Mechanics
& Industry

High temperature gas effect on the Prandtl-Meyer function with application for supersonic nozzle design

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Abstract – This work is to develop a new computational program to determine the effect of using gas propellant of the combustion chamber at high temperature on the calculation of the value of the Prandtl Meyer function and application to supersonic nozzles design. The selected gases are the H_2, O_2, N_2, CO, CO_2, H_2O, NH_3, CH_4 and air. Prandtl Meyer function depends on the stagnation temperature, Mach number and the gas used. The specific heat at constant pressure and the enthalpy of the gases vary with the temperature and the selected gas. The gas is still considered as perfect and it will be calorically imperfect and thermally perfect below the threshold of dissociation of molecules. A calculation of the difference between the Prandtl Meyer function for different gases with the air is made for the purpose of comparison. The application is made for designing the supersonic *MLN* nozzle at high temperature.

Key words: Supersonic flow / supersonic nozzle / design / high temperature / Prandtl Meyer function / calorically imperfect gas / numerical integration / specific heat at constant pressure / gas / error

1 Introduction

Gases play an important role in the propulsion of aerospace vehicles. They influence the behavior of the supersonic flows and in particular on all design parameters [1,2]. For supersonic nozzles of rockets, missiles and supersonic aircraft engines, it is desirable to have small masses of the craft and high possible thrust. For supersonic blowers, it is desired to have nozzles with low temperature distribution and a large possible of exit section to place the prototype and the tools and measuring devices without damage [1,2].

In the design of aerospace propulsion several types of supersonic nozzle are encountered, each with practical value, performance and favored scope. Examples bell shaped nozzle, *MLN* nozzle, plug nozzle, expansion deflexion nozzle, annular nozzle, dual bell nozzle, Laval nozzle, etc. [2–8]. Each constructor uses a specific form according to the performance delivered by the nozzle [2–8] which falls in accordance with the need for construction. We are interested in the present work to make an application to the *MLN* nozzle to see its potential advantage in what has minimum length and weight and high possible thrust from the majority of the indicated nozzles [2–8].

We have to obtain the performances indicated previously (small weight, large C_F), using a propellant gas

that meets these requirements. Usually these properties are found for the propulsion of gas called cold gas [9,10]. Air meets the need for more reason because it exists in quantities in nature.

References [5,11–17] have studied at high temperature for air on the supersonic nozzles design by the method of characteristics. This method is based on the *PM* function. For air at high temperature, it is presented in references [18,19].

At low temperatures, C_P and γ are assumed to be constant and do not depend on temperature. Studies on the design of supersonic nozzles and the *PM* function are shown in references [1,2].

The aim of this work is to develop a new computational program to study the effect of using gas of propulsion rest on at high temperature on the *PM* function and to make application for designing the supersonic *MLN* nozzle. The molecules of the selected gases have one or two kinds of atoms which are H_2, O_2, N_2, CO, CO_2, H_2O, NH_3, CH_4 and air. For substances still the gas temperature must be in a specific range for not having a dissociation of the molecules. This interval varies from one gas to another [20–27], usually between 200 K and 6000 K [20–27] except for air under 3550 K [1,2]. The gas is considered as perfect, including the equation of state ($P = \rho RT$) still valid. It is considered calloricaly imperfect seen the change in the function $C_P(T)$ varying with temperature.

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Nomenclature

M	Mach number
V	Velocity
μ	Mach angle
x	Abscissa of the a section in the nozzle
y	Radius of the nozzle section
T	Temperature
ρ	Density
A	Section area of the nozzle
R	Thermodynamic constant of gas
H	Enthalpy
C_P	Specific heat at constant pressure
γ	Specific heat ratio
θ	Deviation of the flow at the throat of the nozzle
ν	Prandtl Meyer function
ε	Error
a', b', c'	Coefficients of $C_P(T)$ in Equation (1)
C_F	Pressure force coefficient for the nozzle
C_{Mass}	Mass of the nozzle in adimensional value
L	Length of the nozzle
PM	Prandtl Meyer
MLN	Minimum Length Nozzle
HT	High Temperature
PG	Perfect Gas
Indices	
0	Stagnation state (combustion chamber)
*	Critical condition
S	Supersonic state
E	Exit section

It is considered again thermally perfect because the temperature of the dissociation of molecules is not reached. For the chosen gas in a variation according to the relation (1) of $C_P(T)$ by references [20–27] is found.

$$C_P(T) = a' + b'T + \frac{c'}{T^2} \qquad (1)$$

Nine gases are selected, indicated by Table 1, They are selected from the group of gases found in the literature [20–27]. We focus on the specific heat $C_P(T)$ at high temperature and R gas constant [20–27]. The coefficients of the function (1) for the selected gas are shown in Table 1.

Concerning the air, the variation of $C_P(T)$ is chosen as a polynomial of 9th degree [4,5,11–19].

The values of C_P and γ at low temperature under an ideal gas, and for calorically and thermally perfect gas for 9 selected gases and the constant R are shown in Table 2 [20–27].

2 Mathematical formulation

The deviation of a supersonic flow can cause expansion or compression. When the intensity of the shock $\Delta P/P$ approaches zero, provide the necessary tool for the study

Table 1. Coefficients of $C_P(T)$ function for selected gases.

N	Gas	a' J.K^{-1}.mol^{-1}	b' J.K^{-2}.mol^{-1}	c' J.K.mol^{-1}
1	H_2	27.28	3.26	0.50
2	O_2	29.96	4.18	-1.67
3	N_2	28.58	3.76	-0.50
4	CO	28.41	4.10	-0.46
5	CO_2	44.22	8.79	-8.62
6	H_2O	30.54	10.29	0.08
7	NH_3	29.75	25.10	-1.55
8	CH_4	23.64	47.86	-1.92
9	Air	Polynomial of 9th degree [4,5,11–19]		

of this phenomenon is provided [28,29]. Normal speed to the wave is the speed of sound. A small flow deviation $\Delta\theta$ downstream of the wave is linked to a speed difference ΔV by the following relationship [28,29]:

$$d\theta = -\frac{\cot g(\mu)}{V}dV. \qquad (2)$$

By inserting the relationship between the Mach angle μ and the Mach number M in Equation (2) yields:

$$d\theta = d\nu = \left(M^2 - 1\right)^{1/2}\frac{dV}{V} \qquad (3)$$

The relationship (3) is very important for to the study of the supersonic isentropic flow external or internal. Note that the angle $d\theta$ is counted positive when the flow is away from the direction normal to the wave (compression wave) and negative when the flow approaches the direction normal to the wave (expansion wave).

The energy conservation equation in differential form [3,4,11–17,28] is written:

$$C_P dT + V dV = 0 \qquad (4)$$

Which can be written after doing mathematical transformations:

$$\frac{dV}{V} = -\frac{C_P(T)}{2H(T)}dT \qquad (5)$$

For the integration of Equation (3), it should be expressed dV/V by the expression (5) [18,19]. After integration between the sonic state ($T = T_*$, $M = 1.00$) and the supersonic state ($T < T_*$, $M > 1.00$) we obtain:

$$\nu(T) = \int_T^{T_*} \frac{C_P(T)}{2H(T)}\sqrt{M^2(T) - 1}dT \qquad (6)$$

The values of M and T are connected by [3,4,11–19].

$$M(T) = \sqrt{\frac{2H(T)}{\gamma(T)RT}} \qquad (7)$$

With

$$\gamma(T) = \frac{C_P(T)}{C_P(T) - R} \qquad (8)$$

$$H(T) = \int_T^{T_0} C_P(T)\,dT \qquad (9)$$

Table 2. Values of CP and γ for perfect gas assumptions and thermodynamics constants R for some selected gases.

N	Gas	Molecular weight (g)	C_P J.kg^{-1}.K^{-1}	γ J.kg^{-1}.K^{-1}	R
1	H_2	2.01588	14304.594	1.4051	4124.506
2	O_2	31.99880	918.1079	1.3947	259.8381
3	N_2	28.01348	1039.6548	1.3995	296.8038
4	CO	28.01040	1040.3705	1.3992	296.8385
5	CO_2	44.00980	844.22714	1.2883	188.9240
6	H_2O	18.01528	1864.4012	1.3289	461.5254
7	NH_3	17.03056	2092.1466	1.3043	488.2111
8	CH_4	16.04276	2224.7529	1.3037	518.2717
9	Air	28.96003	1001.2893	1.4020	287.1029

To demonstrate the relationship (7), we use the definition of Mach number depending on the speed of sound and the flow velocity and to demonstrate the relationship (9) it is used for energy conservation equation in differential form and to make integration between a supersonic state having the temperature T and the stagnation state having the temperature T_0.

The value of T_* and T_S corresponding, respectively, to $M = 1.00$ and $M = M_S$ can each one be obtained by the resolution of Equation (7) by using the bipartition algorithm [30, 31]. The two values depend on T_0, R and $C_P(T)$ of each gas. The value of ν_S can be obtained by integration of the function (2) in the interval $[T_S; T_*]$ by using the Simpson's quadrature [18] or by special Gauss Legendre quadrature [19] for fast calculation.

To make a comparison between the case at high temperature and the case of an ideal gas, we recall the Prandtl-Meyer function given by the theory of an ideal gas [29, 32]:

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \text{arctg} \sqrt{\frac{\gamma+1}{\gamma-1}(M^2-1)} - \text{arctg}\sqrt{M^2-1} \qquad (10)$$

The PM function (10) is explicitly connected with the Mach number, which is the main variable for the ideal gas, on the contrary to our model High Temperature, where the main variable is the temperature due to the Equation (7) linking M and T; we cannot determine an analytical expression of its inverse.

Then the relation (6) obtained is a generalization of the PM function of the case of an ideal gas (10) with constant specific heats. Thus the expression of the PM function of the high temperature gas (Gas calorically imperfect and thermally perfect) is obtained when the variation of the specific heat C_P depending on the temperature is held in account. The most T_0 or M increases, the error between the relations (6) and (10) also increases and becomes significant.

The most application and use of the PM function are in the compatibilities and characteristics equations for designing of supersonic nozzle, like MLN by the method of characteristics. Figure 1 shows the various regions of the flow that can have MLN nozzle to have uniform and parallel flow to the exit section. So the area OAB is appointed by Kernel region. It is of not simple type. The region ABE
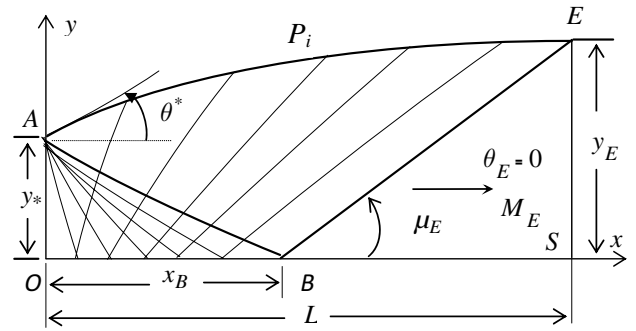
Fig. 1. Presentation of the flow field in the nozzle.
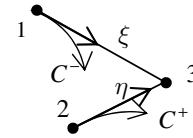
Fig. 2. Illustration Mach lines upward and downward.

is named by transition region. It is a simple region and the BSE region is appointed by uniform region. However, the Mach number is constant at all points in this region. The wall of the nozzle is a priori unknown. It is determined numerically for the desired condition. The search of the wall and the calculation of the internal flow are done by the method of characteristics in the case assumptions at HT [3, 13, 14].

The calculation is based on the use of the method of characteristics at HT. The equations of compatibilities and characteristics respectively valid on upward and downward characteristic, as presented in Figure 2, are represented by [3, 13, 14]:

$$\begin{cases} -\frac{C_P(T)}{2\,H(T)}\sqrt{M^2(T)-1}\,dT + d\theta = 0 \\ \frac{dy}{dx} = tg(\theta-\mu) \end{cases} \qquad (11)$$

$$\begin{cases} -\frac{C_P(T)}{2\,H(T)}\sqrt{M^2(T)-1}\,dT - d\theta = 0 \\ \frac{dy}{dx} = tg(\theta+\mu) \end{cases} \qquad (12)$$

The discretization of the system (11) and (12) gives four algebraic equations with four unknowns $(x, y, T$ and $\theta)$ at each point of the flow field. The resolution was made by the finite difference method with predictor corrector [3, 4, 13, 17].
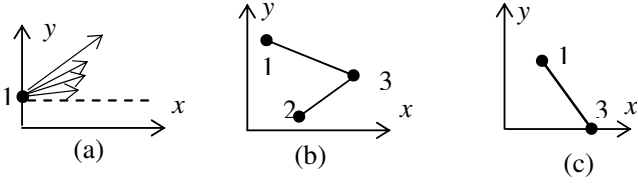
Fig. 3. Different points of the flow field. (a) Point at the expansion center (Point A in Fig. 1). (b) Internal point. (c) Point on the axis of symmetry.

We can have three types of points in the flow as presented in Figure 3.

The flow parameters in the points 1 and 2 of the Figure 3 are known. The problem is to determine the parameters in the point 3 by solving the system of Equations (11) and (12) simultaneously.

Parameters at a point depend only on the properties at the upstream points. It is a property of a supersonic flow [3, 4, 11, 13–17, 29, 32].

3 Comparison with air

The air is generally used as a propellant gas in most aerospace applications considering its existence in quantity in nature. The studies presented in references [3–11] are made only for air. To see the advantage or disadvantage of such gas with respect to air, we determine the difference between the function ν of air and other gases by the following relation.

$$\varepsilon\,(\%) = \left| 1 - \frac{\nu_{\mathrm{Gas}}}{\nu_{\mathrm{Air}}} \right| \times 100 \qquad (13)$$

4 Applications

The case of the flow diversion to the throat of a supersonic nozzle-type MLN giving a uniform and parallel flow at the exit section can be taken as a good example of application. In the 2D case, the deviation of the nozzle θ^* at the throat is connected to the exit Mach number M_E by the following relationship [3, 13, 14].

$$\theta^* = \frac{\nu\,(M_E)}{2} \qquad (14)$$

For axisymmetric case, the angle θ^* is determined numerically based on M_E [8].

Relation (14) is very advantageous in the field of gas dynamics. It connects the exit Mach number M_E of the nozzle with the initial deflection θ^* (by the use of PM function) that must have the nozzle at the nozzle throat. This formula is valid only if the nozzle gives a uniform and parallel flow to the exit section to have, therefore, a complete expansion of the flow through the nozzle, for avoiding the thrust losses caused by the tilting of the nozzle at the exit and for not yet the birth of the shock wave
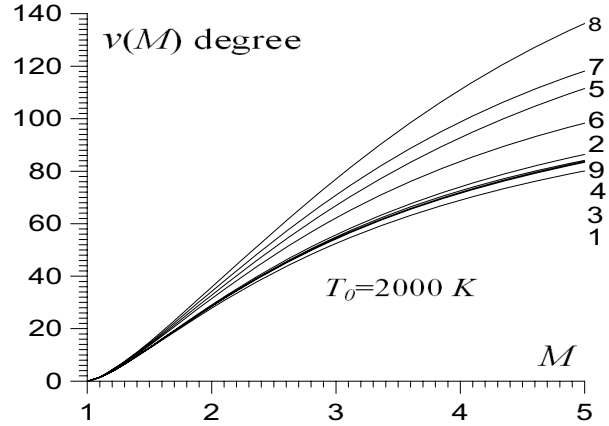

Fig. 4. Gas effect at high temperature on the variation of ν versus M.

in the interior of the nozzle. In such cases if it is the dimensioning of the nozzle giving M_E at the exit can be known the inclination of the wall. This inclination is maximum and there will be a recovery of the wall to the exit section, which is not the case for the axisymmetric nozzle, wherein the nozzle has an inflection point. For the formula (14) can be used the compatibility equations in 2D from conservations equations of gas dynamics by applying the method of characteristics because the flow is supersonic.

We see the influence of the PM function in Equation (14). Then the angle θ^* is half the value of ν corresponding to the ν_S of the nozzle.

The second application is for external flow. The taken example is the dihedron deviation. If the deviation with respect to the dihedron upstream flow is given by θ for example, the Mach number M_2 just after the expansion will be calculated with respect to the upstream Mach number M_1, by solving the following equation.

$$\theta = \nu\,(M_2) - \nu\,(M_1) \qquad (15)$$

The values of M_1 and θ are known and M_2 is unknown. The problem is the inversion of the PM function to determine the value of M_2.

5 Results and comments

Figures 4, 5 and 10 each contain 9 curves for 9 selected gases as shown in Table 1. The curve 1 shows the variation for the H_2 gas. Curve 2 for O_2. Curve 3 for N_2. Curve 4 for CO. Curve 5 CO_2. Curve 6 for H_2O. Curve 7 for NH_3. Curve 8 for CH_4 and the curve 9 for air.

Table 3 presents some values of gas effect on θ^* according to the Mach number for $T_0 = 2000$ K.

Figure 4 shows the effect of propelling gas on the variation of PM according to M. The example chosen is for $T_0 = 2000$ K.

We note that the most gas has a smaller γ ratio, the most value of ν is large, which is the is case of CH_4. The H_2 gas gives the smallest value of ν.

Table 3. Gas effect on θ^* for $T_0 = 2000$ K.

N	Gas	$M_E = 2.00$	$M_E = 3.00$	$M_E = 4.00$	$M_E = 5.00$
1	H_2	13.90 deg	26.22 deg	34.47 deg	40.06 deg
2	O_2	14.52 deg	27.88 deg	37.01 deg	43.21 deg
3	N_2	14.23 deg	27.10 deg	35.79 deg	41.68 deg
4	CO	14.30 deg	27.23 deg	35.96 deg	41.87 deg
5	CO_2	16.38 deg	33.44 deg	46.38 deg	55.74 deg
6	H_2O	15.71 deg	31.07 deg	41.85 deg	49.16 deg
7	NH_3	17.07 deg	35.44 deg	49.38 deg	59.08 deg
8	CH_4	17.90 deg	38.50 deg	55.52 deg	68.15 deg
9	Air	14.31 deg	27.46 deg	36.26 deg	42.06 deg

Fig. 5. Gas effect at high temperature on the variation of ν versus T_0.

Fig. 6. Effect of caloric imperfections on the Prandtl Meyer function for H_2 gas.
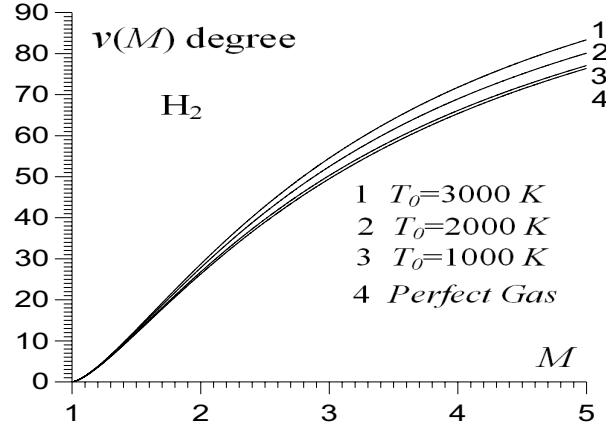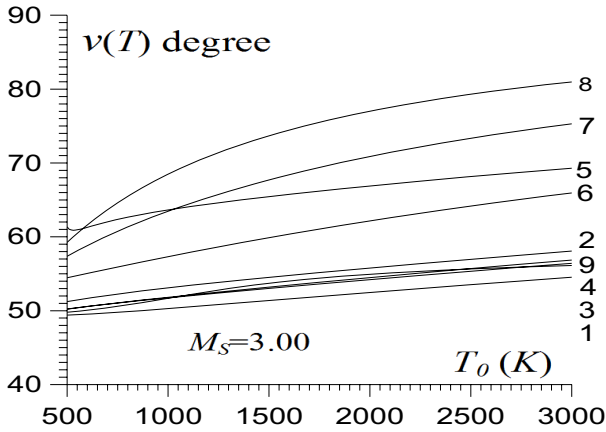
The results for the air shown in curve 9, can be found in references [3, 13, 14]. They are presented for purposes of comparison with other gases.

Figure 5 shows the variation of ν as function of T_0 for the different gases when $M_S = 3.00$. This figure illustrates the effect of gas and T_0 on the value of ν. The most T_0 increases, the most ν is high. Then the most T_0 and M_S are large and γ is small (gas effect), the most the length, weight and thrust coefficient of the nozzle will be high.

In Figures 6−8 are shown the variation of the function ν and the imperfection respectively for all selected gases (H_2, O_2, N_2, CO, CO_2, H_2O, NH_3, CH_4 and air) as a function of M at high temperature for $T_0 = 1000$ K, 2000 K and 3000 K and for the case of perfect gas (low temperature 298.15 K, curve 4). It is noted a difference between the values found but the rate of change is the same for the different gases. For example, when $T_0 = 2000$ K and $M_S = 5.00$, $\nu_{CH4} = 136.31$ degree and $\nu_{H2} = 80.12$ degree.

About imperfection, we notice that it is always greater than unity. This shows that the results of the *PM* function for high temperature case are always greater than those of the ideal gas model case at low temperature. Then the most T_0 is longer, the imperfection will be high, which further shows the influence of the temperature T_0 on the *PM* function. The imperfection can reach 1.72 for CH_4 when $T_0 = 3000$ K, that is to say that the gap may reach 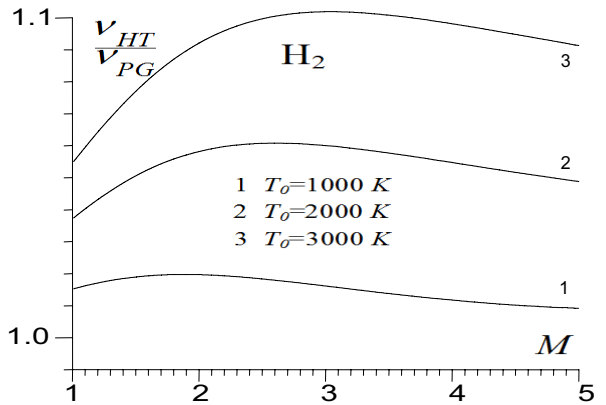72% between the results of the two models. The maximum rate of imperfection, to H_2, O_2, N_2, CO, CO_2, H_2O, NH_3, CH_4 and air can reach respectively to 10.19%, 16.84%, 13.56%, 14.44%, 28.88%, 24.87%, 48.35%, 71.16% and 14.24% for $T_0 = 3000$ K. The most T_0 increases, the most the rate of defect also increases. Higher the rate of imperfection, the most the gas concerned is not convenient for use at this temperature as a propellant gas for aerospace vehicles. This is the case, for example, for CH_4, NH_3, H_2O and CO_2. As a possible solution for these gases, use a stagnation temperature T_0 the smallest possible for aerospace propulsion, but not recommended. While for H_2, O_2, N_2 and air, and since the rate of imperfection is small, we can use a high value of T_0 if necessary.
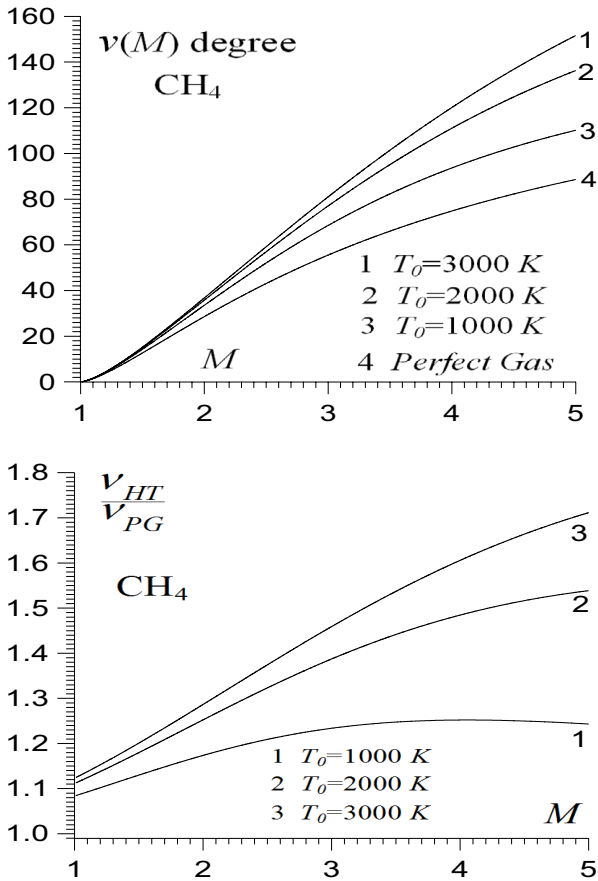
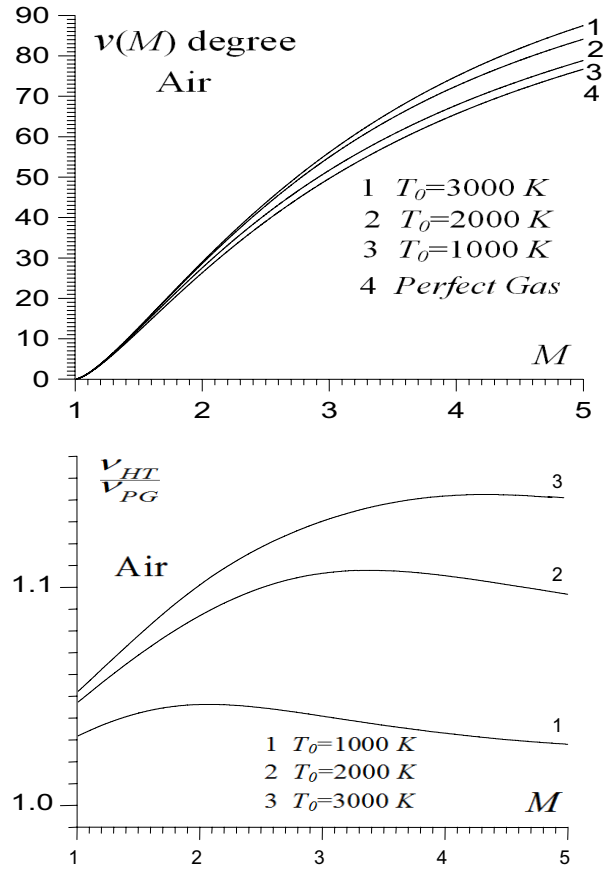**Fig. 7.** Effect of caloric imperfections on the Prandtl Meyer function for CH_4 gas.

Fig. 8. Effect of caloric imperfections on the Prandtl Meyer function for M for air.

Fig. 9. Variation of error on ν for different gases with respect to the air depending on M.

Yet the supersonic Mach number affects the rate of imperfection.

By comparing the results in Figure 8 obtained for the air by our numerical program with results on PM function presented in [18, 19], we note that there is a maximum relative error that does not exceed 2.47%

On Table 4, the effect of gas on the Mach number is shown when the function ν is given for $T_0 = 2000$ K. The calculation of the inverse function ν has practical interest for example in the flow expansion over a dihedral supersonic applied to the external flows. The nozzle is another example for internal flows.

The purpose of calculating the value of M_2, is linked to the importance of Table 4. Is noted from. The Table 4 the influence of gas on the downstream Mach number, will inevitably influence on thermodynamic and all geometric parameters of downstream flow P/P_0, T/T_0, ρ/ρ_0, A/A^*.

Figure 6 shows the error between the ν for different gases and is the air depending on M, where $T_0 = 2000$ K. For H_2, the difference could go to 4.77% and 62.00% for CH_4 when $M_S = 5.00$. This variation depends on M_S, T_0 and the gas itself, considering the PM function equal to zero when $M = 1.00$ ($T = T_*$) from Equation (6). Then the error analysis has problem of calculation by computer (zero under zero). In this case, the obtained error is presented in Table 5.
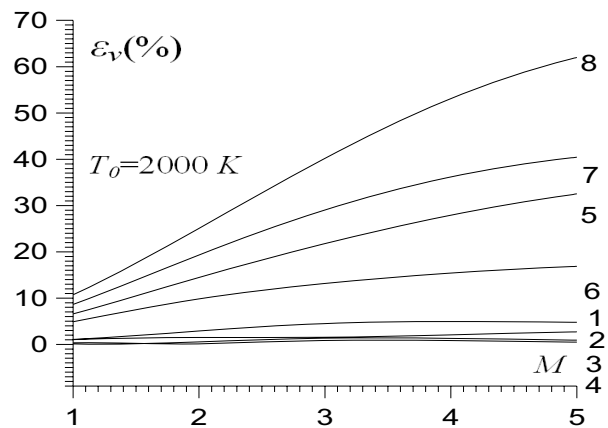
Figure 10 shows an example of the effect of propelling gas on the shape of the supersonic nozzles kind MLN 2D giving all exit Mach number $M_S = 3.00$ for $T_0 = 2000$ K. The purpose of this figure is to show the effect of gas on the initial deflection of the wall at the throat, presented by the angle θ^*, which depends on the PM value, shown in Table 6. Note well the opening and closing of the wall at the throat according to the type of gas. For example, the

Table 4. Gas effect on the value of M when the function ν is given for $T_0 = 2000$ K.

N	Gas	$\nu = 10$ deg	$\nu = 20$ deg	$\nu = 30$ deg	$\nu = 40$ deg	$\nu = 50$ deg	$\nu = 60$ deg
1	H_2	1.420	1.742	2.076	2.448	2.881	3.408
2	O_2	1.410	1.718	2.030	2.369	2.752	3.199
3	N_2	1.414	1.728	2.051	2.404	2.809	3.291
4	CO	1.413	1.726	2.046	2.397	2.798	3.275
5	CO_2	1.386	1.661	1.926	2.196	2.477	2.778
6	H_2O	1.393	1.679	1.959	2.251	2.567	2.918
7	NH_3	1.378	1.643	1.896	2.148	2.408	2.681
8	CH_4	1.370	1.626	1.864	2.098	2.333	2.572
9	Air	1.414	1.726	2.045	2.391	2.782	3.247

Table 5. Error on ν for $M = 1.00$.

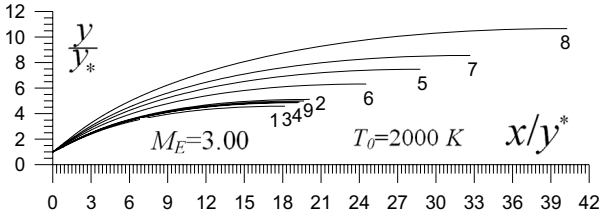| Gas	ε (%)	Gas	ε (%)	Gas	ε (%)
H_2	1.061	CO	0.335	NH_3	0.335
O_2	1.029	CO_2	6.573	CH_4	6.573
N_2	0.082	H_2O	4.893		

Fig. 10. Gas effect at high temperature on the shape of the *MLN* 2D nozzle for nine selected gases.

Table 6. Design values for the nozzles of Figure 10.

N	Gas	θ^* (deg)	L/y^*	C_{Mass}	C_F	y_E/y_*
1	H_2	26.228	18.173	18.724	0.323	4.582
2	O_2	27.885	20.134	20.796	0.357	5.113
3	N_2	27.101	19.175	19.782	0.341	4.853
4	CO	27.239	19.326	19.942	0.344	4.895
5	CO_2	33.441	28.783	29.970	0.482	7.476
6	H_2O	31.078	24.550	25.476	0.426	6.319
7	NH_3	35.440	32.685	34.126	0.529	8.556
8	CH_4	38.503	40.277	42.218	0.607	10.661
9	Air [2]	27.465	19.671	20.306	0.349	4.986

CH_4 gas gives the largest opening, while the H_2 gas gives the smallest opening. This angle will necessarily affect all design parameters, such as Table 6 shows it. The results for aerospace supersonic nozzles applied to missiles and supersonic aircraft are recommended for use for example the H_2 gas, N_2 and CO instead of air. For example, CH_4 and NH_3 gas are not recommended.

In Figures 4, 5 and 10, curves 1, 3, 4, 9 and 2 are quite close to each other, as the calorific value of the gas H_2, N_2, CO, O_2 and air is the same order of greatness course with a difference.

Figures 11−13 show the effect of the stagnation temperature T_0, respectively for H_2 gas, O_2, N_2, CO, CO_2, H_2O, NH_3, CH_4 and air on the design of the supersonic nozzle type *MLN* 2D, giving a uniform flow and parallel to the exit section with $M_E = 3.00$, including the
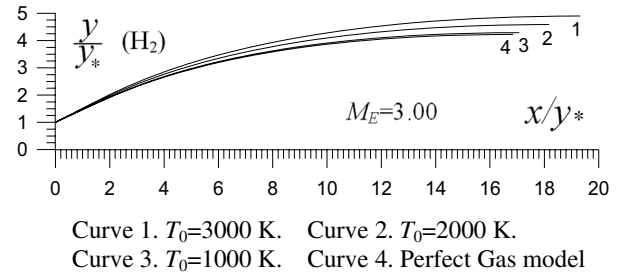

Curve 1. $T_0 = 3000$ K.   Curve 2. $T_0 = 2000$ K.
Curve 3. $T_0 = 1000$ K.   Curve 4. Perfect Gas model

Fig. 11. Effect of T_0 on the shape of the 2D *MLN* nozzle for H_2 gas.

Curve 1. $T_0 = 3000$ K.   Curve 2. $T_0 = 2000$ K.
Curve 3. $T_0 = 1000$ K.   Curve 4. Perfect Gas model
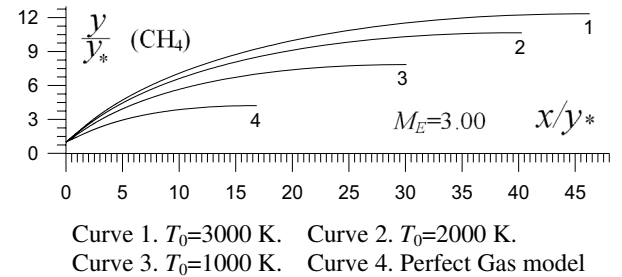
Fig. 12. Effect of T_0 on the shape of the 2D *MLN* nozzle for CH_4 gas.

Curve 1. $T_0 = 3000$ K.   Curve 2. $T_0 = 2000$ K.
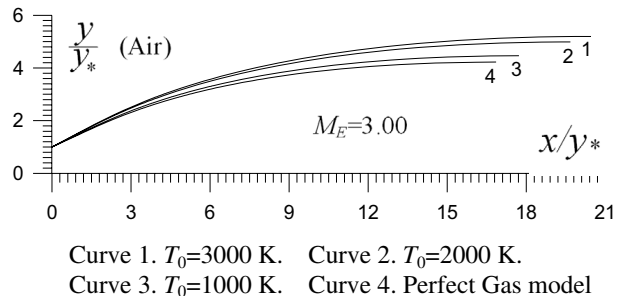Curve 3. $T_0 = 1000$ K.   Curve 4. Perfect Gas model

Fig. 13. Effect of T_0 on the shape of the 2D *MLN* nozzle for air.

case of the ideal gas for constant γ. The application is for $T_0 = 1000$ K, 2000 K and 3000 K. The results for constant γ (curve 4) on Figures 11−13 can be compared with the results of references [9,10].

The design results of the nozzles of these figures are presented respectively in Tables 7−9. For air (Fig. 13 and Tab. 9), the results can be found in reference [2].

Table 7. Design values for the nozzles of Figure 11.

T_0 (K)	θ^* (deg)	L/y^*	C_{Mass}	C_F	y_E/y_*
1000	25.1420	17.0834	17.5724	0.3037	4.2859
2000	26.2289	18.1739	18.7247	0.3239	4.5829
3000	27.2642	19.3177	19.9342	0.3441	4.8941
Perfect	24.8258	16.8522	17.3268	0.2989	4.2200

Table 8. Design values for the nozzles of Figure 12.

T_0 (K)	θ^* (deg)	L/y^*	C_{Mass}	C_F	y_E/y_*
1000	34.2401	30.0875	31.3622	0.4986	7.8421
2000	38.5030	40.2773	42.2181	0.6075	10.6612
3000	40.4941	46.2765	48.6236	0.6608	12.3329
Perfect	24.8258	16.8522	17.3268	0.2989	4.2200

Table 9. Design values for the nozzles of Figure 13.

T_0 (K)	θ^* (deg)	L/y^*	C_{Mass}	C_F	y_E/y_*
1000	25.8410	17.7220	18.2482	0.3161	4.4620
2000	27.4658	19.6718	20.3066	0.3490	4.9861
3000	28.0588	20.4559	21.1349	0.3626	5.1973
Perfect	24.8258	16.8522	17.3268	0.2989	4.2200

It is clearly noticed the effect of T_0 and gas on the shape of the nozzle. So, the most temperature T_0 is higher, it will have a broader form of the nozzle to keep the uniform and parallel flow to the exit section. Therefore, the temperature T_0 affects the thrust coefficient and generally the performances of the nozzle. For air (Fig. 13), the results can be compared with the results of references [3,13,14].

For a small mass of the nozzle we must choose a value of T_0 as small as possible, which is the case for some aerospace applications. But if applications do not allow, in this case we must design the suitable shape to the temperature T_0. For the selection of gas, it is noted that the CH_4 gas, NH_3, CO_2, H_2O not agree for use as aerospace propulsion gases. The best choice is the H_2 (Fig. 11), air (Fig. 13) and N_2. The same comments are valid for any exit Mach number M_E.

6 Conclusion

This work allows us to study the effect of using gas of propulsion at high temperature on the *PM* function and makes applications for designing the supersonic 2D *MLN* nozzle. We can draw the following conclusions:

- The developed numerical program can study any gas found in the literature. In this case, we must add the function $C_P(T)$ and the gas constant R even calculation the enthalpy $H(T)$.
- For the calculation of the supersonic flow parameters as function of stagnation temperature, Supersonic Mach number and the used gas are necessary.
- $C_P(T)$, R and $\gamma(T)$ characterize the calorific value of the gas, and necessarily influence the *PM* function.
- The presented relation is valid for any interpolation form chosen for the function $C_P(T)$ of the selected

gases. The essential one is that the interpolation gives an acceptable small error.
- The stagnation temperature and the Mach number also influence the *PM* function.
- The most ratio γ of the gas is small, the *PM* function becomes larger. Among the selected gas, it is the case for CH_4.
- In addition to T_0 and M_S affecting the opening of the nozzle to the throat (which affects the size and shape of the nozzle), the gas itself also influences.
- The propellant gas affects all thermodynamic and design parameters.

In the same line of research, we can use the present work to study the effect of gas propulsion at high temperature on the design of various forms of supersonic nozzles as *MLN*, plug nozzle, end nozzle, bell shaped nozzle, dual nozzle for 2D, and 3D and axisymmetric geometries by the method of characteristics.

One can even consider the propulsion gas effect on the determination of the drag force caused by friction and in particular the drag coefficient, the thickness of displacement, momentum and energy thickness of the boundary layer developed through the wall of the nozzle.

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