2\ ;\ t_i/2]$	–
5	–1	3.125	0.05	*No requirement*		–
6	1	4.505	0.03	1.7	As dim. 1, 2, 3	1.94
7	1	5.985	0.09	1.8		2.8
8	–1	21.535	0.11	0.38		1.32
9	–1	8.195	0.11	0.38		1.32
10	1	5.985	0.09	1.8		2.8
11	1	12.305	0.03	1.7		1.94
12	–1	10.325	0.05	*No requirement*	As dim. 4,5	–
13	–1	5.125	0.05	*No requirement*		–
14	1	3.75	0.1	1.8	As dim. 1, 2, 3	2.6
15	1	0.825	0.05	1.05		1.85
16	1	8.8	0.1	1.8		2.6
17	–1	0.4	0.2	1.8		2.6

6.3 APTA results, comparison with bibliography and conclusion

APTA defect probability is computed according to the assumptions about the δ, σ density functions. The results are presented in Table 4. For comparison, $P_{\mathrm{D}}^{\mathrm{U}}$ is computed, producing a significant overestimation of P_{D}. P_{D} computations are compared with literature proposals for X_i distribution: uniform distributions within t_i, non shifted Gaussian distribution with $\sigma_i = t_i/6$ and shifted Gaussian distribution with $\eta = 0.2$. The results can be compared in Table 4 for $S = 1.75$. Defect probability predictions are quite different. With no knowledge of either manufacturing process or δ, σ density function, the upper bound $P_{\mathrm{D}}^{\mathrm{U}}$ does not require the joint density function, but it is not interesting in this case, since it leads to a very high overestimation. In this application, literature proposals give overestimations of defect probability. For a customer quality requirement of 10 ppm, $P_{\mathrm{D}}^{\mathrm{U}}$ and all the bibliography examples would lead to the design's being rejected, even if a more precise analysis of (δ, σ) distribution would give rise to a relevant design. In addition, Figure 13 provides the sensitivity of P_{D} and $P_{\mathrm{D}}^{\mathrm{U}}$ on a logarithmic scale with respect to S. It represents the cumulative probability function of Y $\quad F_Y(S) = \mathrm{Prob}(Y \leq S)$ in the cases of the upper bound and of the APTA computation.

7 Non-linear industrial application: contact clearance

7.1 Presentation of the study

This application concerns a non-linear industrial case provided by the RADIALL Company. The f function is composed of 14 dimensions (see characteristics in Tab. 5) in a non-linear analytical expression including basic mathematical functions: quotient, trigonometry, etc. It concerns the maximum amplitude Y of the extremity of an electrical pin (Fig. 14) possibly leading to a problem of non-assembly. The amplitude Y is function of the dimensions of parts 1 and 2. Its mathematical expression is provided hereafter:

$$\begin{aligned}
c &= \frac{cm_4 + cm_8 + cm_7}{2} \\
i &= \frac{ima_{17} + ima_{19} + ima_{20}}{2} \\
h &= ima_2 + ima_3 \\
\alpha &= \arccos\left(\frac{c}{\sqrt{i^2 + h^2}}\right) - \arccos\left(\frac{i}{\sqrt{i^2 + h^2}}\right) \\
r_2 &= \frac{\frac{ima_{19} + ima_{20}/2}{2} - \frac{cm_8 + cm_7/2}{2\cos(\alpha)}}{\tan(\alpha)} \\
z &= \frac{r_2}{\cos(\alpha)} + ((cm_9 + cm_{10}/2)/2 + cm_7/4)\tan(\alpha) \\
J_1 &= (cm_1 - cm_3 - z)\sin(\alpha) \\
J_2 &= \frac{cm_7}{4}\cos(\alpha) \\
J_3 &= \frac{cm_5 + cm_6}{2}\cos(\alpha) \\
Y &= J_1 + J_2 + J_3
\end{aligned} \tag{27}$$

This relation is simply obtained from geometrical considerations. The contact points between each part are assumed to be the same whatever the part dimensions (Fig. 14). The functional requirement imposes that $Y < S$, with $S = 0.52$.

7.2 (δ, σ) joint density function

The density functions proposed in Table 5 were supplied by RADIALL from statistical analyses on process

Table 4. Linear industrial application – Defect probability computations, comparison with literature proposals for $S = 1.75$.

Proposed methology results		Bibliography results		
P_{D}	$P_{\mathrm{D}}^{\mathrm{U}}$	Uniformly distributed statistical model	Non-shifted Gaussian distribution with $\sigma_i = t_i/6$	Shifted Gaussian distribution with $C_{pki} = C_{pki}^{(r)}$ and $\eta_i = 0.2$
8 ppm	987 732 ppm	7698 ppm (95% conf. interval 460 ppm)	44 ppm	216 140 ppm

Table 5. Non-linear industrial application – dimension details.

Dim.	T_i	t_i	$C_{pi}^{(r)} = C_{pki}^{(r)}$	(δ, σ) Density function	$C_{pi}^{(0)}$
$cm1$	10.53	0.2	1.1		1.6
$cm3$	0.75	0.04	1.1		1.6
$cm4$	0.643	0.015	1.1		1.6
$cm5$	0.1	0.2	1.1	σ, δ	1.6
$cm6$	0	0.06	1.1	uniformly	1.6
$cm7$	0	0.2	1.1	variable	1.6
$cm8$	0.72	0.04	1.1	in the	1.6
$cm9$	1.325	0.05	1.1	variability	1.6
$cm10$	0	0.04	1.1	domain	1.6
$ima2$	3.02	0.06	0.86	bounded by	1.36
$ima3$	0.4	0.06	0.86	$C_{pi}^{(0)}$	1.36
$ima17$	0.72	0.04	0.86		1.36
$ima19$	0.97	0.04	0.86		1.36
$ima20$	0	0.04	0.86		1.36

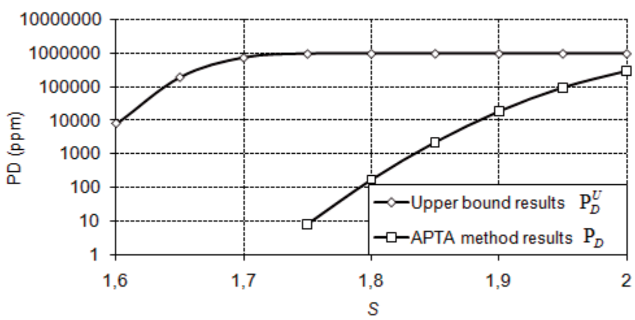

Fig. 13. Linear industrial application – sensitivity of the defect probability regarding S.

devices that are involved in the manufacturing of the electrical system. All dimensions have uniform variable mean shift and standard deviation in a variability domain defined by a triangle in the δ, σ diagram (as in Fig. 11) and involving a $C_{pi}^{(0)}$ upper capability bound mentioned in Table 5.

7.3 APTA results, comparison with bibliography and conclusion

For this application, since the f function has a non-linear expression, $P_{\mathrm{D}|\delta,\sigma}(\delta_i, \sigma_i)$ is evaluated using the FORM algorithm. The APTA evaluation of P_{D} consequently requires several FORM computations according to the (δ, σ) density function. Results are shown in

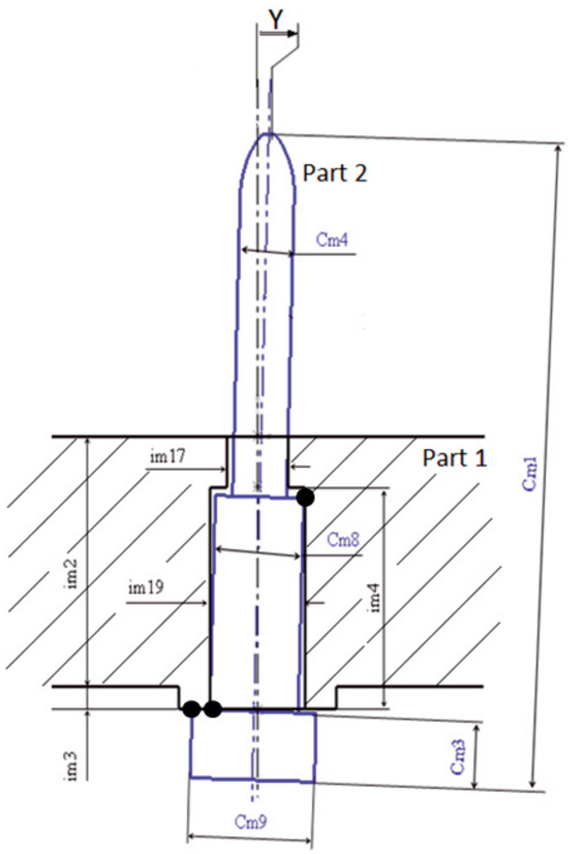

Fig. 14. Non-linear industrial case, contact points in black circles.

Table 6. Non-linear industrial application – comparison with literature proposal for $S = 0.52$.

Proposed methodology results		Bibliography results		
P_D	P_D^U	Uniformly distributed statistical model	Non-shifted Gaussian distribution with $\sigma_i = t_i/6$	Shifted Gaussian distribution With $C_{pki} = C_{pki}^{(r)}$ and $\eta_i = 0.2$
26 ppm	43 855 ppm	8629 ppm (conf. interval 470 ppm)	300 ppm	3994 ppm

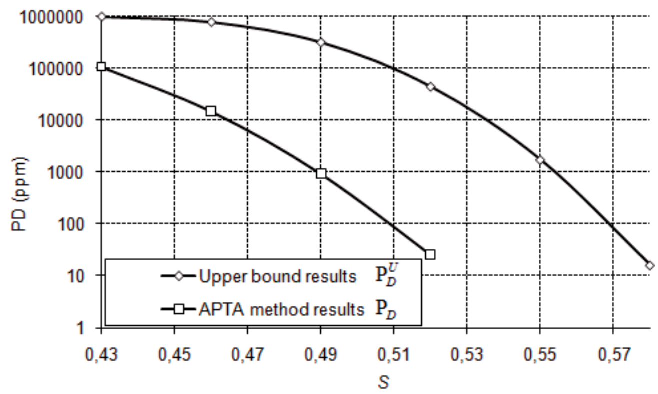

Fig. 15. Non-linear industrial case – sensitivity of defect probability with respect to S.

Table 6. P_D computations are compared with literature proposals: the defect probability evaluations are quite different. A Monte Carlo simulation of 5×10^6 samples is used for the uniform distribution. Without knowledge concerning the δ, σ density functions, the upper bound P_D^U gives a high overestimation of defect probability. The bibliography models also provide overestimations of defect probability. For a customer quality requirement of 30 ppm, P_D^U and all the bibliography examples would lead to the rejection of the design, even if a more precise analysis of (δ, σ) distribution would conclude that the design is relevant. This underlines the interest of having knowledge about the joint density function $h_{\delta,\sigma}$. In addition, Figure 15 provides the sensitivity of P_D and P_D^U on a logarithmic scale with respect to S. It represents the complementary cumulative probability function of Y $F_Y(S) = 1 - \text{Prob}(Y \leq S)$ in the cases of the upper bound and of the APTA computation.

8 Conclusion

The APTA method is implemented to assess the defect probability of an assembled product during the design stage. A general synoptic is proposed in Figure 9. It is based on the knowledge of the production devices that will be used, taking into account the future production batch parameter (δ, σ) variations in the prediction of P_D. The characterization of the (δ, σ) joint density function for dimensions of new parts is not trivial, and requires the capitalization of the joint density function of similar existing part dimensions (same kind of part, same process, same supplier, ...). This characterization enables an accurate tolerance analysis of the mechanical product and can:

- validate the use of a production device with its (δ, σ) parameters;
- demonstrate the conformance of the design to the customer quality requirements;
- manage the best compromise between production wastage and very narrow and expensive part tolerances in an optimization scheme;
- test the sensitivity of the P_D defect probability using finite differences with respect to the increase in a particular part tolerance.

This defect probability approach can be used whatever the type of dimensional chain (linear or not). It is also possible to use these methods in three-dimensional chains with non-explicit f functions produced by complex CAD software, like in [1]. For this kind of f function, the methods used in reliability analysis to build a dialogue between FORM algorithms and any other mechanical scheme [20] could be explored. Moreover, this approach seems to be able to take into account multi-modal (δ, σ) density functions that can come from multi-cavity tools or from parts manufactured by several suppliers.

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