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Cost optimization of reliability testing by a Bayesian approach

S. Beleulmi[1,a], A. Bellaouar[2] and M. Lachi[3]

[1] Laboratoire ingénierie des transports et environnement Département de Génie Mécanique Faculté des sciences de la technologie Université Constantine 1 Constantine, Algeria
[2] Laboratoire ingénierie des transports et environnement, Faculté des sciences de la technologie Université Constantine 1Constantine, Algeria
[3] Laboratoire GRESPI, UFR Sciences, Université de Reims Champagne Ardenne, Reims, France

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Abstract – The Bayesian approach is a stochastic method, allowing to establish trend studies on the behavior of materials between two periods or after a break in the life of these materials. It naturally integrates the inclusion of the information partially uncertain to support in modeling problem. The method is therefore particularly suitable for the analysis of the reliability tests, especially for equipment and organs whose different tests are costly. Bayesian techniques are used to reduce the size of estimation tests, improving the evaluation of the parameters of product reliability by the integration of the past (data available on the product concerned) and process, the case "zero" failure observed, difficult to treat with conventional statistical approach. This study will concern the reduction in the number of tests on electronic or mechanical components installed in a mechanical lift knowing their a priori behavior in order to determine their a posteriori behavior.

Key words: Bayesian approach / reliability / optimisation / failure rate / accelerated tests

1 Introduction

Tests for estimating the reliability are to assess the reliability phase of design and production. Determining the parameters of a reliability law for a system requires knowledge of the failure time of a sample of n size of systems in sufficient number. But, in the case of a very reliable system, it will be necessary to wait very long to get all the failure time. In an industrial application, it is inconceivable to have important test duration. Thus, among the tests that estimate the reliability; one finds the accelerated and Bayesian tests.

Reliability is defined as the probability that a system performs its function in a given period and under given operating. It has become an essential element for safety and performance issues in companies. In order to improve the reliability of industrial equipment, as well as its availability, one solution is to carefully analyze the history of their behavior over time, in an operational situation. These are feedback experience analyses called FEX. Besides the difficulty in obtaining reliable information on the readiness concept, one must also know how to handle data, so how to choose the appropriate statistical method. Among these existing techniques, the Bayesian statistic techniques (using Bayes' theorem), are interesting and make possible the combination of these different types of information.

Bayesian techniques [1], are used to reduce the size of estimation tests, to improve the parameters estimation of product reliability by the integration of the past (data available on the product concerned) and handle the case "zero" failure observed, difficult to treat with conventional statistical approach which would require a large sample size. There are many works dealing with Bayesian methods. The following fundamental project works [2–8] are related to the reliability applications: [9–12]. In a Bayesian test plan, the test results carried out during the development cycle are combined with a reliability model "a priori", to obtain a model "a posteriori", by means of Bayes theorem. The prior model is constructed using known information acquired before the implementation of the first test (expert opinion, information from previous studies such as predictive analytics, FMEA...).

The reliability standard testing is intended to provide samples to estimate reliability characteristics (parameters of a law of life, failure rates, etc.). The information contained in these samples is called objective information. The Bayesian methods may be given in a dynamic dimension. This characteristic allows the modeling of complex systems reliability for optimizing maintenance strategies,

[a] Corresponding author: Salima_b05@Yahoo.fr

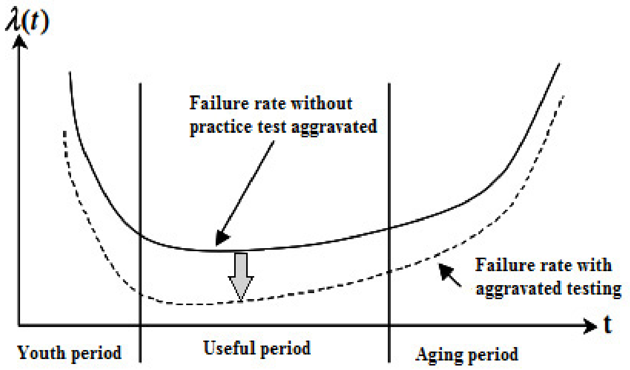

Fig. 1. Failure rates in compounded tests.

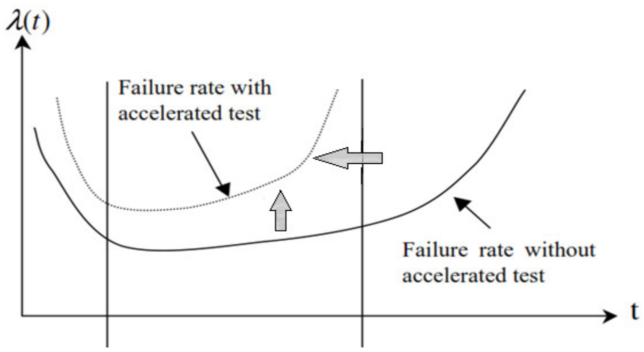

Fig. 2. Failure rates in accelerated tests.

and the simulation of the behavior evolution of a system in a prognostic approach.

2 Failure rates

The failure rate is a reliability estimator ratio often noted λ(t) and represents a proportion of devices still operational at time t. It should not take into account the intrinsic defects. At a certain threshold λ, the device is not in use. It is then downgraded, then repelled and sometimes repaired. The decision of its reform or renewal, results from a techno-economic study. Failure rates are variable following the considered system [13]:

- In mechanics $\lambda(t) \sim 10^{-7}$ à 10^{-4} fail/h.
- In electronics $\lambda(t) \sim 10^{-9}$ à 10^{-6} fail/h.
- Human failure $\lambda(t) \sim 10^{-4}$ à 10^{-2} fail/h.

Today, several tests are used to assess the equipment reliability in particular for equipment with an important life span. The most performed tests are presented below.

2.1 The compounded tests

They are also called highly accelerated tests; they are used in the design phase before the qualification phase of the first sub sets available to the state models (very recent design components). Their application, allows in a short time to accelerate the maturation of product performance, built and improve their robustness, to accelerate the growth of reliability, from the first prototypes, and so control the failure sources so as to make corrections.

The principle of this type of test is to apply to the product a step stress, progressive and increasing, until the occurrence of a failure, and then perform technological and corrective analysis of that failure in order to increase the product reliability and therefore reduce the failure rate (see Fig. 1).

2.2 The accelerated tests

When the material is considered very reliable, failures are rare events and the feedback processing for estimation parameters of reliability does not allow obtaining a sample involving failures. This will be more sensitive to the population size of the material.

Thus, the necessity to know the product behavior with a long life span before being put into service, often leads to apply accelerated experiments or accelerated life testing in qualification and production.

The principle of accelerated testing is to subject the product to operating stresses or amplified environment with respect to the expected values in operational use to estimate behavioral characteristics (law of reliability, operational performance, . . .) of the product in normal conditions of use from accelerated conditions of use and that within a calendar compatible with the constraints associated with the development phase.

The passage under accelerated conditions (or severe) under normal conditions regarding life span occurs with a law called the law of acceleration [14–17]).

2.3 The Bayesian tests

Bayesian techniques are used to reduce the size of test estimation, improve the parameters estimation of product reliability by the integration of the past (data available on the product concerned) and handle the case "zero" failure observed difficult to treat with conventional statistical approach which would require a large sample size.

There are many works dealing with Bayesian techniques. The fundamental can be shown as follows: [18]. . . and the work on reliability applications: [19]. . . In a Bayesian test plan, the achieved test results during the development cycle are combined with a reliability model "a priori", to obtain a model "a posteriori", using the Bayes theorem. The prior model is constructed using known information acquired before implementation of the first test (expert opinion, information from previous studies such as predictive analytics, the FMEA. . .).

The conventional reliability testing is intended to provide samples to estimate reliability characteristics (parameters of life span law, failure rates, etc.). The information contained in these samples is called objective information. In addition to objective information, there is sometimes additional information of subjective nature (subjective information) such as data on equipment of

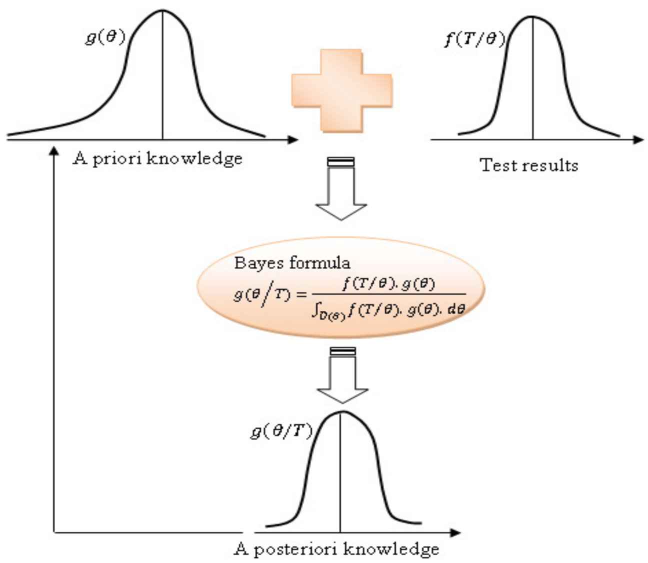

Fig. 3. Principe of Bayesian test.

technologies similar to that of the equipment being studied. If it is possible to translate this subjective information by assigning parameters to estimate a probability distribution of "a priori", it is possible, using Bayes theorem, to combine subjective information and objective information in order to build a distribution "a posteriori" of the considered parameters. It is necessary to use a simple model "a priori" allowing analytical treatment of the model "a posteriori". The procedure to be followed in the implementation of a Bayesian test plan is a combined procedure using the expertise and previous results. It can be summarized in three main steps:

- Collect all available data on the product concerned: expert opinion, foresight studies, system architecture, equipment similar technologies. . .
- Translate these data into a reliability model "a priori" (Construction of a prior distribution law on the parameters). It is noted that the most difficult problem to solve in the Bayesian techniques is the a priori knowledge modeling.
- Combine this model "a priori" with test results (information from tests) to obtain the model "a posteriori" using the theorem of Bayes. Figure 3 summarizes the main steps of the Bayesian approach.

At the end of a Bayesian test, the distribution of the parameter θ given by Bayes theorem can be obtained. The distribution is given, in the case of continuous random variables, by the formula:

$$g\left(\theta/T\right) = \frac{f\left(T/\theta\right) g\left(\theta\right)}{\int\limits_{D(\theta)} f(T/\theta) g(\theta) \mathrm{d}\theta} \tag{1}$$

In our case θ is none other than the failure rate λ, and the expression (1) becomes:

$$g\left(\lambda/T\right) = \frac{f\left(T/\lambda\right) g\left(\lambda\right)}{\int\limits_{D(\lambda)} f(T/\lambda) g(\lambda) \mathrm{d}\lambda} \tag{2}$$

where:

λ : failure rates, repair rates, etc.;
T: experiment results (times of failure . . .);
$g(\lambda)$: a priori probability density of parameter λ;
$f(T/\lambda)$: likelihood function of the sample;
$g(\lambda/T)$: posterior probability density of parameter λ (this is a conditional probability that depends on the available information);
$D(\lambda)$: domain of the random variable λ.

A posteriori estimation of the parameter λ can be calculated by conventional methods as well as the optimal size of the sample to be tested.

The Bayesian test results allow:

1. To improve the estimation of unknown parameters and its accuracy (minimizing the variance estimators).
2. To propose a region (or range) of credibility to a certain level $(1-\alpha)$, is a sub set of accessible values of parameter θ such that the probability to contain θ is equal to à $(1-\alpha)$. This region can therefore be concluded when the parameter is close to a critical value established by contract between the supplier and the customer.
3. To demonstrate that assumptions made by analysis concerning parameter θ are accurate or no by making a posteriori assumption tests (e.g.: "the failure rate is less than 10^{-8}").
4. To make better decisions (decision analysis) such as the choice of a corrective action from a set of possible actions (maintenance policy, acceptance of material, invest in new equipment, etc.) taking into account the different costs.

λ is the probability distribution parameter being sought to be estimated and is the failure rate in the case of an exponential law.

The likelihood function gives the probability of each possible outcome of the test for a value of λ.

The law gives the combined probability density λ knowing that test result was obtained.

The Gamma is defined as follows:

$$g(\lambda/k, T) = \frac{\lambda^k T^{k+1} \mathrm{e}^{-\lambda T}}{k!} \tag{3}$$

if k failures occurr during the total duration T.

The gamma distribution is widely used in the Bayesian approach; it is the combination of the natural exponential parameter λ. From the expression (6) expressions of conjugated laws of probability densities can be defined.

$$g(\lambda/K, T) = \frac{\lambda^{K_0} T_0^{(K_0+1)} \mathrm{e}^{(-\lambda T_0)}}{K_0!} \tag{4}$$

$$g(\lambda/K, T) = \frac{\lambda^{K_1} T_1^{(K_1+1)} \mathrm{e}^{(-\lambda T_1)}}{K_1!} \tag{5}$$

$$g(\lambda/K, T) = \frac{\lambda^{K} T^{(K+1)} \mathrm{e}^{(-\lambda T)}}{K!} \tag{6}$$

Table 1. Data of test at $K_0 = 1$.

$\lambda_{90\%} = 3.890\ \mathrm{h}^{-1}$ / $\lambda_{90\%\ \mathrm{expert}} = 3.0\ \mathrm{h}^{-1}$	Integration of Gamma law	Inverse Chi-square
$\lambda_{80\%}$ a priori	$3.0 \times 10^{-6}\ \mathrm{h}^{-1}$	$2.994 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ trials	$4.1 \times 10^{-6}\ \mathrm{h}^{-1}$	$4.103 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ a posteriori	$2.5 \times 10^{-6}\ \mathrm{h}^{-1}$	$2.474 \times 10^{-6}\ \mathrm{h}^{-1}$

3 Application

84 inductive proximity sensors of the cable car station of serial 19-302-3BBPKG/SS-000-K/PG36 have been tested for 1 year. One brokes down after 0.3 year. The failure rate at 80% confidence defines that this test is used to justify the upper bound of the confidence interval unilateral to risk $\alpha = 20\%$.

For modeling this interval, The Chi-square law is not used to directly model, but essentially for the calculation of confidence limits during estimates of the confidence interval. Let $\frac{\chi^2_{1-\alpha}(2k+2)}{2T} = \chi^2_{80\%}$ (4)$/(2 \times 83.3 \times 365 \times 24) = 4.103 \times 10^{-6}\ \mathrm{h}^{-1}$ with $T_1 = 83.3$ years and $k_1 = 1$ breakdown.

The failure rate of this type of equipment was estimated at $10^{-6}\ \mathrm{h}^{-1}$ [20] according to the standard and considering that the risk that the actual failure rate is less than three times the value of this estimation is lower than 10%.

This expert judgment can be converted into virtual testing such as:

$$\lambda_{\mathrm{moy}} = k_0/T_0 = 10^{-6}\ \mathrm{h}^{-1} \quad \text{and}$$
$$\lambda_{\mathrm{sup}90\%} \geqslant 3 \times 10^{-6}\ \mathrm{h}^{-1}.$$
$$\lambda_{\mathrm{sup}90\%} = \chi^2_{90\%}(2k_0+2) \times \lambda_{\mathrm{moy}}/2k_0$$
$$= 3.890 \times 10^{-6}\ \mathrm{h}^{-1} \quad \text{for} \quad k_0 = 1$$

Only the configuration $k_0 = 1$ breakdown and $T_0 = 1\,000\,000$ h satisfies this condition. Table 1, gives the values of $\lambda_{\mathrm{sup}90\%}$ depending on the number of breakdowns K_0.

4 Results and discussion

The tests were carried out according to experts' opinion relating to product examined. The curve in Figure 4 expresses the result of the test failure ($K_0 = 1$), where $\lambda_{\mathrm{sup}90\%} \geqslant 3 \times 10^{-6}\ \mathrm{h}^{-1}$. Table 1 gives the corresponding values. According to the expert judgment this is the most favourable case because it saves time estimated at 60.30%. Figure 5 shows the densities evolution in terms of percentage of test failures and a good agreement can be noted.

To check the consistency of product information, according to expert opinion, other configurations that are not available have been tested. The density values and their percentage are shown in Figures 6 and 7 respectively. Table 2 gives the corresponding values.

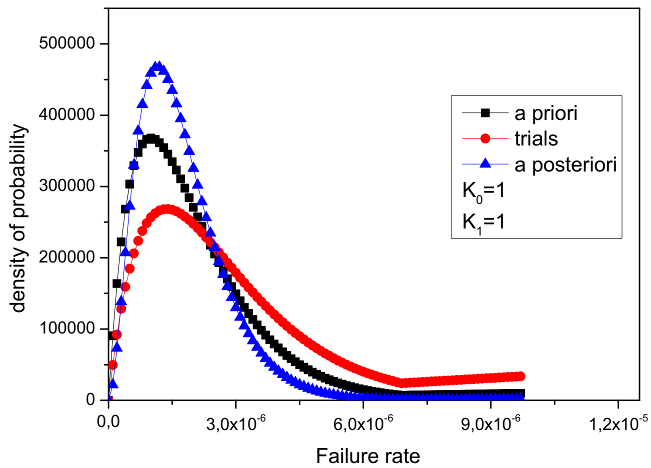

Fig. 4. Density of probability at $K_0 = 1$.

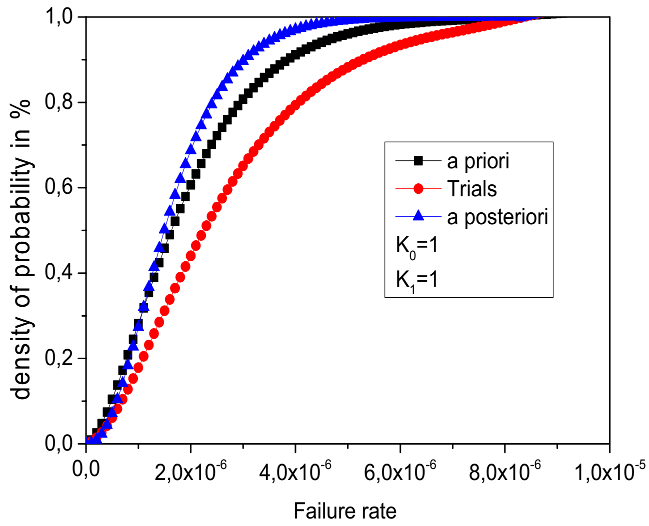

Fig. 5. Pourcentage of densities at $K_0 = 1$.

If the test is done with a number of virtual tests different from that suggested by the experts for this kind of material, it can be noted, from Figures 8 and 9, the non-conformity of results and in particular the testing. This confirms the hypothesis for the equipment behavior to the actual testing. The data of this are summarized in Table 3.

Table 2. Data of test at $K_0 = 5$.

$\lambda_{90\%} = 1.855\ \mathrm{h}^{-1}$ / $\lambda_{90\%\ \mathrm{expert}} = 3.0\ \mathrm{h}^{-1}$	Integration of Gamma law	Inverse Chi-square
$\lambda_{80\%}$ a priori	$1.6 \times 10^{-6}\ \mathrm{h}^{-1}$	$1.581 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ trials	$4.1 \times 10^{-6}\ \mathrm{h}^{-1}$	$4.103 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ a posteriori	$2.5 \times 10^{-6}\ \mathrm{h}^{-1}$	$1.584 \times 10^{-6}\ \mathrm{h}^{-1}$

Table 3. Data of test at $K_1 = 2$.

$\lambda_{90\%} = 3.890\ \mathrm{h}^{-1}$ / $\lambda_{90\%\ \mathrm{expert}} = 3.0\ \mathrm{h}^{-1}$	Integration of Gamma law	Inverse Chi-square
$\lambda_{80\%}$ a priori	$3 \times 10^{-6}\ \mathrm{h}^{-1}$	$2.994 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ trials	—	$5.864 \times 10^{-6}\ \mathrm{h}^{-1}$
$\lambda_{80\%}$ a posteriori	$2.9 \times 10^{-6}\ \mathrm{h}^{-1}$	$3.188 \times 10^{-6}\ \mathrm{h}^{-1}$

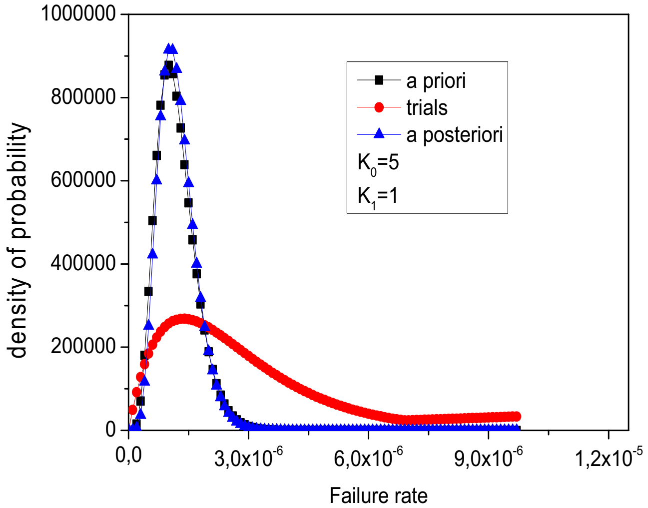

Fig. 6. Densities at $K_0 = 5$.

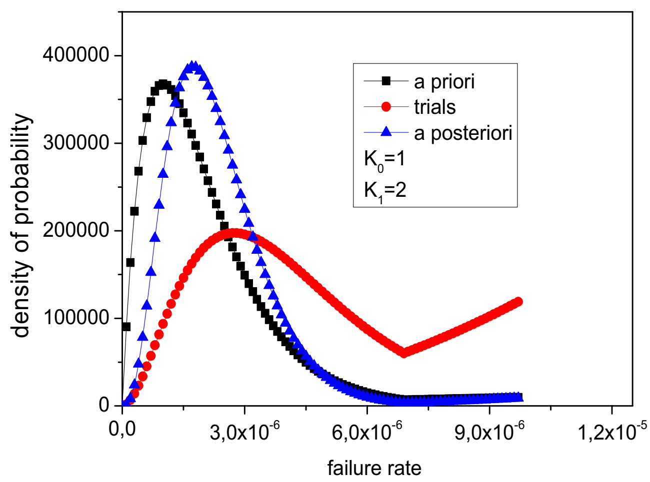

Fig. 8. Densities at $K_1 = 2$.

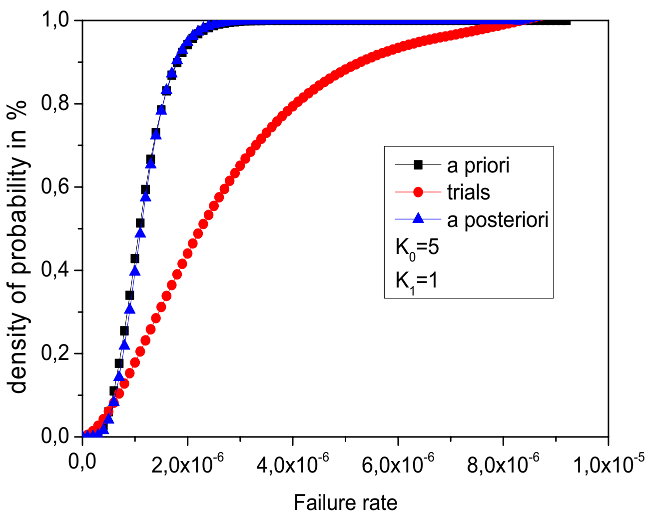

Fig. 7. Percentage of density at $K_0 = 5$.

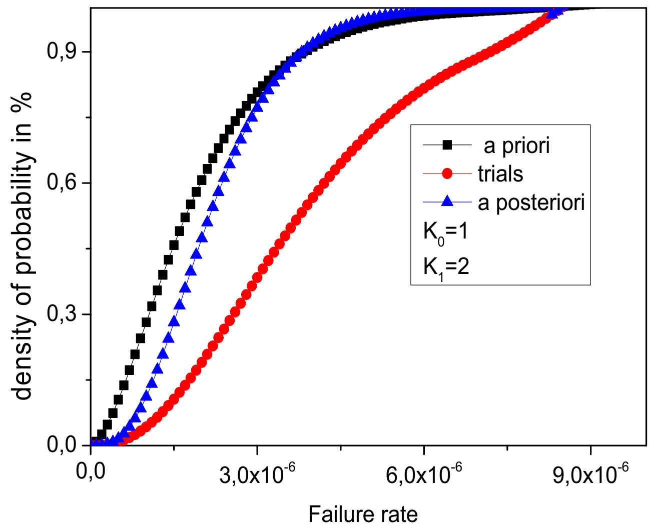

Fig. 9. Percentage of densities at $K_1 = 2$.

5 Conclusion

The Bayesian approach has the advantage of integrated components in the past. It applies to more complex components and with high reliability.

The Bayesian modeling has allowed to obtain a failure rate of 2.474×10^{-6} h^{-1} at 80% instead of 4.103×10^{-6} h^{-1} after testing. A gain of 60.30% is therefore obtained. The Bayesian estimation allows also:

- To improve the evaluation of unknown parameters and accuracy (minimizing the variance estimators).
- Propose a credibility region (or interval) to a certain level $(1-\alpha)$, with a sub set of accessible values of the parameter λ such that the probability to contain λ be equal to $(1-\alpha)$. This region can therefore be concluded when the parameter is close to a critical value established by contract between the supplier and the customer.
- Demonstrate that the assumptions made by the analysis on the parameter λ are accurate or not by carrying out of tests a posteriori hypothesis (e.g.: "the failure rate is less than 10^{-7}").
- Make better decisions (decision analysis) such as the choice of a corrective action from a set of possible actions (maintenance policy, acceptance of equipment, invest in new equipment, etc.) taking into account different costs.

References

[1] D.C. Li, F.M. Chang, K.C. Chen, Building Reliability Growth Model Using Sequential Experiments and the Bayesian Theorem for Small Datasets, Expert Systems with Applications (2009), DOI: 10.1016/j.eswa.2009.10.001

[2] G. Maurizio, P. Gianpaolo, Bayesian Analysis Of Repairable Syst. Showing A Bounded Failure Intensity, Reliab. Eng. Syst. Safe. 91 (2006) 828–838

[3] A. Makota, Bayesian Analysis For Results Of Fatigue Test Using Full-Scale Models To Obtain The Accurate Failure Probabilities Of The Shinkansen Vehicle Axle, Reliab. Eng. Syst. Safe. 75 (2002) 321−33

[4] F. Corset, Aide à l'optimisation de maintenance à partir de réseaux bayésiens et fiabilité dans un contexte doublement censuré. Thèse de doctorat, Université Joseph Fourrier, Grenoble 1, 2003

[5] A. Muller, Contribution à la maintenance prévisionnelle des systèmes de production par la formalisation d'un processus de pronostic, Thèse de doctorat, Université Henri Poincaré, Nancy 1, 2005

[6] P. Weber, L. Jouffe, Reliability modeling with Dynamic Bayesian Networks. 5th IFAC Sysmposium on Fault Detection, Supervision and Safety of Technical Processes, D.C. Washington, USA, 2003

[7] C. Robert, L'analyse statistique bayésienne. Edition Economica, 1992

[8] C.P. Robert, The Bayesian choice. Ed. Springer, 2001

[9] P. Congdon, Bayesian statistical modelling. Ed. Wiley, 2001

[10] J. Ibrahim, M. Chen, D. Sinha, Bayesian survival analysis. Ed. Springer, 2001

[11] J. Ringler, Utilisation des techniques bayésiennes dans le cas de système complexe. Revue de Statistique Appliquée XXVII (1979)

[12] H. Procaccia, P. Morilhat, Fiabilité des structures des installations industrielles. Ed. Eyrolles, (1996)

[13] P. Gondran, Fiabilité des systèmes, Eyrolles, 1980

[14] W. Nelson, Accelerated Testing: Statistical Models, Test Plans and Data Analysis, Eds. Wiley Series in Probability and Mathematical Statistics, 1990

[15] P. O'Connor, Testing for reliability, Qual. Reliab. Eng. Int. 19 (2003) 73–84

[16] H. Caruso, A. Dasgupta, A fundamental overview of accelerated testing analytical models. In IEEE Proceedings Annual Reliability and Maintainability Symposium, Tutorial notes, 1998

[17] A. Vassilious, A. Mettas, Understanding accelerated life testing analysis. In IEEE Proceedings Annual Reliability and Maintainability Symposium, Tutorial notes, USA, 2001

[18] H. Procaccia, L. Piepszownik, C. Clarotti, Fiabilité des équipements et théorie de la décision statistique fréquentielle et bayésienne. Ed. Eyrolles, 1992

[19] T.D. Ross, Accurate confidence intervals for binomial proportion Poisson rate estimation, Comput. Biol. Med. 33 (2003) 509–531

[20] Military Handbook-Reliability Prediction of Electronic Equipment. United States Department Of Defense, notice 2. [MIL-HDBK-217 F], 1995