Mechanics
& Industry

Vibratory diagnosis by finite element model updating and operational modal analysis

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Abstract – In this paper, a subspace fitting method is proposed to update, in the time domain, the finite element model of a rotating machine. The procedure is achieved by minimizing an error norm, leading to the comparison between experimental and theoretical observability matrices. Experimental observability matrix is obtained through a MOESP subspace identification algorithm, by projecting the output signal onto some appropriate subspaces, resulting in a cancellation of input excitations and noises. The theoretical observability matrix is obtained from modal parameters of a finite element model of the structure. The minimization procedure is carried out through a Gauss-Newton algorithm. The method is applied to determine the foundation stiffness of an experimental rotating machine subject to a random noise.

Key words: Subspace fitting / operational modal analysis / finite elements model updating / vibratory diagnosis / rotating machine

Nomenclature

Ω	Angular velocity (rad.s^{-1})
ρ	Density (kg.m^{-3})
E	Young modulus of the shaft (N.m^{-2})
G	Shear modulus of the shaft (N.m^{-2})
I	Inertia moment of the shaft (kg.m^2)
$I_{Dy,z}$	Inertia moments of the disk (kg.m^2)
k_B	Bearing stiffness (N.m^{-1})
L	Length of the shaft (m)
m_D	Mass of the disk (kg)
S	Cross-sectional area of the shaft (m^2)

1 Introduction

Evaluating damages occurring in mechanical systems constitutes a tough task. Their emergence and evolution are characterized by variations (those can be small) of the dynamic properties of structures [1]. Many damage diagnosis methods have been proposed to carry out this issue. The methods based on Finite Element (FE)

model updating [2] perform the comparison between the modal parameters obtained experimentally with those of a numerical model. For industrial processes, Operational Modal Analysis (OMA) [3] approaches aim at extracting the structural parameters in operating conditions. Among these approaches, Subspace Identification (SubID) techniques [4–6] appear highly efficient to determine the modal parameters of structures in the time domain. The framework of SI techniques is summarized as follows. From the consideration of input-output data, a so-called experimental observability matrix is obtained by projecting the output signal onto some appropriate subspaces. The observability matrix contains the modal parameters of the structure considered, which are extracted using a subspace fitting method [7, 8]. The motivation behind this work is to improve further on the accuracy of SI techniques to predict the modal parameters of any mechanical system. To this aim, a FE model of the structure is considered to carry out the subspace fitting procedure in a Least Squares (LS) sense. The modal parameters of the system are then updated by minimizing an error norm which depends on some unknown parameters of the structure (e.g., stiffnesses...). The proposed approach is applied to a rotating machine excited by a random noise. The experimental observability matrix, as obtained using the MOESP SubID technique, is used to update the parameters of the structure. The accuracy of the method is highlighted.

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2 Deterministic-stochastic modal analysis

The purpose of SI techniques [5] is to consider a discrete modal state-space representation of the form

$$q_{k+1} = \Lambda q_k + B_{\mathrm{mod}} u_k + w_k \qquad (1)$$

$$y_k = \Phi_{\mathrm{obs}} q_k + v_k \qquad (2)$$

where u_k and y_k are the vectors of input and output data, respectively; Λ is a diagonal matrix of eigenvalues; B_{mod} and Φ_{obs} are matrices expressed in terms of the mode shapes of the structure; also, w_k and v_k are vectors of noises while q_k is a vector of generalized coordinates. In (1), the subscripts k and $k+1$ refer to vectors of data measured at two different times t_k and $t_{k+1} = t_k + \Delta t$. From the linear state space model, an input-output matrix equation is derived as [6]

$$Y = \Gamma_{\mathrm{exp}} Q + H^d U + H^s R + N \qquad (3)$$

where Y is a matrix of output data that are measured over different time intervals $\{[t_k t_{k+1} \ldots t_{k+\alpha-1}]\}_k$. Q, H R and N are the related matrices of generalized coordinates, input data and noises; also, Γ_{exp} represents the experimental observability matrix; otherwise, H^d and H^s are Hankel matrices. The basic idea behind SubID techniques is to identify the experimental observability matrix Γ_{exp} from the knowledge of Y. This is done by eliminating the terms $H^d U + H^s R + N$ in (3) by means of projection and weighting procedures. Clearly, a projection of the row space of Y onto the orthogonal complement $U^\perp$ of the row space of U enables one to remove the influence of inputs. In addition, the fact to left and right multiply (3) with some matrices W_1 and W_2 having some specific properties regarding noise uncorrelation enables those noise terms to be removed as well. Considering such procedures yields

$$O = W_1 Y / U^\perp W_2 = W_1 \Gamma_{\mathrm{exp}} Q / U^\perp W_2 \qquad (4)$$

More specifically, SI techniques deal with the matrix $W_1 \Gamma_{\mathrm{exp}} = U_1 S_1^{1/2}$ [5], where the matrix S_1 results from a SVD of O, i.e.

$$O = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & \simeq 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \qquad (5)$$

The matrices of eigenvalues Λ and mode shapes Φ_{obs} are determined from the experimental observability matrix Γ_{exp} in different ways [6]. All the methods make use of the invariance property of the matrix Γ_{exp}.

3 Subspace fitting for finite element model updating

3.1 Objective function

The subspace fitting procedure [7,8] is a concept that aims at correlating a theoretical matrix $\Gamma(\theta)$ with the experimental observability matrix Γ_{exp} as

$$\Gamma_{\mathrm{exp}} = \Gamma(\theta) T \qquad (6)$$

where T is a similarity matrix. Here, the matrix $\Gamma(\theta)$ is supposed to be dependent from a set of parameters (denoted as θ) which are to be identified. The subspace fitting procedure can be formulated through the following LS problem:

$$\{\theta, T\} = \mathrm{argmin} ||\Gamma_{\mathrm{exp}} - \Gamma(\theta) T||_F^2 \qquad (7)$$

where $||.||_F$ denotes the Frobenius norm. This LS problem can be simplified by determining the matrix T in a preprocessing step as $T = \Gamma(\theta)^+ \Gamma_{\mathrm{exp}}$ ($\Gamma(\theta)^+$ being the pseudo-inverse of $\Gamma(\theta)$), which yields

$$\theta = \mathrm{argmin} ||r(\theta)||_2^2 \qquad (8)$$

where $r(\theta) = \mathrm{vec}\{(I - \Gamma(\theta)\Gamma(\theta)^+)\Gamma_{\mathrm{exp}}\}$.

The key idea behind the present work is to express the theoretical observability matrix $\Gamma(\theta)$ by means of a FE model of the considered mechanical system. In doing so, the spatial dynamics of the system is taken into account to carry out the minimization procedure of $r(\theta)$ with a view to identifying the parameters θ in an accurate and unique way.

Regarding rotating machines, a related FE based eigenproblem is considered as

$$\left(\mu_j^2 M + \mu_j(\gamma + \Omega G) + K \right) \Phi_j = 0 \qquad (9)$$

where M, K and γ refer to the mass, stiffness and damping matrices; also, G is the matrix that reflects the gyroscopic effects. The solutions of the eigenproblem are $\{\mu_j, \Phi_j\}$ which stand for complex eigenvalues and right eigenvectors, respectively. Thus a theoretical FE-based observability matrix can be expressed as

$$\Gamma(\theta) = \begin{bmatrix} \Phi_{\mathrm{obs}} \\ \Phi_{\mathrm{obs}} \Lambda \\ \vdots \\ \Phi_{\mathrm{obs}} \Lambda^{\alpha-1} \end{bmatrix} \qquad (10)$$

where $\Lambda = \mathrm{e}^{\Delta t \, \mathrm{diag}(\mu_i)}$ and Φ_{obs} is the matrix of eigenvectors $\{\Phi_j\}$ at the observation points of output signals.

3.2 Optimization algorithm

The Gauss-Newton algorithm [9] is used to solve the minimization problem (9). This algorithm is based on the following iterative scheme

$$\theta_{f+1} = \theta_f - \beta_f H^{-1} g \qquad (11)$$

where β_f is a step size, while g and H are Gradient and Hessian matrices of $||r(\theta)||_2^2$, defined as

$$g_i = 2\mathrm{Re}\left\{ r^H \frac{\partial r}{\partial \theta_i} \right\} \qquad (12)$$

and

$$H_{ij} = 2\mathrm{Re}\left\{ \frac{\partial r^H}{\partial \theta_i} \frac{\partial r}{\partial \theta_j} \right\} \qquad (13)$$

$$\boldsymbol{K}_{\mathrm{S}} = \frac{EI}{(1+a)L^3} \begin{bmatrix} 12 & 0 & 0 & 6L & -12 & 0 & 0 & 6L \\ & 12 & -6L & 0 & 0 & -12 & -6L & 0 \\ & & 4L^2+a & 0 & 0 & 6L & 2L^2-a & 0 \\ & & & 4L^2+a & -6L & 0 & 0 & 2L^2-a \\ & & & & 12 & 0 & 0 & -6L^2 \\ & \text{sym} & & & & 12 & 6L & 0 \\ & & & & & & 4L^2+a & 0 \\ & & & & & & & 4L^2+a \end{bmatrix} \tag{18}$$
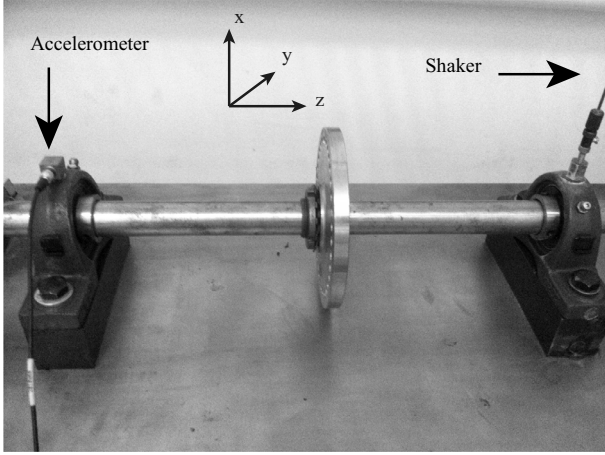

Fig. 1. Experimental structure.

Table 1. Properties of the structure.

	Geometrical proprieties	Physical proprieties
	Length $= 0.50$ m	$E = 193 \times 10^9$ N.m^{-2}
Shaft	Diameter $= 0.0254$ m	$G = 74.2 \times 10^9$ N.m^{-2}
		$\rho = 7818$ kg.m^{-3}
Disk	Thickness $= 0.01$ m	$\rho = 7818$ kg.m^{-3}
	Diameter $= 0.13$ m	

4 Experiments

4.1 Description of the structure

The proposed method is applied to update the FE model of the rotating machine depicted in Figure 1. The structure is composed of a shaft attached to one rigid disk and supported by two flexible bearings. The properties of the structure are reported in Table 1. The output signal of the structure is measured by means of one accelerometer which is attached to the first bearing. On the second bearing, a shaker generates a random noise. The experimental observability matrix is obtained using the MOESP SubID technique [6]. Some of identified experimental eigenfrequencies are reported in Table 2.

4.2 FE model of the structure

A FE model is considered which is composed of six Timoshenko beam elements for the shaft, with two

translations (u and v) and two rotations (ψ and ϕ) per node, along the x and y-axis.

The nodal displacement vector is denoted as

$$\boldsymbol{\delta} = \left\{ u \ v \ \phi \ \psi \right\}^T \tag{14}$$

The mass matrices $\boldsymbol{M}_{\mathrm{S}}$ and $\boldsymbol{M}_{\mathrm{S}}^r$ of a beam element, related to translational and rotational displacements, are respectively expressed as

$$\boldsymbol{M}_{\mathrm{S}} = \frac{\rho SL}{420} \begin{bmatrix} 156 & 0 & 0 & 22L & 54 & 0 & 0 & -13L \\ & 156 & -22L & 0 & 0 & 54 & 13L & 0 \\ & & 4L^2 & 0 & 0 & -13L & -3L^2 & 0 \\ & & & 4L^2 & 13L & 0 & 0 & -3L^2 \\ & & & & 156 & 0 & 0 & -22L \\ & \text{sym} & & & & 156 & 22L & 0 \\ & & & & & & 4L^2 & 0 \\ & & & & & & & 4L^2 \end{bmatrix} \tag{15}$$

and

$$\boldsymbol{M}_{\mathrm{S}}^r = \frac{\rho I}{30L} \begin{bmatrix} 36 & 0 & 0 & 3L & -36 & 0 & 0 & 3L \\ & 36 & -3L & 0 & 0 & -36 & -3L & 0 \\ & & 4L^2 & 0 & 0 & 3L & -L^2 & 0 \\ & & & 4L^2 & -3L & 0 & 0 & -L^2 \\ & & & & 36 & 0 & 0 & -3L \\ & \text{sym} & & & & 36 & 3L & 0 \\ & & & & & & 4L^2 & 0 \\ & & & & & & & 4L^2 \end{bmatrix} \tag{16}$$

Also, the related matrix of gyroscopic effects is given by

$$\boldsymbol{G}_{\mathrm{S}} = \frac{\rho I}{15L} \begin{bmatrix} 0 & -36 & 3L & 0 & 0 & 36 & 3L & 0 \\ & 0 & 0 & 3L & -36 & 0 & 0 & 3L \\ & & 0 & 4L^2 & 3L & 0 & 0 & L^2 \\ & & & 0 & 0 & 3L & L^2 & 0 \\ & & & & 0 & 36L & 3L & 0 \\ & \text{skew} - \text{sym} & & & & 0 & 0 & 3L \\ & & & & & & 0 & 4L^2 \\ & & & & & & & 0 \end{bmatrix} \tag{17}$$

The element stiffness matrix is given by

see equation (18) *above*

where $a = \frac{12EI}{GSL^2}$ is introduced to take into account the shear deformation effects. Finally, the whole equation of

Table 2. Eigenfrequencies of the system (rotating machine – foundation).

	Experimental frequency (Hz)	Before updating		After updating	
		Frequency (Hz)	Error (%)	Frequency (Hz)	Error (%)
1	631	394.7	37	635.8	0.8
2	884	759.5	14.1	878.9	0.6
3	1551	1294.3	16.6	1552.1	0.1
4	2399	2064.6	13.9	2284.9	4.8
5	3001	2606.1	13.2	2962.9	1.3

motion for the shaft element is expressed as

$$(M_S + M_S^r) \begin{Bmatrix} \ddot{\delta}_1 \\ \ddot{\delta}_2 \end{Bmatrix} + \Omega G_S \begin{Bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \end{Bmatrix} + K_S \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = 0 \quad (19)$$

where structural damping is neglected.

Otherwise, the disk is modeled by means of concentrated mass and gyroscopic effects using the following matrix term:

$$M_D \ddot{\delta} + \Omega G_D \dot{\delta} = \begin{bmatrix} m_D & 0 & 0 & 0 \\ 0 & m_D & 0 & 0 \\ 0 & 0 & I_{Dx} & 0 \\ 0 & 0 & 0 & I_{Dx} \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{\psi} \\ \ddot{\phi} \end{bmatrix}$$

$$+ \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_{Dz} \\ 0 & 0 & I_{Dz} & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} \quad (20)$$

Finally, the foundation of the bearings is modeled by means of the following stiffness matrix

$$K_B = \begin{bmatrix} k_B & 0 & 0 & 0 \\ 0 & k_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

4.3 Updating procedure

The updating procedure of the FE model is carried out considering the bearing stiffness k_B as an unknown parameter. This parameter is updated through the subspace fitting procedure described previously, whose flowchart is postponed in Figure 2.

The procedure is initialized with a value of 10×10^6 N.m^{-1} for k_B. The eigenfrequencies of the system (rotating machine – foundation), obtained for this value, are reported in Table 2 and compared with the experimental eigenfrequencies. Thus the updating procedure is carried out which yields a value of 33×10^6 N.m^{-1} for k_B. In that case, the errors between FE-based and experimental eigenfrequencies appear quite small, as expected. In a more general view, the updating procedure can be used to determine the variation of the stiffness parameter k_B over the time domain to carry out structural health monitoring. This yields an efficient way to detect the occurrence of defects as well as machine breakdowns.
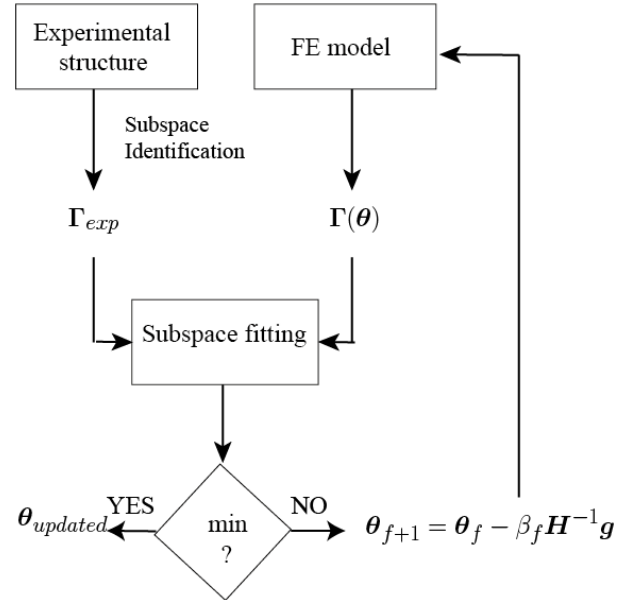
Fig. 2. Flowchart of the subspace fitting procedure.

5 Conclusion

A finite element model updating procedure has been proposed. The procedure used a subspace fitting approach to adjust a FE-based observability matrix with an experimental observability matrix obtained from a subspace identification technique. The method has been successfully applied to determine the bearing stiffness of a rotating machine. In a more general view, this method seems to constitute an efficient tool to carry out the structural health monitoring of mechanical systems.

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