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The influence of hygrothermal effects on the cross-ply composite laminate with transverse cracking in transient mode

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Abstract – The stiffness reduction of symmetric hygrothermal cracked and aged cross-ply laminates is predicted by using two different models, shear-lag and variational approach. The material properties are considered to be dependent on temperature and transient moisture distribution in desorption case. This transient hygrothermal effect is taken into account to access the changes in the longitudinal modulus due to transverse cracking. The obtained results illustrate well the dependence of the stiffness degradation on the cracks density and transient environmental conditions, especially at high temperature. Through the presented study, we hope to contribute to the understanding of hygrothermal behaviour of cracked composite laminate.

Key words: Damage / stiffness reduction / transverse crack / hygrothermal aging / cross ply / moisture desorption

1 Introduction

In the case of cross-ply laminates submitted to uniaxial loading, the first damage which appears is often matrix cracking in the off-axis plies. It is clear, therefore, that the capability to predict the occurrence of transverse cracking and its effect on stiffness degradation is necessary in the design and utilization of cross-ply laminate. Matrix cracks and their effects on material properties degradation gained much attention both experimentally, numerically and analytically due to their practical importance.

Groves et al. [1], have examined experimentally the matrix cracking phenomenon in polymer composite cross-ply laminates. The first results show a degradation of the mechanical properties due to transverse cracks. Many analytical models have been developed which attempt to evaluate the stress distribution and the stiffness degradation due to transverse cracks. A more refined method is the shear lag method used by Berthelot et al. [2], Nairn and Mendels [3], Yokozeki and Aoki [4], and recently modified by Tounsi et al. [5], Adda Bedia et al. [6] by introducing the stress perturbation function. The simplicity of the method and the good accuracy are the strong points of this model. Another method based on the principal of minimum complementary potential energy is the variational approach which was used by Hashin [7], Vinogradov et al. [8] Barbero and Cosso [9], Hajikazemi et al. [10], Katerelos et al. [11] This method can provide acceptable values, for upper and lower bounds of the stiffness.

A generalised plane strain model for the evaluation of the stress fields in $[0/90]_s$ laminates, loaded in tension with a regular crack array in the 90° ply was proposed by McCartney [12]. The in-plane stresses were considered constant through the thickness of each layer, and they were written as the sum of their nominal value, computed with a stress perturbation, function of the transverse coordinate of the 90° ply and the Classical Lamination Theory (CLT). A fourth order differential equation was obtained for the stress perturbation function by solving the 2D equilibrium equations. The result was identic to that obtained by Hashin [7], a part from the fact that a generalised plane strain hypothesis was assumed by McCartney.

Crack-Faces-Displacement (CFD) models are based on the micromechanical theorem, which states that the global laminate strains can be computed by averaging the strains over the volume of each layer. By means of the divergence theorem it turns out that this process is equivalent to averaging the displacement components over the surfaces of each layer. Lundmark and Varna [13] have developed calculation scheme to predict stiffness reduction

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Nomenclature

$A_0(x)$	Displacement parameter in the 0° ply
$A_{90}(x)$	Displacement parameter in the 90° ply
D_z	Diffusivity in the transverse direction
E_{fx}	Young modulus of fibre in longitudinal direction
E_{fy}	Young modulus fibre in transverse direction
E_m	Young modulus of the matrix
$f(z)$	Function of displacement
υ_{fx}	Poisson's ratio in the direction of the fibre
υ_m	Poisson's ratio of the matrix
G_{fx}	Shear modulus of the fibre
G_m	Shear modulus of the matrix
g	Ratio temperature concentration
h	Thickness of plate
V_f	Volume of fibres
E_x	Longitudinal Young's modulus damaged cross-ply
E_{x0}	Longitudinal Young's modulus undamaged cross-ply
E_0	Young modulus of 0° -layer
E_{90}	Young modulus of 90° -layer
G_0	Shear modulus of 0° -layer
G_{90}	Shear modulus of 90° -layer
η_t	Transfer parameter of mechanical loading across the thickness of 0° -layer
υ_0	Poisson's ratio of 0° -layer
υ_{90}	Poisson's ratio of 90° -layer
σ_c	Applied tensile stress
σ_{xx}^0	Longitudinal stress of 0° -layer
σ_{xx}^{90}	Longitudinal stress of 90° -layer
σ_0	Applied stress in 0° -layer
σ_{90}	Applied stress in 90° -layer
t_0	Thickness of 0° -layer
t_{90}	Thickness of 90° -layer
t	Time
T_{opr}	Operational temperature
T_g	Transition temperature
T_{0g}	Transition temperature to the reference temperature
T_{rm}	Room temperature
$\overline{\varepsilon_{xx}^0}$	Average strain
l	Half-length between two consecutive cracks
$\overline{l_0}$	Half-length between two consecutive cracks standardized
α	Stacking parameter of layer 0° and 90°

in the entire crack density region. The degradation of thermo-elastic properties of a laminate strongly depends on the intralaminar crack surface opening and sliding during loading. Convergence has been proved to occur after a very small number of iterations, but the final result was not in satisfactory agreement with FE models.

Another method which uses to analyse the dependence of the COD (crack opening displacement) on the crack density based on the finite element model was used by Joffe and Krasnikovs [14], Singh and Talreja [15], Akula and Garnich [16]. This model has shown quite interesting results which converge accurately to the experimental data. Li et al. [17] presented a technique for modelling multidirectional laminates in the presence of regular crack arrays in multiple layers with no more than two different fibres orientations. The main disadvantage of this method is that it allows to model no more than two crack directions.

The increase of the moisture inside the composite materials at the time of absorption or desorption phase, can modify the mechanical properties of these structures, Han and Nairn [18], Shen and Springer [19]. Moreover, following to prolonged exposure in a wet atmosphere and at variable temperatures, the degradation of these properties with time under various conditions during the use of structures, can be more severe [20, 21]. Recently, Amara et al. [22], Tounsi et al. [23,24] examined theoretically the changes of the longitudinal Young's modulus on a cracked composite, using the shear-lag method and Hashin model. The material properties are considered to be dependent on temperature and moisture. A simple hygrothermal model (with constant temperature and moisture), was used to predict the stiffness degradation on the transverse crack density. They concluded that the degradation of elastic properties depends on the cracks density and hygrothermal conditions.

In this paper, two analytical models have been studied and compared with experimental test [1]. The shear-lag model, modified by introducing the stress perturbation function and the variational approach, are used to predict the effect of transverse cracks on the longitudinal Young's modulus degradation. Then, the cross-ply laminates are initially exposed to the hygrothermal aging, and submitted to transient and non-uniform moisture concentration distribution, for desorption case. For that the model which will enable us to introduce ageing and to see its development on the fibre and matrix scales, is the transient Tsai model [25]. This model allows us to predict the best representation of hygrothermal effects, on cracked composite laminate, compared to other works already done [22–24].

2 Analysis

We consider a symmetric cross-ply laminate which is subjected to uniaxial loads. It is assumed that the 90° ply has developed continuous intralaminar cracks in fibre direction which extend from edge to edge in z direction (Fig. 1).

2.1 Shear lag model

In first-order shear deformation theory (FSDT), the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. Thus, a shear correction factor, which is hard to find because it depends on many parameters as cited by Menaa et al. [26], is required to compensate error because of assumption in FSDT. Recently, new plate theories, based on non-linear variation in the in-plane displacements through

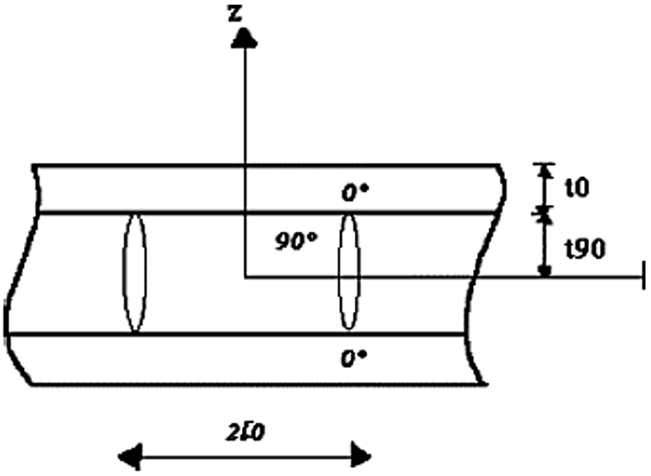

Fig. 1. Transverse cracked cross ply laminate and geometric model.

the thickness and do not require shear correction factor, are developed for plates by Tounsi et al. [27], Ait Yahia et al. [28], Ait Amar et al. [29], Belabed et al. [30], Bousahla et al. [31], Hebali et al. [32], and for beams by Bourada et al. [33], Ould Larbi et al. [34].

We have used in this study the model developed by Berthelot [2]. This later is modified by introducing the stress perturbation function. The axial stress is transferred by shear in thin layer. The longitudinal displacement in both layers 0° plies and 90° ply is [2]:

$$u_0(x,z) = \overline{u_0}(x) + f(z)A_0(x) \tag{1}$$

$$u_{90}(x,z) = \overline{u_{90}}(x) + \left(z^2 - \frac{t_{90}^2}{3}\right)A_{90}(x) \tag{2}$$

where $\overline{u_0}(x)$ and $\overline{u_{90}}(x)$ are the average longitudinal displacement in 0° plies and the 90° ply, respectively. $A_0(x)$, $A_{90}(x)$ and $f(z)$ to be determined.

The average longitudinal stresses in 90° and 0° layers are in the following forms:

$$\overline{\sigma_{xx}^{90}} = \sigma_c \frac{E_{90}}{E_x^0}\left(1 - \frac{\cosh \eta a \frac{x}{l}}{\cosh \eta a}\right) \tag{3}$$

$$\overline{\sigma_{xx}^{0}} = \sigma_c \frac{E_0}{E_x^0}\left(1 + \frac{t_{90}E_{90}}{t_0E_0}\frac{\cosh \eta a \frac{x}{l}}{\cosh \eta a}\right) \tag{4}$$

with $a = \frac{l}{t_{90}}$ is the cracking aspect ratio and $\eta^2 = 3\left(1+\frac{1}{\alpha}\right)\frac{GE_x^0}{E_0E_{90}}$ is the shear transfer parameter.

The longitudinal Young's modulus E_x of the cross-ply laminate relates the average stress σ_c to the average longitudinal strain ε_c of the laminate

$$\sigma_c = E_X\, \varepsilon_c \tag{5}$$

The average strain ε_c is equal to average longitudinal strain of 0° -layer [2]:

$$\varepsilon_c \approx \overline{\varepsilon_{xx}^0} = \frac{1}{l}\int_0^l \frac{\overline{\sigma_{xx}^0}}{E_0}\,\mathrm{d}x \tag{6}$$

The final expression of the longitudinal Young's modulus can be written as [2]:

$$\frac{E_x}{E_x^0} = \frac{1}{1 + \frac{t_{90}}{t_0}\frac{E_{90}}{E_0}\frac{1}{\eta a}\tanh \eta a} \tag{7}$$

Two different analytical functions of the variation function have been considered [2]:

(a) A complete parabolic model with :

$$f(z) = z^2 - 2(t_0+t_{90})z + \frac{2}{3}t_0^2 + 2t_0t_{90} + t_{90}^2 \tag{8}$$

(b) The variation of the longitudinal displacement is supposed progressive in 0°-layer with:

$$f(z) = \frac{\sinh \alpha\eta_t}{\alpha\eta_t} - \cosh \eta_t\left(1 + \alpha - \frac{z}{t_{90}}\right) \tag{9}$$

2.2 Variational approach

There is another method, relatively simple, the variational approach [7], which satisfy equilibrium and all boundary and interface conditions to find an optimal approximation to the principle of minimum complementary energy. By assumption, normal stresses in load direction σ_{xx} are constants depending on the thickness (z) and the width (y) in the 90° and 0° layers respectively and may be expressed in the form:

$$\sigma_{xx}^{90} = \sigma_{90}(1 - \phi_1(x)) \tag{10}$$

$$\sigma_{xx}^{0} = \sigma_{0}(1 - \phi_2(x)) \tag{11}$$

$\phi_1(x)$ and $\phi_2(x)$ are unknown functions.

Next, we denote

$$\phi_1(x) = \phi(x) \tag{12}$$

And express $\phi_2(x)$ in term of $\phi(x)$ due to equilibrium condition in x direction:

$$\sigma_0t_0\phi_1(x) + \sigma_{90}t_{90}\phi_2(x) = 0 \tag{13}$$

The final expression for complementary energy will be in form of:

$$u_c' = \frac{1}{2}\sigma_{90}^2t_{90}^2\int_{-a}^{+a}\left(C_{00}\phi^2 + C_{02}\phi\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} + C_{22}\phi\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2}\right)^2 + C_{11}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2\right)\mathrm{d}\xi \tag{14}$$

where

$$C_{00} = \frac{1}{E_{90}} + \frac{1}{\alpha E_0} \tag{15}$$

$$C_{02} = \frac{\upsilon_{90}}{E_{90}}\left(\alpha + \frac{2}{3}\right) - \frac{\alpha\upsilon_0}{3E_0} \tag{16}$$

$$C_{22} = \frac{1}{60E_{90}}(\alpha+1)(3\alpha^2 + 12\alpha + 8) \tag{17}$$

$$C_{11} = \frac{1}{3}\left(\frac{1}{G_{90}} + \frac{\alpha}{G_0}\right) \tag{18}$$

The function ϕ which minimizes the complementary energy $u_c^{'}$ is the fourth order differential equation of Euler-Lagrange:

$$\frac{\mathrm{d}^4\phi}{\mathrm{d}\xi^4}+p\frac{\mathrm{d}\phi^2}{\mathrm{d}\xi^2}+q\phi=0 \tag{19}$$

where

$$p=\frac{C_{02}-C_{11}}{C_{22}} \text{ and } q=\frac{C_{00}}{C_{22}} \tag{20}$$

when $4q>p^2$.

Then the final solution is:

$$\phi=A_1\mathrm{ch}\alpha a\frac{x}{l}\cos\beta a\frac{x}{l}+A_2\mathrm{sh}\alpha a\frac{x}{l}\sin\beta \tag{21}$$

A_1 and A_2 are determined by the boundary conditions:

$$\text{and } \alpha=q^{1/4}\cos\theta/2 \;\; \beta=q^{1/4}\sin\theta/2\theta=\mathrm{arctg}\sqrt{\frac{4q}{p^2}-1} \tag{22}$$

The stiffness reduction E_x can be expressed by [7]:

$$\frac{E_x}{E_{x0}}=\frac{1}{1+\frac{E_{90}}{E_0}\frac{t_{90}}{t_0}\frac{1}{\overline{l_0}}g(\overline{l_0})} \tag{23}$$

where $g(\overline{l_0})$ is expressed by:

$$g(\overline{l_0})=\frac{2\delta\beta}{\delta^2+\beta^2}\frac{\cosh(2\delta\,\overline{l_0})-\cos(2\beta\,\overline{l_0})}{\delta\sin(2\beta\,\overline{l_0})+\beta\sinh(2\delta\,\overline{l_0})} \tag{24}$$

3 Hygrothermal aging model

The study, here has been focused on the stiffness reduction due to transverse ply cracking in cross ply laminate, when this latter is initially exposed to the transient hygothermal aging. Graphite/epoxy composite material [25] was selected in the present example. Tsai [25] proposes the adimensional temperature T^*, which is the essential parameter for evaluation of the hygrothermal effect in stress distribution:

$$T^*=\frac{T_g-T_{\mathrm{opr}}}{T_g-T_{\mathrm{rm}}} \tag{25}$$

We further assume that moisture suppresses the glass transition temperature by simple temperature shift.

$$T_g=T_g^0-gc \tag{26}$$

Let us consider a laminated plate of thickness h made of polymer matrix composite, submitted on it two sides to the same dry environment. The plate is considered to be infinite in both x and y directions and the moisture varies only in the z direction. The initial moisture concentration C_{init} is uniform at $t=0$. Both sides of the plate are suddenly exposed to a zero moist environment (Fig. 2). The moisture concentration inside the plate is described by Fick equation with diffusivity D_z.

$$\frac{\partial C}{\partial t}=D_z\frac{\partial^2 C}{\partial z^2} \tag{27}$$

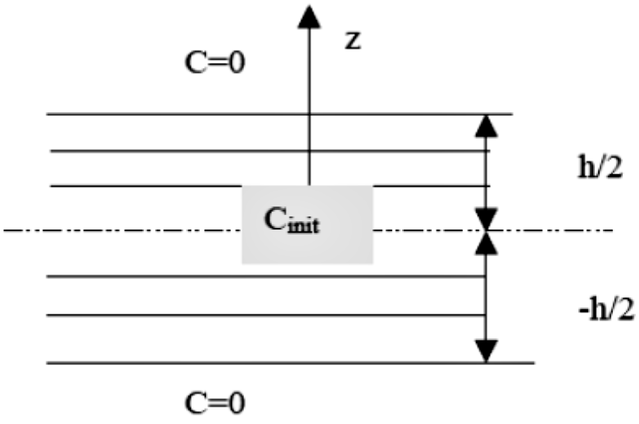

Fig. 2. Desorption phase.

with the initial conditions:

$$C=C_{\mathrm{init}} \text{ for } -h/2\le z\le h/2 \text{ and } t=0$$
$$C=0 \text{ for } z=-h/2;z=h/2, \text{ and } t>0$$

The initial conditions being uniform and the boundary conditions are constants. The unidimensional solution of Fick equation can be expressed as [35]:

$$C(z_k,t)=\left[\frac{4C_{ini}}{\prod}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos\left(\frac{(2n+1)\prod z_k}{h}\right)\times\exp\left(\frac{-D_z(2n+1)^2\prod^2 t}{h^2}\right)\right] \tag{28}$$

4 Results and discussion

4.1 Reduction of longitudinal Young's modulus

In this section, we will validate the results without taking into account the hygrothermal effect on the material properties. The results are compared with experimental data for graphite/epoxy AS4-3502, where the material properties are summarized in Table 1.

The reduction of stiffness due to the transverse crack for AS/3502 laminate is shown in Figures 3 and 4. The figures exhibit the comparison between the analytical models and the experimental data (Groves et al. 1987). It can be seen that good agreement is obtained between the shear-lag model and experimental data. On the other hand, the variational approach gives greater reduction than the experimental tests [1]. Finally, it can be deduced that the reduction of stiffness is higher as we have more 90° layers in laminates.

4.2 Influence of hygrothermal conditions on the reduced stiffness

The stiffness reduction is represented in cracked cross ply laminate exposed to hygrothermal conditions. Two

Table 1. Material properties of composite AS4-3502 systems used in calculations [36].

Properties Material	E_0 (GPa)	E_{90} (GPa)	G_0 (GPa)	G_{90} (GPa)	υ_0	υ_{90}	t_0 (mm)
AS4-3502	144.8	9.58	4.79	4.2	0.31	0.4	0.127

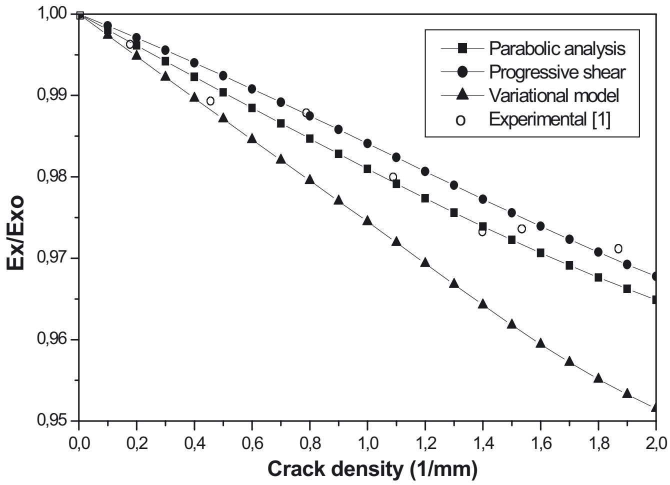

Fig. 3. Longitudinal Young's modulus degradation due to transverse cracks in a $[0/90]_s$ AS/3502 laminate.

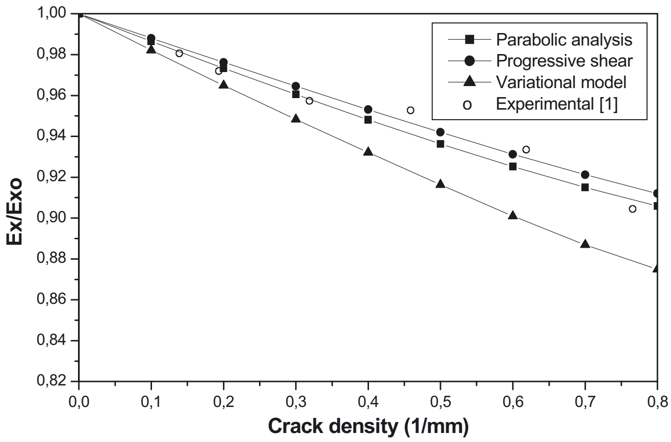

Fig. 4. Longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ AS/3502 laminate.

different hygrothermal modes, uniform and transient moisture concentration have been selected, to represent the effect of temperature and moisture in the cracked cross-ply laminates. The time chosen for simulation is taken equal to t_{sat}, with t_{sat} equal to 4222 h, and is the moisture saturation for T300/5208. The characteristics of fibre and matrix of graphite/epoxy (T300/5208) $[0/90_3]_s$ laminate are exposed in Table 2.

4.3 Moisture effect on normalized axial modulus

The loss of stiffness in the laminate as of crack density is evaluated compared to the initial stiffness of the same uncracked laminate and for the same environmental case.

Table 2. Fiber and matrix characteristics of graphite/epoxy material (T300/5208) [36].

E_{fx} (Gpa)	E_{fy} (Gpa)	ν_{fx}	E_m (Gpa)	ν_m	G_m (Gpa)	G_{fx} (Gpa)	V_f
259	18.69	0.25	3.4	0.35	1.26	19.69	0.7

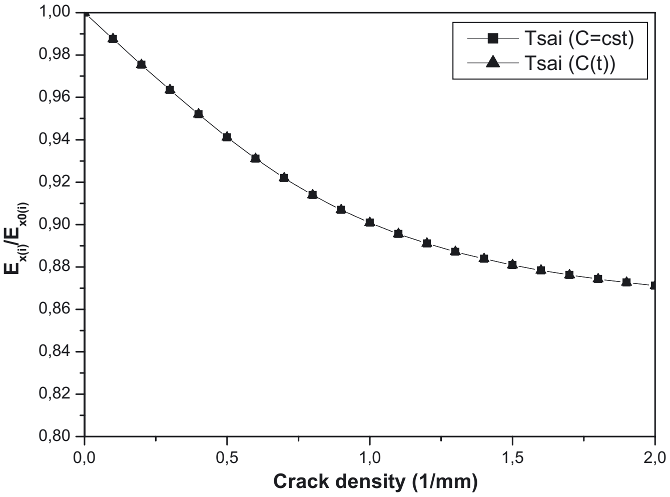

Fig. 5. Moisture effect on the longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ Graphite/epoxy laminate T300/5208 ($T_{\text{op}} = 22$ °C and $C = 0.5\%$).

We note that this initial stiffness of the uncracked laminate is function of temperature and moisture distribution. Consequently, Equations (7) and (23) become:

$$\frac{E_{x(i)}}{E^0_{x(i)}} = \frac{1}{1 + \frac{t_{90}}{t_0}\frac{E_{90(i)}}{E_{0(i)}}\frac{1}{\eta_{(i)}a}\tanh \eta_{(i)}a} \tag{29}$$

$$\frac{E_{x(i)}}{E^0_{x(i)}} = \frac{1}{1 + \frac{E_{90(i)}}{E_{0(i)}}\frac{t_{90}}{t_0}\frac{1}{l_0}g_{(i)}(\overline{l_0})} \tag{30}$$

The index (i) represents the considered case of the environmental conditions.

The obtained results are reported in Figures 5 to 7 using a parabolic variation of longitudinal displacement in both 0° and 90° layers. The longitudinal Young's modulus is represented with two different hygrothermal models, uniform concentration, and transient moisture distribution in desorption case. The results show that the moisture has no effect on the normalized axial modulus. Only the crack density influences this latter.

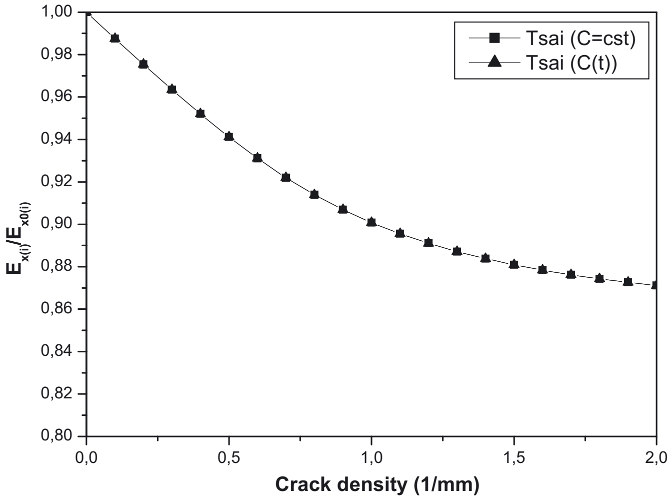

Fig. 6. Moisture effect on the longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ Graphite/epoxy laminate T300/5208 (T_{op} = 22 °C and C = 1%).

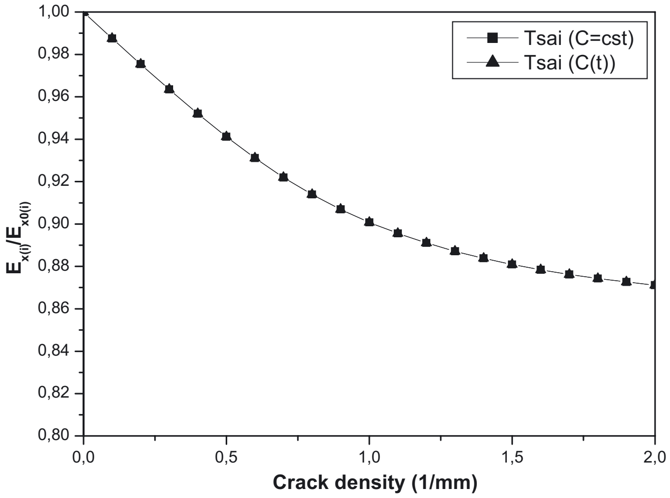

Fig. 7. Moisture effect on the longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ Graphite/epoxy laminate T300/5208 (T_{op} = 22 °C and C = 1.5%).

4.3.1 Temperature effect on normalized axial modulus

In the same as presented above, two sets of environmental conditions are considered. For environmental case 1, $T_{\text{op}} = 60°$ and for environmental case 2, $T_{\text{op}} = 120°$. In the two cases, the moisture is supposed to be equal to $C = 1.5\%$. The results in Figures 8 and 9 show that starting from T_{op} = 60 °C, a little difference is noticed between transient Tsai model and Tsai. Increasing the operational temperature, the transient Tsai model gives more reduction of the relative stiffness, especially at high crack density.

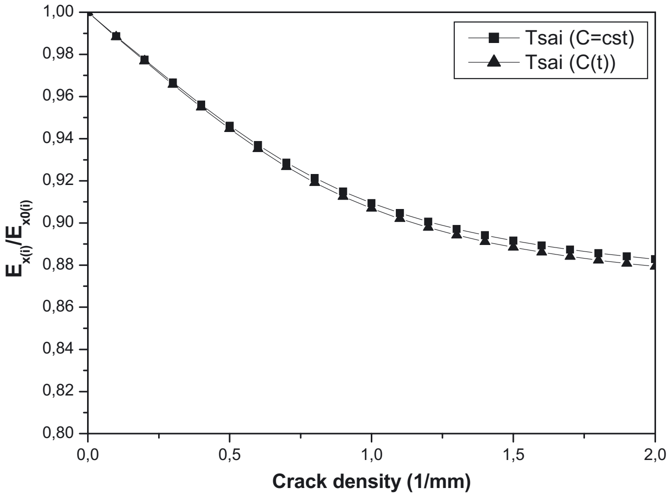

Fig. 8. Temperature effect on the longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ Graphite/epoxy laminate T300/5208 (T_{op} = 60 °C and C = 1.5%).

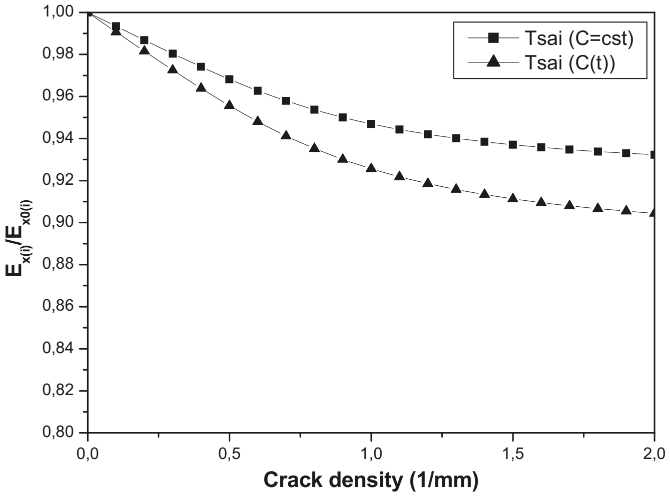

Fig. 9. Temperature effect on the longitudinal Young's modulus degradation due to transverse cracks in a $[0/90_3]_s$ Graphite/epoxy laminate T300/5208 (T_{op} = 120 °C and $C = 1.5\%$).

5 Conclusion

Different analytical models have been proposed, to predict the effect of transverse cracks on the stiffness reduction of hygrothermal aged composite laminates. The material properties are considered to be dependent on temperature and moisture distribution, which are given explicitly in terms of the fibre and matrix properties and fibre volume ratio. The result shows that the moisture has no effect on the normalized axial modulus. However, at high temperatures, the transient Tsai model gives more reduction of the relative stiffness than obtained with uniform moisture concentration. For that, in desorption case, the chosen material will be weaker at high crack density and hygrothermal conditions. Finally, at high environmental conditions, the transient Tsai model seems more accurate to predict the true hygrothermal behaviour of cracked cross-ply laminates.

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