

Balancing of flexible rotors based on evolutionary algorithms

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Abstract – An unbalance in a rotating flexible rotor causes excessive vibration and elastic deformations with subsequent malfunction and failure. In spite of different techniques deployed to reduce or eliminate rotor unbalance, it is impossible to remove the unbalance completely. The unbalance will only be reduced to a residual level. Hence, any other method that can reduce this residual level further can be considered as an alternative. In this article, Differential Evolution (DE) and Genetic Algorithm (GA) were successfully applied as optimization techniques to balance rotating flexible rotors. The unbalancing challenge is formulated as an optimization problem with an objective function of minimizing the rotor unbalance by identifying the optimum correction parameters. Modeling and response analyses were performed in ANSYS while optimizations were conducted in MATLAB. The results of four balancing cases show that the approaches are robust at both balancing speed and beyond. Also, the results obtained show that GA performs slightly better than DE in terms of optimization time and effective reduction of vibration amplitude.

Key words: Optimization / rotor unbalance / evolutionary algorithms / ANSYS / MATLAB

1 Introduction

Rotating machinery accounts for majority of present-day machines. While the dynamics of such machinery has been important, the need for higher efficiency has consequently led to design of machines which are increasingly liable to vibration at higher speeds. A level of unbalance in a rotor that is tolerable at a low speed might be intolerable at a higher speed because the centrifugal force due to an unbalance could be coupled with an elastic deformation of the rotor. An unbalance in a rotating system can have devastating effects during start-up, slow down or when operating at or near critical frequency. Such an unbalance may be due to material faults, manufacturing and assembly errors, transportation and commissioning as well as operational conditions. Consequently, noises, wears, high level of vibrations, reduction in machine life, etc, are expected. In flexible rotors (such as high-speed turbines, generators and paper-machine rolls), unbalance problem becomes more complicated due to the interaction with the elastic deformations of the rotor, and thus calls for a special balancing procedure.

Rotor balancing can be done either by adding or removing a correction mass of which its quantity and location can be quantified either by modal or influence coefficient method [1–7]. Modal balancing is a progressive or mode-by-mode balancing method [2]. The influence

coefficient method is a data processing technique aiming at minimizing rotor's vibration to an acceptable level and a relationship between the two approaches has been developed by Tan and Wang [3]. It was stated in reference [3] that for the influence coefficient approach, this relationship requires that the rotor-bearing systems should be undamped or have insignificant damping and the effect of run speeds on its natural frequencies should be negligible. However, the applicability of the method has not been demonstrated and it can be inferred that their proposed relationship between modal and influence coefficient methods is not applicable to all rotors because there are rotors with significant damping. Other methods employed for balancing rotors available in the literatures include: holo-balancing method by Liu and Qu [4] where two sensors were used at each measuring point and the method is limited to the rotors that are operating between the first and second critical speeds. Xu et al. [6] used transfer matrix and genetic algorithm to balance the rotor and two sensors were utilized at each measurement point. Active vibration control scheme was proposed by Das et al. [8] in which electromagnetic exciters were employed. Nevertheless, their approach cannot be applied to all rotors because the exciters will constraint the bearing or disk movement where the shaft is not long or disks are closed together. Other available method is the minimization of Hermitian matrix of influence coefficients by Kang et al. [9].

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Nomenclature

Notation	Unit	Description
$\ddot{\mathbf{y}}$	N.m ⁻²	Acceleration vector
$\mathbf{A}m$	m	Amplitude matrix
Y_i	m	Amplitude of \mathbf{Y}_i
C_{xx}, C_{yy}	Ns.m ⁻¹	Bearings damping coefficients
K_{xx}, K_{yy}	N.m ⁻¹	Bearings Stiffness coefficients
\mathbf{C}	Ns.m ⁻¹	Damping matrix for bearings and gyroscopic of the shaft and disks
r	m	Distance from unbalance mass to shaft/rotor centerline
\mathbf{q}	N	Force vector
i		Generation number
m_l	kg	Lower boundary of trial correction mass
ϕ_l	°	Lower boundary of trial correction mass angular position
\mathbf{M}	kg	Mass/Inertia matrix for rotor shaft and disks
J	m	Objective function
N		Population size
Ng		Maximum number of generation
\mathbf{y}	m	Rotor displacement vector
ω_o, ω_b	rpm	Rotor operation and balancing speeds
ω	rad.s ⁻¹	Speed
\mathbf{K}	N.m ⁻¹	Stiffness matrix for shaft and bearings
t	s	Time
m	kg	Trial correction mass
\emptyset	°	Trial correction mass angular position
θ	radian	Unbalance angular position
m_u	kg	Unbalance mass
\mathbf{f}_{ud}	N	Unbalanced force
m_h	kg	Upper boundary of trial correction mass
ϕ_h	°	Upper boundary of trial correction mass angular position
\mathbf{q}^o	N	Vector of other force
\mathbf{q}^u	N	Vector of unbalanced force
$\dot{\mathbf{y}}$	m	Velocity vector

Although the above techniques enable balancing procedures and minimize operating cost, they still have a few inherent shortcomings. For example, the techniques are non-universal, some approaches require two measurement instruments at one measurement point, and they take longer time and require high balancing speed as well as high professional training. In order to minimize these bottlenecks, the present work utilizes differential evolution (DE) and genetic algorithm (GA) as optimization techniques for obtaining correction parameters. The optimization methods consider gyroscopic effects, low balancing speed and measurements are taken in one direction only at each measuring point. The optimum correction parameters obtained do not only balance the rotor at balancing and operation speeds but also at higher speeds. Furthermore, the study presents the influence of correction parameters and compares the performance of the two optimization techniques.

2 Formulation and implementation of unbalanced rotor problem

Equation of motion of a rotor-bearing system with forced damped vibration can be assembled in general

form as:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{q} \quad (1)$$

Then, for an unbalanced rotor-bearing system, Equation (1) can be written as:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{q}^u + \mathbf{q}^o \quad (2)$$

\mathbf{f}_{ud} due to m_u located at node j (make angle θ with respect to Y -axis) and at $t = 0$ (when unbalanced rotor is at rest and position of m_u is as shown in Fig. 1) can be defined as:

$$(\mathbf{f}_{ud})_j = m_u r \omega^2 \begin{bmatrix} \sin(\omega t + \theta) \\ \cos(\omega t + \theta) \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

Furthermore, equation (3) can be expressed as:

$$\{\mathbf{q}^d\}_j = m_u r \omega^2 \begin{bmatrix} \sin(\omega t) \cos \theta \\ -\sin(\omega t) \sin \theta \\ 0 \\ 0 \end{bmatrix} + m_u r \omega^2 \begin{bmatrix} \cos(\omega t) \sin \theta \\ \cos(\omega t) \cos \theta \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

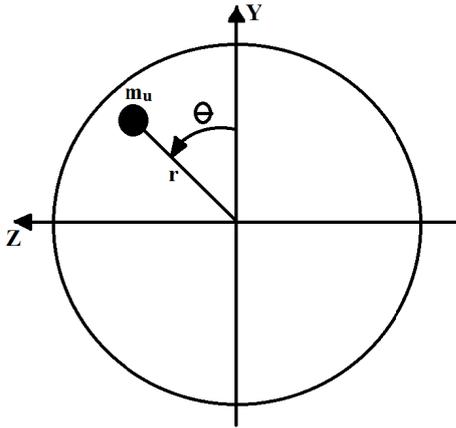


Fig. 1. Mass unbalance, m_u : unbalance mass, r : distance from unbalance mass to shaft/rotor centerline, θ : angular position.

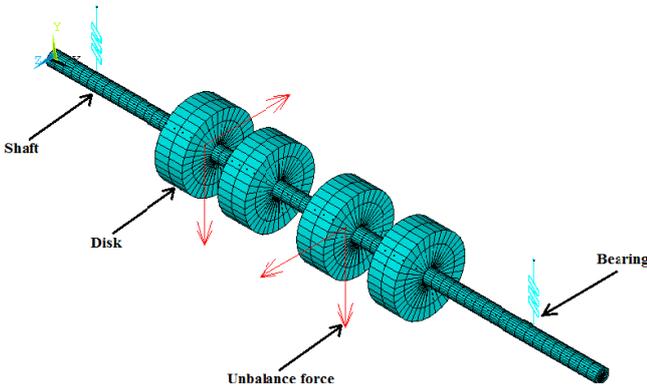


Fig. 2. Unbalance four-disk rotor model.

If the rotor bearing system contains unbalance, then $q^u = q^d$, else $q^u = 0$. This work is focused at reducing the effect of q^u on y using the two optimization techniques mentioned earlier. The procedures are implemented in ANSYS and MATLAB platforms.

An unbalanced rotor was modeled in ANSYS (Fig. 2) and the harmonic response analysis, based on equation (2), was conducted. More information on harmonic response analysis on ANSYS can be found in reference [10]. The rotor and its bearings were discretized using BEAM188 and COMBIN14 elements, respectively. BEAM188 is based on Timoshenko beam theory and it includes shear-deformation effects and can be used to analyze slender and stubby/thick beam structures. Each node has about six degrees of freedom which include three translations and three rotations. COMBIN14 has spring and damping capabilities and it is capable of longitudinal or torsional behaviour in 1, 2 and 3D applications. CORIOLIS command was deployed to take care of the gyroscopic effect, SYNCHRO command was used to indicate that the force on the system is due to unbalance and stepped loading was utilized to determine how this unbalance force is being added to the system. In addition, ANSYS Parametric Design Language (APDL) script was used to perform both modal and harmonic analyses.

3 Formulation of the optimization problem and evolutionary optimization techniques

At each measurement point on the rotor-bearing system showing in Figure 2, a vector of amplitude of y will be obtained due to frequency steps. For n measurement points and j frequency steps, the Am for each trial parameter in the i th generation (where $i = 1, 2, \dots, N_g$) can be obtained as follows:

$$Am = [Y_1 Y_2 \dots Y_n]_{j \times n} \quad (5)$$

For each generation i with N solution, Equation (5) becomes:

$$Am = (Am)_{T \times n}, T = N * j \quad (6)$$

Therefore, the objective J can be defined as:

$$J = \text{Minimize} (\max (Am)) \quad (7)$$

which is subject to:

$$m_l \leq m \leq m_h, \theta_l \leq \theta \leq \theta_h \quad (8)$$

In equation (7), the minimization will be done at any range of frequencies but the size of Am for any i th generation depends on the frequency steps employed and speed under consideration.

Various techniques based on evolutionary algorithm (EA) have been described and can be found in reference [11]. These include genetic algorithms (GA), genetic programming (GP), evolutionary programming (EP), differential evolution (DE), evolution strategies (ESs), cultural evolution (CE) and co-evolution (CoE). The well-known areas of applications include optimization of doublet [12], realistic process engineering optimization problems [13], Circuit Design Problems [14] among others. In this work, both DE and GA were used because they have been proven to be effective for evolutionary optimization. Interested reader is referred to the following references [11, 15–18].

4 Proposed balancing methodology

In DE the following processes are implemented: population generation, mutation, crossover, reproduction (selection) and elitism while those implemented for GA include: population generation, reproduction (selection), crossover, mutation and elitism in those orders. The flowchart for DE is shown in Figure 3.

Both techniques were deployed to minimize the maximum amplitudes (see Eq. (7)) at all measurement points in order to obtain optimal correction parameters. The number of correction parameters so obtained depends on the number of correction planes that are capable of reducing the unbalance to an acceptable level. Both ANSYS and MATLAB are used simultaneously and iteratively as summarized in Figure 4. The rotor modelling and analysis were done in ANSYS while MATLAB was used for

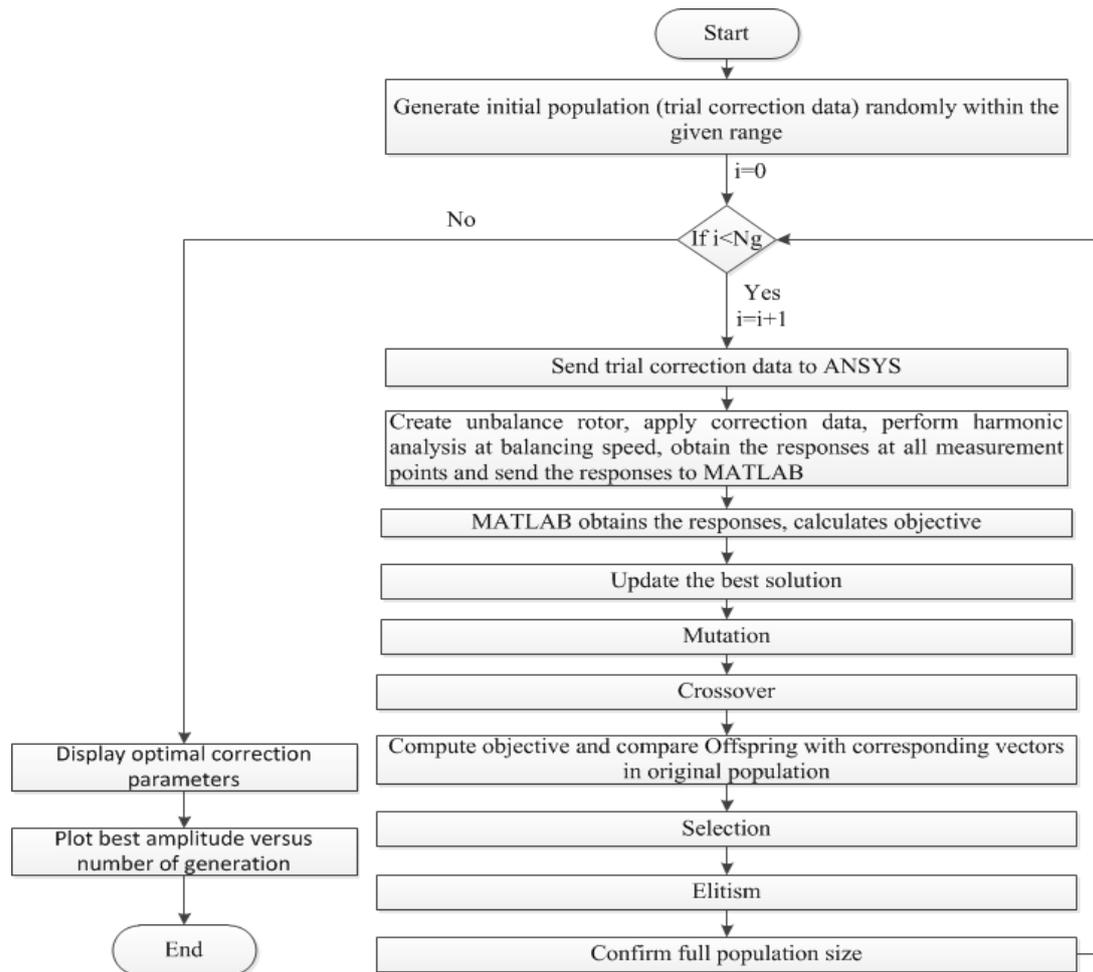


Fig. 3. Flowchart for balancing processes for DE.

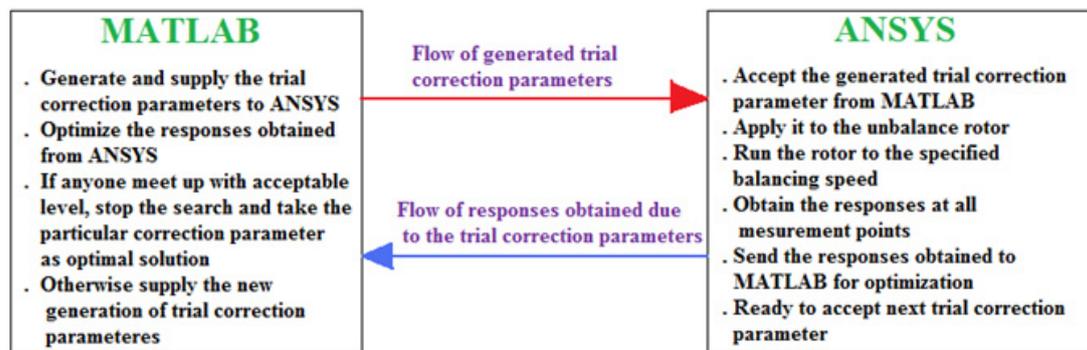


Fig. 4. The back-and-forth process for balancing methodology based on DE and GA.

the optimization process. For each iteration, the optimized correction parameters are generated in MATLAB and then transferred to ANSYS for the rotor analysis. The rotor responses obtained at measurement points in ANSYS are sent back to MATLAB for further processing (Fig. 4). This back-and-forth procedure continues until a stopping criterion (maximum number of generation or iteration) is met. Finally, the solution which gives the lowest value of the objective function is considered as the optimal correction parameters.

5 Results and discussion

To demonstrate the applicability of the proposed approaches, a four-disk rotor (Fig. 5) previously modelled in reference [6] was used for both DE and GA. The disk diameter, disk thickness, shaft diameter, density and elastic modulus are 0.08 m, 0.03 m, 0.015 m, 7800 kg.m⁻³ and 2.1×10¹¹ N.m⁻², respectively. Also, K_{xx} and K_{yy} are 140106 and 175133 N.m⁻¹ while C_{xx} and C_{yy} are 12.26 and 17.51 N.s.m⁻¹, respectively. In Figure 5,

Table 1. Different cases of unbalance and the corresponding proposed correction parameters.

Cases	Unbalance disks	Unbalance with location (g @°)	Proposed correction planes	Correction radii (mm)	Measurement points
1	A and C	$U_A = 0.8@100$ $U_C = 1.2@230$	19 & 32, 25 & 32 and 25 & 38	36	1 and 2
2	A and C	$U_A = 0.8@100$ $U_C = 1.2@230$	300 & 302, 301 & 302 and 301 & 303	40	1, 2 and 3
3	A, B, C and D	$U_A = 0.8@100$ $U_B = 2.0@230$ $U_C = 0.4@0$ $U_D = 0.5@310$	19, 25 & 32; 25, 32 & 38 and 19, 25 & 38	36	1 and 2
4	A, B, C and D	$U_A = 0.8@100$ $U_B = 2.0@230$ $U_C = 0.4@0$ $U_D = 0.5@310$	300, 301 & 302; 301, 302 & 303 and 300, 301 & 303	40	1, 2 and 3

Table 2. Comparison of the optimized correction parameters with those in reference [6].

Correction planes	[6]		Present study						
	Optimized correction parameters		Correction planes	1		Correction planes	2		
	Masses (g)	Angles (Deg.)		Masses (g)	Angles (Deg.)		Masses (g)	Angles (Deg.)	
Unknown	0.80	279.68	DE	18	0.70	0.05	19	0.10	14.10
	0.19	50.29	GA	31	0.32	38.88	32	0.80	11.58
				18	0.30	70.61	19	0.92	15.65
				31	0.81	360.00	32	0.09	0.00

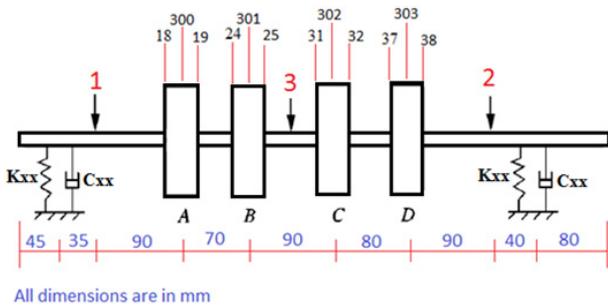


Fig. 5. The four-disk rotor configuration considered for this work A–D: Rotor disks, 1, 2, 3: measuring points on the shaft, 18, 24, 31, 37; 19, 25, 32, 38 and 300, 301, 302, 303: nodes at the left, right and middle edges of the disks where the correction masses can be attached.

A, B, C, and D are rotor disks while 1, 2, and 3 denote measurement points in which point 3 is located at centre of the distance between disks B and C. The proposed correction planes are identified as 300, 301, 302 and 303, and 18, 19, 24, 25, 31, 32, 37 and 38. However, it was assumed that ω_o is 4000 rpm. The ANSYS model of a rotor with unbalances on two disks (A and C) is shown in Figure 2. The modal analysis of the rotor was initially performed and the first two critical speeds are 1874.7 and 4582.26 rpm. These critical speeds imply that ω_o (which is 4000 rpm) is supercritical (i.e. above the first critical speed).

5.1 Validation of DE and GA results

Simulations were carried out based on 4 cases presented in Table 1. In each case, the distance r between the mass unbalance m_u and the rotor center (see Fig. 1) is 0.036 m while ω_b , N and N_g are 2000 rpm, 40 solutions and 20 generations, respectively. Prior to this, the approach was compared with that of Xu et al. [6] by considering the case 1 as defined in Table 1.

The ranges of m and their ϕ were 0 to 0.003 kg and 0 to 360 degree, respectively. While reference [6] used 6 different ω_b (1300, 2100, 2500, 3000, 4600, 5600 rpm), N of 80 and N_g of 20, ω_b of 2100 rpm and the same N and N_g were selected for the validation. Two trials, each having two correction planes (18 & 31 and 19 & 32), were considered for balancing. Detailed results are presented in Table 2.

The first 2 critical speeds (1874.7 and 4582.26 rpm) are comparable to those obtained by reference [6] (i.e. 1871 and 5374 rpm). It is also observed that by considering the correction planes 18 and 31 for both DE and GA, the correction masses are close to those of reference [6] while the correction angles are different. The correction planes that give similar results are opposite for the 2 approaches and the 2 cases considered. The slight difference between our results compared to those of reference [6] may be attributed to the facts that neither their exact locations of the unbalance on the 2 disks nor the correction planes were known. Also, the exact speed

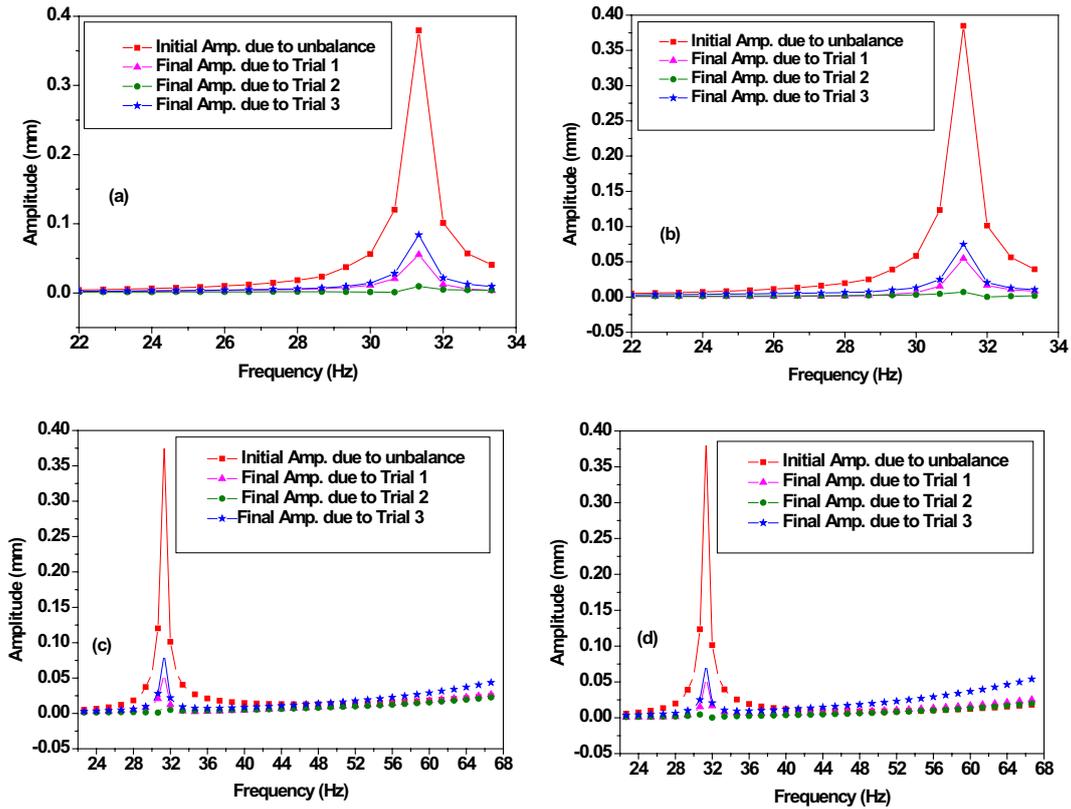


Fig. 6. Amp.: Amplitude, (a) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at first measurement point for Case 1 under DE, (b) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at second measurement point for Case 1 under DE, (c) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at first measurement point for Case 1 under DE, (d) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at second measurement point for Case 1 under DE.

used for the simulation is not known. However, it was noted that N_g of 20 and N of 80 took a long time to implement without a significant difference in the results, hence N of 40 were considered for the rest of this work.

5.2 Results of differential evolution (DE) based on cases 1 and 3

To obtain the optimized correction parameters, DE was applied to all the cases listed in Table 1. While all the results are promising only the results of cases 1 and 3 are discussed here. In case 1, disks A and C contain unbalance. Three trials were carried out and the optimized correction parameters obtained are presented in Table 3.

Each of the trial conditions in Table 3 was applied to the unbalanced rotor and tested at both ω_b and ω_o . The responses are plotted against the initial response for the unbalanced rotor (Fig. 6). It is noted that the rotor is balanced at ω_b with all the 3 correction parameters (Figs. 6a and 6b). However, only one out of these 3 correction parameters balanced the rotor at ω_o , while the other trial conditions failed prior to reaching ω_o (Figs. 6c and 6d). The responses are the same for case 2 at ω_b but 2 out of 3 correction parameters balanced the rotor at ω_o . This makes case 2 superior to case 1.

Table 3. Optimized correction parameters for Case 1 based on DE.

Trial S/N	Correction planes	Optimized correction parameters	
		Masses (g)	Angles (Deg.)
1	19	0.4140	64.5597
	32	0.8040	0.0001
2	25	0.3070	41.8438
	32	0.6780	0.0001
3	25	0.9750	181.1602
	38	2.2680	12.8803

In Figures 6–8, the response of unbalanced rotor is indicated as *Initial Amp. due to unbalance* while the responses of balanced rotor are designated as *Final Amp. due to Trial 1, Trial 2 and Trial 3*. The best of the correction masses from which balanced rotor responses were obtained are considered as desired correction masses.

In the case 3, disks A, B, C and D contain unbalance and following the procedures described earlier, the optimization results are presented in Table 4.

In the same manner, each of the trials in Table 4 was applied to the unbalanced rotor and tested at

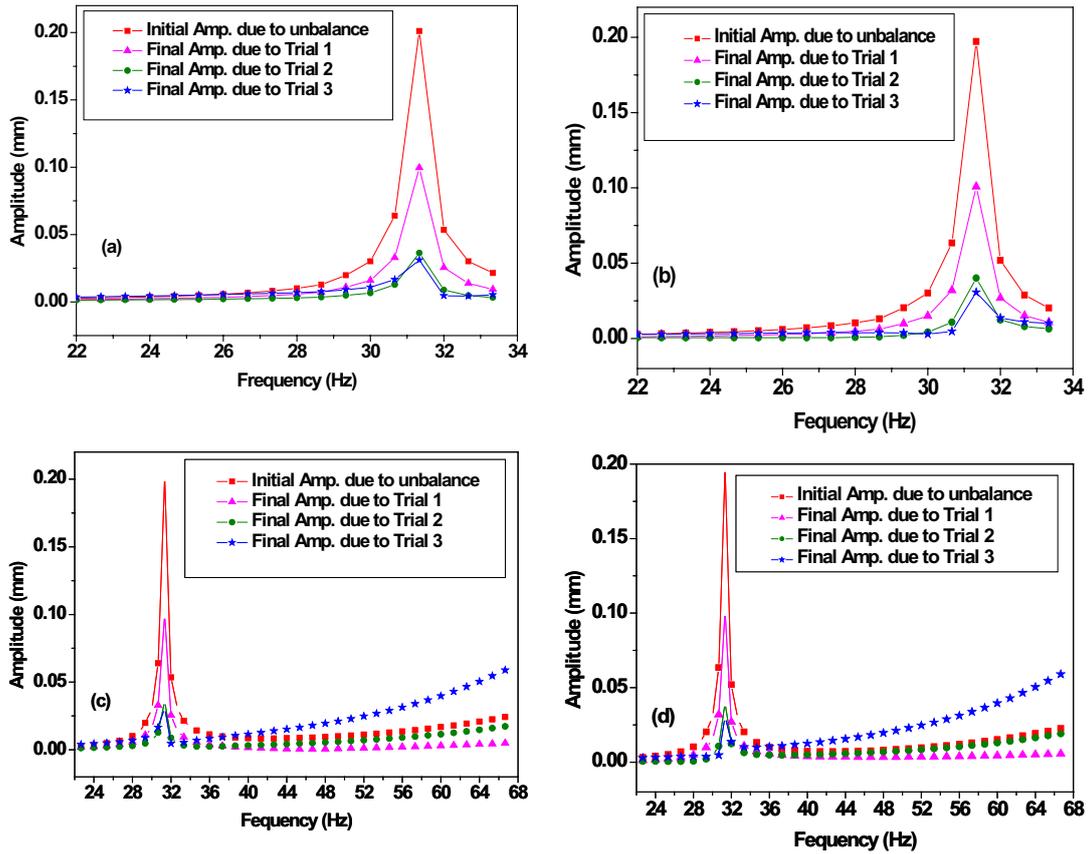


Fig. 7. Amp.: Amplitude, (a) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at first measurement point for Case 3 under DE, (b) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at second measurement point for Case 3 under DE, (c) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at first measurement point for Case 3 under DE, (d) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at second measurement point for Case 3 under DE.

Table 4. Optimized correction parameters for Case 3 based on DE.

Trial S/N	Correction planes	Optimized correction parameters	
		Masses (g)	Angles (Deg.)
1	19	1.7300	203.0158
	25	2.9000	0.0001
	32	2.1800	164.7053
2	25	1.9400	217.5462
	32	2.0400	0.0001
	38	1.5200	136.3986
3	19	2.5100	158.5715
	25	2.1600	297.3341
	38	1.3100	55.2829

Table 5. Optimized correction parameters for Case 1 based on GA.

Trial S/N	Correction planes	Optimized correction parameters	
		Masses (g)	Angles (Deg.)
1	19	1.0450	11.2000
	32	0.0630	132.8600
2	25	0.1090	112.6700
	32	0.9540	7.5900
3	25	0.8620	7.7300
	38	0.1340	71.5500

5.3 Comparison between GA and DE

both ω_b and ω_o . Based on the responses (Figs. 7a and 7b), all the correction parameters balanced the rotor at ω_b . However, Figures 7c and 7d show that 2 of the 3 correction parameters balanced the rotor at ω_o . Similar responses were obtained at both ω_b and ω_o for the case 4, although unbalances are reduced more in the case 3 compared to the case 4.

For the GA optimization technique, the result of case 1 only is described here. Three trials were performed and the optimized correction parameters are presented in Table 5.

Each trial condition in Table 5 was applied to the unbalanced rotor and tested at both ω_b and ω_o . By examining the responses (Fig. 8), all the correction parameters

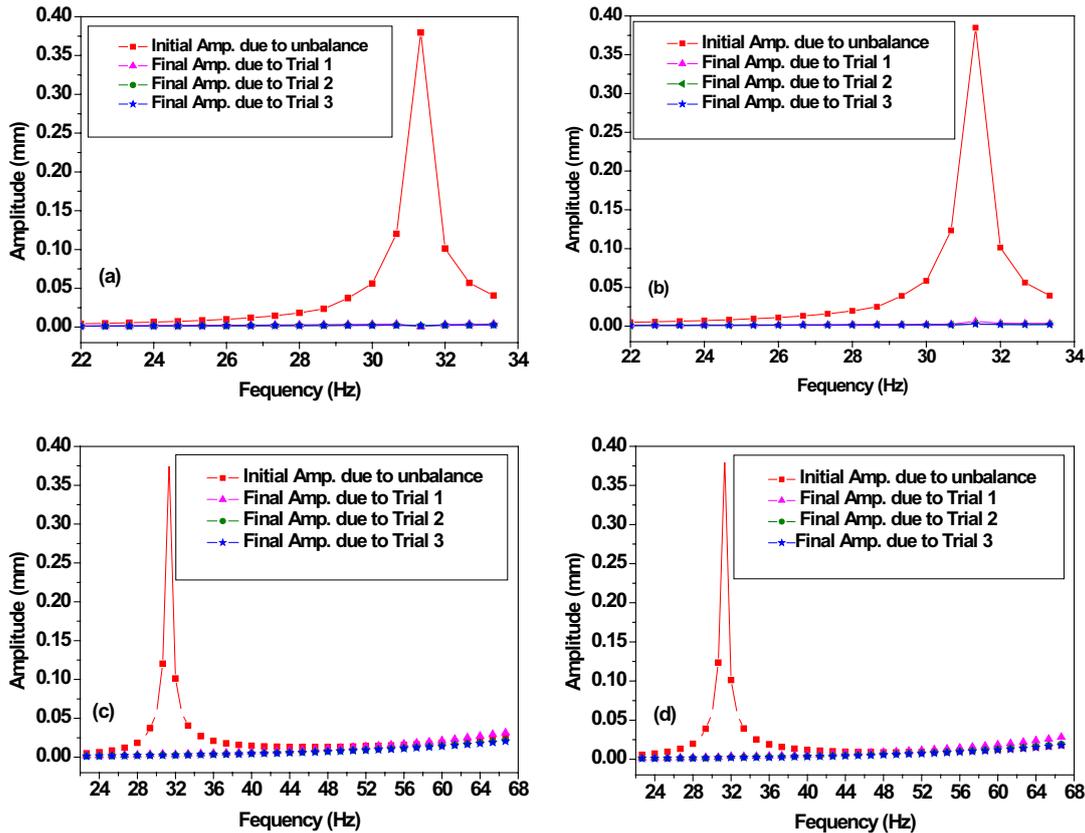


Fig. 8. Amp.: Amplitude, (a) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at first measurement point for Case 1 under GA, (b) comparison of the unbalanced and balanced rotors responses at ω_b of 2000 rpm at second measurement point for Case 1 under GA, (c) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at first measurement point for Case 1 under GA, (d) comparison of the unbalanced and balanced rotors responses at ω_o of 4000 rpm at second measurement point for Case 1 under GA.

balanced the rotor at ω_b (Figs. 8a and 8b) while 2 of the 3 conditions balanced the rotor at ω_o (Figs. 8c and 8d).

As previously discussed, both DE and GA are excellent techniques for optimizing balancing parameters in unbalanced rotors. However, our results show that GA is more efficient than DE. One, the rotor responses (amplitudes of vibration) are much reduced in GA compared to that of DE (see Fig. 9 for the case 1). Two, optimization based on GA takes lesser time compared to DE. Three, the number of successful trials is more for GA than for DE.

6 Conclusions

Both DE and GA were successfully deployed as optimization techniques to balance an unbalanced flexible rotor. The balancing problem is formulated as an optimization problem with an objective of minimizing vibration amplitudes by applying different correction parameters. The optimal correction parameters so obtained keep the rotor balanced at both balancing speed and beyond. Our results also show that GA is slightly superior to DE. Also, unlike the available balancing techniques in the literature, the proposed method can be used to balance the

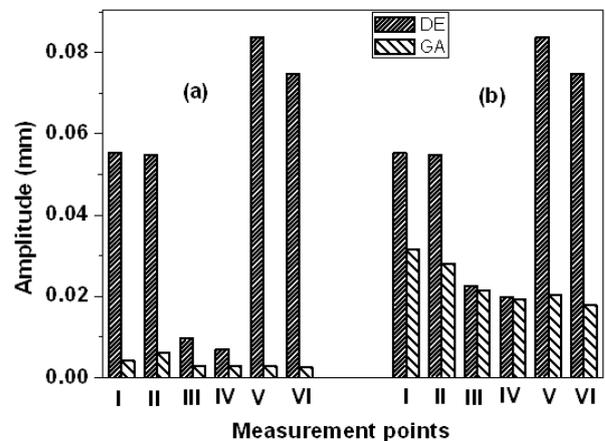


Fig. 9. Comparison between DE and GA at (a) ω_b of 2000 rpm, (b) ω_o of 4000 rpm using Case 1.

rotor at low or high balancing speed, it requires only one measurement instrument at each measurement point, it saves cost and time, it does not require high professional training and the proposed approach follows neither modal nor influence-coefficient method approaches. However,

experimental part validating the proposed technique will be carried out in the future work.

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