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REGULAR ARTICLE

A calculation method of thermal deformation for double helical gear

Cheng Wang[*]

School of Mechanical Engineering, University of Jinan, Jinan 250022, PR China

Abstract. Thermal deformation caused by gear transmission is an important factor causing gear impact, vibration and partial load. Gear modification can effectively improve the effects caused by thermal deformation. The calculation of thermal deformation is the first problem to be solved before gear modification. This paper takes double helical gear as the research object and a calculation method of thermal deformation is proposed. Firstly, temperature of instantaneous meshing points on the tooth surface are measured and these discrete temperature values are fitted by the linear interpolation. Calculation formula of temperature distribution along gear radial direction is introduced. Combining both, tooth surface temperature field is obtained. Secondly, equation of tooth surface for double helical gear before and after thermal deformation is derived according to the tooth surface temperature field. Finally, an example is given. Compared with the given modification of thermal deformation, the calculated thermal deformation is almost equal to theoretical value. On this basis, the thermal deformation of double helical gear considering the installation error and machining error is calculated, which provides a theoretical basis for thermal deformation modification of double helical gear.

Keywords: Thermal deformation / double helical gear / temperature field / modification / error

1 Introduction

Thermal deformation caused by gear transmission is an important factor causing gear impact, vibration and partial load. Gear modification is an effective way to improve its transmission performance [1–4]. The calculation of thermal deformation provides a basis for gear modification.

When calculating thermal deformation of gear, the temperature field should be determined first. It consists of two parts: the gear body temperature and the instantaneous surface temperature (it is also known as flash temperature). The instantaneous surface temperature depends on the body temperature. The traditional calculation of gear body temperature has the experience formula of ISO flash temperature criterion and integral temperature criterion, but the calculation results cannot determine the temperature distribution of gear teeth, and cannot meet the requirements of deformation calculation. Bobach [5] analyzed the regularities of distribution of temperature field inside the gear teeth. Qiu Liangheng [6] used finite element method to calculate and analyze the temperature and thermal deformation of gear. The calculated results can be used as the basis for analyzing and calculating the lubricating oil film thickness and gear

tooth profile modification. Gong Xiansheng [7] applied the theories of Hertz contact, tribology, gear engagement and heat transfer, established the finite element analysis model of the temperature field of gear body, and obtained the steady-state temperature field of the planetary gear tooth. Patir [8] and Wang [9] used finite element method to estimate the temperature field of spur gear tooth, and made theoretical analysis and numerical calculation on dynamic load and oil film thickness. Li [10] proposed the concept of non-involute characteristics. When the temperature of gear is changed, the theoretical tooth profile and the practical tooth profile are not superposition. On this basis, Wang [11] proposed a method for calculating the thermal deformation of helical gear.

Present experimental measurement technology can get the meshing point temperature value of gear. Reference [1] gives a simplified formula for temperature distribution along gear radial direction, which can be applied in engineering. Based on this, the specific process of the paper is shown in Figure 1.

2 Determination of tooth surface temperature field

Figure 2 showed the schematic diagram of determining tooth surface temperature distribution. The temperature
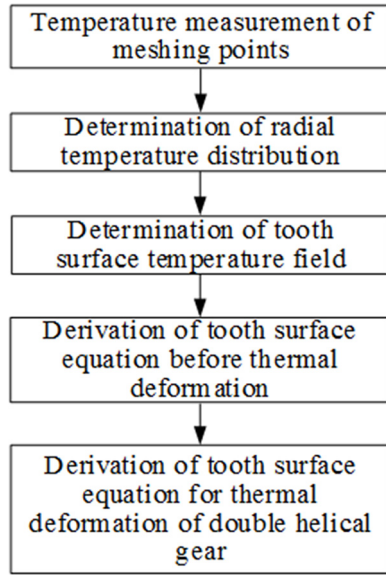
* e-mail: me_wangc@ujn.edu.cn

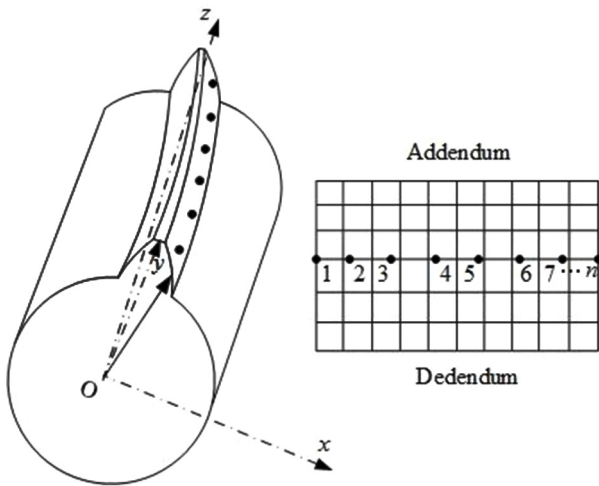**Fig. 1.** Flow diagram for calculation of thermal deformation of double helical gear.

Fig. 2. Schematic diagram of determining tooth surface temperature distribution.

values of instantaneous meshing point 1, 2, …, n are firstly measured and fitted. Together with the given temperature formula along gear radial direction, temperature field is determined.

2.1 Measurement and fitting of temperature of instantaneous meshing points on the tooth surface

Using the infrared technology, the miniature thermocouple automatic recorder and the simulated heat source infrared technology to measure the temperature of instantaneous meshing points on the tooth surface. The temperature curve in the direction of the meshing line along the tooth width is obtained by the linear interpolation.
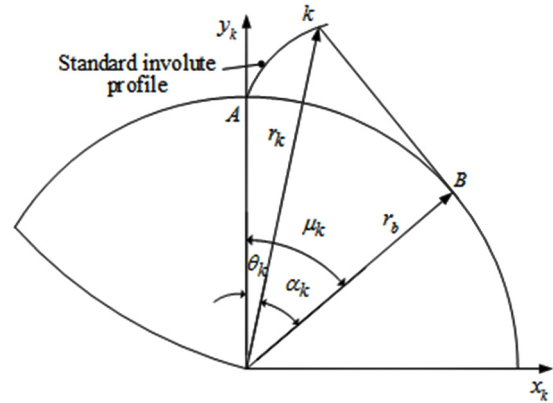

Fig. 3. Coordinate of standard tooth profile.

2.2 Temperature distribution along gear radial direction

Reference [1] gives a simplified formula for temperature distribution along gear radial direction, which can be applied in engineering. The formula can be expressed as

$$t_k = t_c + (t_a - t_c)r_k^2/r_a^2, \tag{1}$$

where r_k is the radius of gear arbitrary circle k, its temperature is t_k; t_c is the temperature of gear center; r_a is the radius of addendum circle, its temperature is t_a; The unit of temperature in the formula is °C.

2.3 Determination of tooth surface temperature field

According to the fitting curve along meshing points and formula (1), the temperature t_a of addendum circle along tooth surface can be obtained. And then through formula (1), the temperature of arbitrary point can be calculated, i.e., the tooth surface temperature field is determined.

3 Determination of thermal deformation of double helical gears tooth surface

According to the structural characteristics of double helical gear, the standard equation for tooth surface is derived. On the basis, the thermal deformation equation of tooth surface is derived according to the tooth surface temperature field.

3.1 Standard tooth surface equation of double helical gear

3.1.1 Standard tooth profile equation of double helical gear

Take one end tooth surface as an example, the coordinate of standard tooth profile is shown in Figure 3. Two tooth profile equation are expressed as

$$\begin{cases} x_{k1} = r_{b1} \sin \mu_{k1} - r_{b1}\mu_{k1} \cos \mu_{k1} \\ y_{k1} = r_{b1} \cos \mu_{k1} + r_{b1}\mu_{k1} \sin \mu_{k1} \end{cases} \tag{2}$$
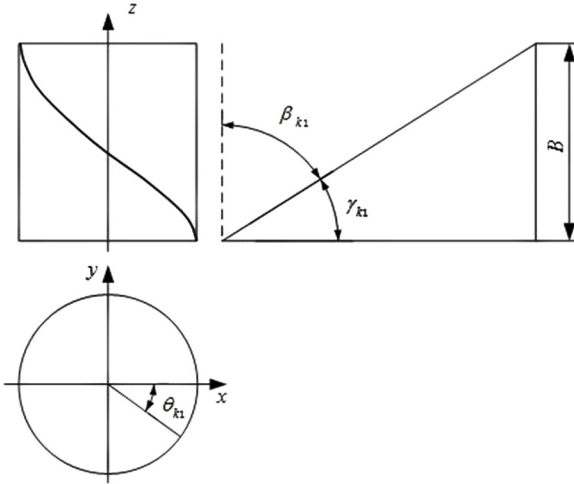
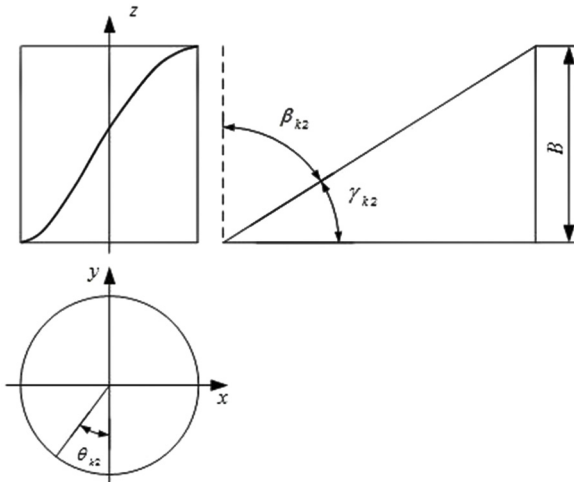**Fig. 4.** Helix diagram of left tooth surface.

Fig. 5. Helix diagram of right tooth surface.

$$\begin{cases} x_{k2} = r_{b2}\sin\mu_{k2} - r_{b2}\mu_{k2}\cos\mu_{k2} \\ y_{k2} = r_{b2}\cos\mu_{k2} + r_{b2}\mu_{k2}\sin\mu_{k2} \end{cases} \quad (3)$$

Formulas (2) and (3) are the profile equation of left tooth and right tooth, respectively. Where $r_{bi}(i=1, 2)$ is the radius of base circle, $\mu_{ki}\,(i=1, 2)$ is the roll angle of point k in the involute.

3.1.2 Standard tooth surface equation

The tooth surfaces of double helical gear is involute helicoid. Figures 4 and 5 are its left and right tooth surface helix diagram, respectively.

The left tooth surface equation is expressed as

$$\begin{cases} x_{k1} = r_{b1}\sin\mu_{k1} - r_{b1}\mu_{k1}\cos\mu_{k1} + r_{k1}\sin\theta_{k1} \\ y_{k1} = r_{b1}\cos\mu_{k1} + r_{b1}\mu_{k1}\sin\mu_{k1} + r_{k1}\cos\theta_{k1} - r_{b1} \\ z_{k1} = r_{k1}\theta_{k1}\tan\gamma_{k1} \end{cases} \quad (4)$$
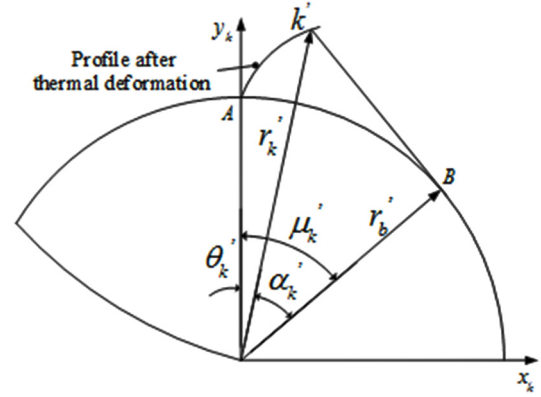

Fig. 6. Left tooth profile after thermal deformation.

The right tooth surface equation is expressed as

$$\begin{cases} x_{k2} = r_{b2}\sin\mu_{k2} - r_{b2}\mu_{k2}\cos\mu_{k2} + r_{k2}\sin\theta_{k2} \\ y_{k2} = r_{b2}\cos\mu_{k2} + r_{b2}\mu_{k2}\sin\mu_{k2} + r_{k2}\cos\theta_{k2} - r_{b2}. \\ z_{k2} = r_{k2}\theta_{k2}\tan\gamma_{k2} \end{cases} \quad (5)$$

Equations (4) and (5) are all the equation of involute helicoid. Where, $r_{bi}\,(i=1, 2)$ is the radius of base circle, $\mu_{ki}\,(i=1, 2)$ is the roll angle of point k in the involute, $r_{ki}(i=1, 2)$ is the radius of point k in the involute, $\theta_{ki}(i=1, 2)$ is the expansion angle of point k in the involute, $\gamma_{ki}(i=1, 2)$ is the helix angle of ascent of point k in the involute, $\beta_{ki}\,(i=1, 2)$ is the helix angle of point k in the involute.

3.2 Tooth surface equation of double helical gears after thermal deformation

3.2.1 Variation of tooth profile equation caused by thermal deformation

Still take the left tooth surface as an example, the left tooth profile after thermal deformation is shown in Figure 6. The tooth profile equation after thermal deformation can be expressed as

$$\begin{cases} x_k' = r_b'\sin\mu_k' - r_b'\mu_k'\cos\mu_k' \\ y_k' = r_b'\cos\mu_k' + r_b'\mu_k'\sin\mu_k' \end{cases} \quad (6)$$

where $r_b' = r_b(1 + \Delta t\lambda)$, $\mu_k' = \tan\alpha_k'$, $\alpha_k' = \arccos\left(\dfrac{r_b'}{r_k'}\right)$, r_b' is the radius of base circle after the thermal distortion, μ_k' is the roll angle of point k in the involute after the thermal distortion.

3.2.2 The tooth surface equation after thermal deformation

The helix diagram of left tooth surface and right surface after thermal deformation is shown in Figures 7 and 8, respectively.

The left tooth surface equation after thermal deformation is expressed as

$$\begin{cases} x_{k1}' = r_{b1}'\sin\mu_{k1}' - r_{b1}'\mu_{k1}'\cos\mu_{k1}' + r_{k1}'\sin\theta_{k1}' \\ y_{k1}' = r_{b1}'\cos\mu_{k1}' + r_{b1}'\mu_{k1}'\sin\mu_{k1}' + r_{k1}'\cos\theta_{k1}' - r_{b1}'. \\ z_{k1}' = r_{k1}'\theta_{k1}'\tan\gamma_{k1}' \end{cases} \quad (7)$$
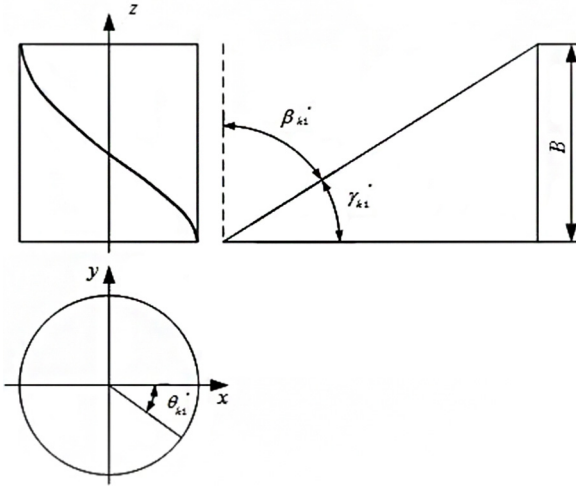
Fig. 7. Helix diagram of left tooth surface after deformation.
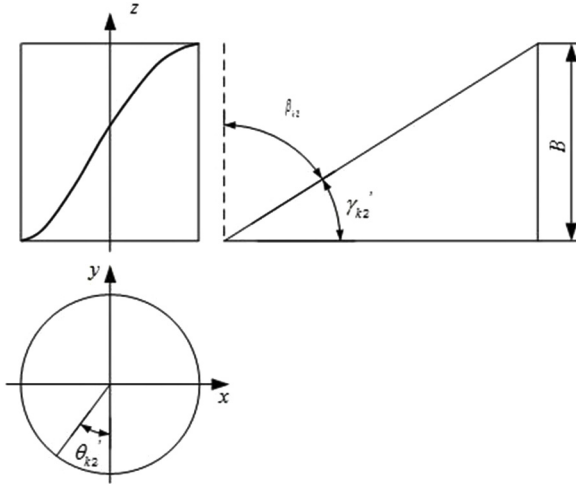

Fig. 8. Helix diagram of left tooth surface after deformation.

The right tooth surface equation after thermal deformation is expressed as

$$\begin{cases} x'_{k2} = r'_{b2}\sin\mu'_{k2} - r'_{b2}\mu'_{k2}\cos\mu'_{k2} + r'_{k2}\sin\theta'_{k2} \\ y'_{k2} = r'_{b2}\cos\mu'_{k2} + r'_{b2}\mu'_{k2}\sin\mu'_{k2} + r'_{k2}\cos\theta'_{k2} - r'_{b2} \\ z'_{k2} = r'_{k2}\theta'_{k2}\tan\gamma'_{k2} \end{cases} \quad (8)$$

where $\theta'_{ki} = \frac{b'_{ki}\cos\beta'_{ki}\sin\beta'_{ki}}{r'_{ki}}$, $\beta'_{ki} = ar\tan\left(\frac{r'_{ki}}{r_i}\tan\beta_i\right)$,

$\gamma'_{ki} = \frac{\pi}{2} - \beta'_{ki}$, r'_{bi} $(i = 1, 2)$ is the radius of base circle after the thermal distortion, μ'_{ki} $(i = 1, 2)$ is the roll angle of point k in the involute after the thermal distortion, r'_{ki} $(i = 1, 2)$ is the radius of point k in the involute after the thermal distortion, θ'_{ki} $(i = 1, 2)$ is the expansion angle of point k in the involute after the thermal distortion, γ'_{ki} $(i = 1, 2)$ is the helix angle of ascent of point k in the involute after the thermal distortion, β'_{ki} $(i = 1, 2)$ is the helix angle of point k in the involute after the thermal distortion, b'_{ki} is the tooth face width of point k in the

Table 1. Parameters of double helical gear.

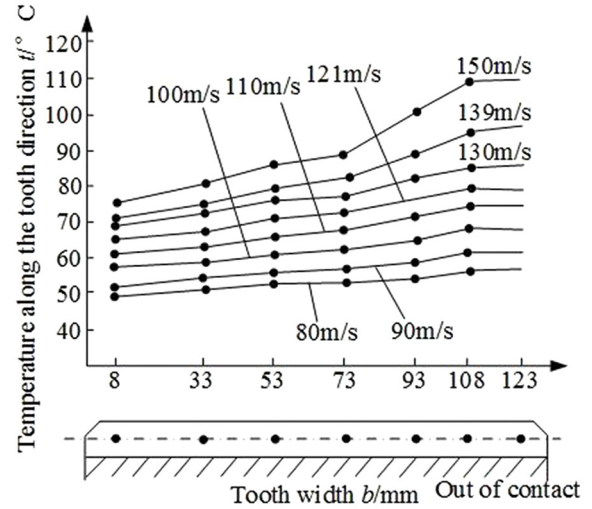|	
z	65
m_n	3
α_n	20°
β	12°
Face width	130×2 mm
Helical direction	Left right
Linear velocity of meshing	130 m/s

Fig. 9. Temperature of instantaneous meshing points.

involute after the thermal distortion, r_i $(i = 1, 2)$ is the radius of reference circle after the thermal distortion.

4 Example

Taking the measured temperature gear as an example [1], the material of gear is 45 steel and the coefficient of linear expansion $\lambda = 11.6e^{-6}c^{-1}$, room temperature $t_0 = 25\,°C$. Table 1 shows other parameters of the gear. The tooth surface temperature distribution measured by Zhengzhou Institute of machinery [1] is shown in Figure 9. Because the installation error and machining error of double helical gear are not considered, the gear tooth temperature distribution of two tooth surfaces is the same.

According to Figure 9, the temperature equation along reference circle axial with a linear velocity of 130 m/s is deduced.

$$\begin{cases} 0 \le b \le 8 & t = 68.5 \\ 8 < b \le 33 & t = 0.16b + 67.22 \\ 33 < b \le 53 & t = 0.2b + 65.9 \\ 53 < b \le 73 & t = 0.1b + 71.2 \\ 73 < b \le 93 & t = 0.25b + 60.25 \\ 93 < b \le 108 & t = 0.24b + 61.18 \\ 108 < b \le 123 & t = 0.1b + 76.3 \\ 123 < b & t = 88.6 \end{cases} \quad (9)$$
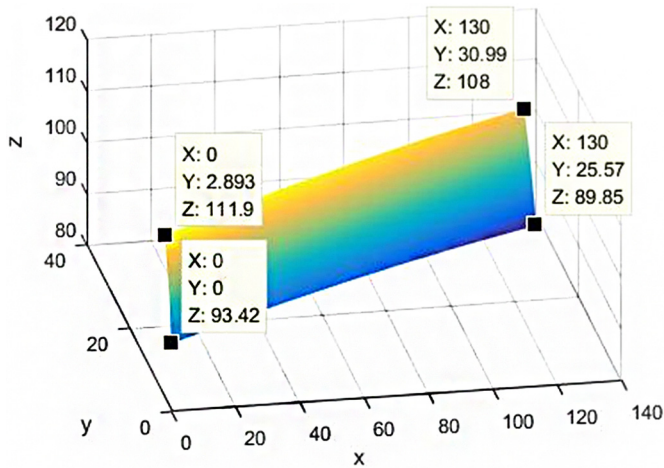
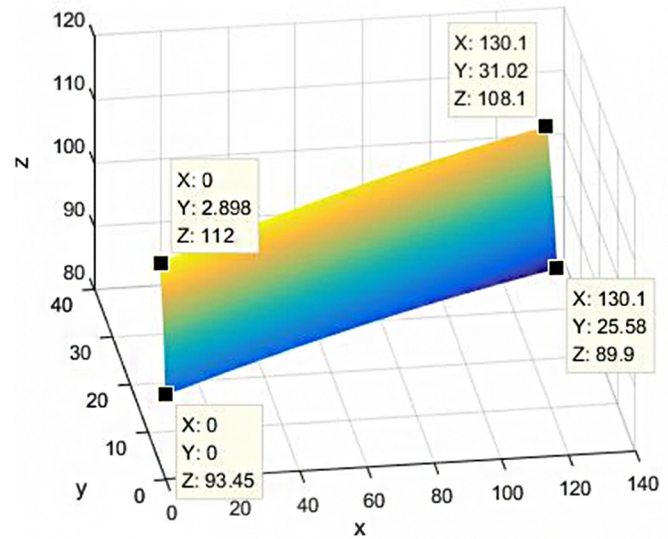**Fig. 10.** Tooth surface 3D drawing before thermal deformation (left tooth surface).

Fig. 12. Tooth surface 3D drawing after the thermal deformation (left tooth surface).
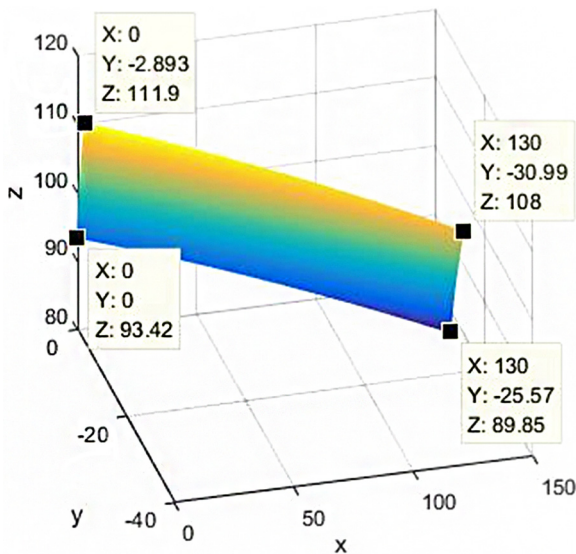

Fig. 11. Tooth surface 3D drawing before the thermal deformation (right tooth surface).
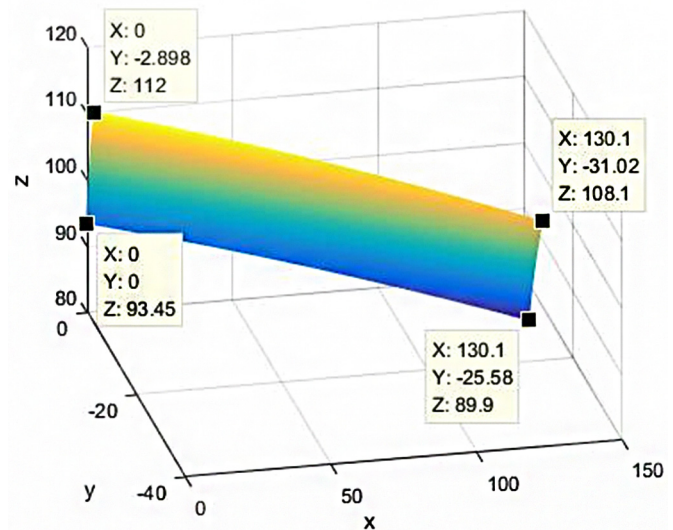

Fig. 13. Tooth surface 3D drawing after the thermal deformation (right tooth surface).

According to formulas (4), (5), (7), (8). the tooth surface before and after thermal deformation are shown in Figures 10–13 (where, the x axis describes longitudinal direction, the y axis describes tooth thickness direction and the z axis describes radial direction).

Because the installation error and machining error of double helical gear are not considered, the thermal deformation of Figures 12 and 13 is the same. Take left tooth surface as an example, the difference of the tooth surface coordinates before and after thermal deformation is shown in Figure 14, the mean value of the difference of the tooth surface coordinates before and after thermal deformation is shown in Table 2. According to reference [1], Table 3 showed the modification of thermal deformation, which corresponds to the deformation of tooth thickness. In this paper, the diameter of reference circle is 199.36 mm, which is close to 200 mm in Table 4. Therefore, the

modification can be taken as 0.009 mm. In Table 3, the mean value of the difference of the tooth surface coordinates along tooth thickness direction is 0.009627 mm. The error is one order of magnitude smaller than them.

When the installation/machining error of gear exists, the meshing path of two tooth surfaces change. Reference [12] gave the meshing path of left and right tooth surfaces under installation/machining error of gear (Fig. 15). The temperature of instantaneous contact points on the left/right tooth surface is assumed that still satisfies formula (9). The thermal deformation of two tooth surfaces is recalculated, the mean value of the difference of tooth thickness direction without and with considering installation error and machining error of gear is shown in Table 4.
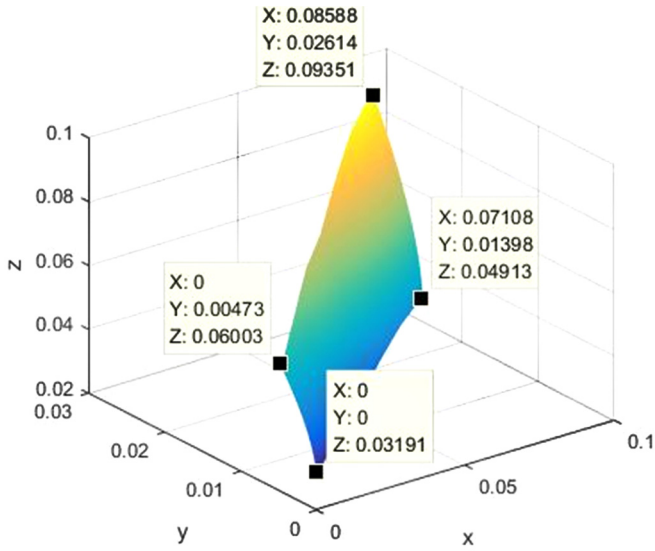
Fig. 14. Difference of the tooth surface coordinates before and after thermal deformation.

Table 2. Mean value of the difference of thermal deformation.

Direction	Longitudinal direction	Tooth thickness direction	Radial direction
Mean value of the difference (mm)	0.0346	0.009627	0.059

Table 3. Modification of thermal deformation.

Linear velocity/(m/s)	Modification of thermal deformation				
	Gear diameter/mm				
	100	150	200	250	300
130	0.005	0.007	0.009	0.012	0.015

Table 4. Mean value of the difference of tooth thickness direction without and with considering installation error and machining error of gear.

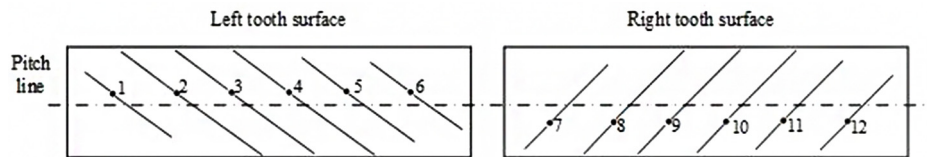|	Tooth thickness direction (left tooth surface)	Tooth thickness direction (right tooth surface)
Mean value of the difference without considering installation error and machining error of gear (mm)	0.009627	
Mean value of the difference under installation error and machining error of gear (mm)	0.00944	0.0098

Fig. 15. Path of tooth surface contact under installation error and machining error of gear.

It can be found that thermal deformation of left and right tooth surfaces is difference and different modifications of thermal deformation are needed.

5 Conclusion

The calculation of gear thermal deformation is one of the keys to gear modification. Double helical gear is taken as the research object and a calculation method of thermal deformation is proposed. The temperature field is determined according the measured temperature of instantaneous meshing point and radial temperature formula. According to the structural characteristics of double helical gear, the standard equation for tooth surface is derived. On the basis, the thermal deformation equation of tooth surface is derived according to the tooth surface temperature field. Taking a temperature measuring gears as an example, the tooth surface has changed obviously before and after thermal deformation. Compared with the relevant data given in reference [1], the result is reasonable which verifies the feasibility of the proposed method. By calculating the thermal deformation of two surfaces when the installation error/machining error of gear exists, it can be found that the deformation of two tooth surfaces is different. That is to say, the left and right tooth surfaces should be modified with different amounts.

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