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Study on the influence of ship collision upon propulsion shafting

Liang-Xiong Dong[*], Yu-Sheng Niu, and Xiao-Fei Wen

Zhejiang Ocean University, Zhoushan, Zhejiang, PR China

Abstract. In this study, a mathematical model of a struck ship based on the dynamic stiffness of the hull was developed; a comparison of the impact of collision in the models of dynamic stiffness and static stiffness, showed that the vibration amplitude in the dynamic stiffness was larger than that in the static stiffness in the stress accumulation stage. A simulation of shafting in colliding ships was performed based on a finite-element model using the ANSYS software, and the numerical simulation values and theoretical values were observed to be similar. To analyze the influence of ship collision on propulsion shafting, a test rig on ship collision based on dynamic stiffness was designed, and the experimental values agreed well with the numerical simulation values. From a practical perspective, this study could generate a map similar to an operating guide.

Keywords: Collision / dynamic stiffness / amplitude / propulsion shafting

1 Introduction

The risk to a ship on a voyage is difficult to predict, and serious damage to the ship would influence the motion of the shaft. The damage to the shaft can-not directly destroy the hull structure; however, it can cause violent hull vibration and mechanical jamming of the shaft, resulting in a severe loss of propulsive power [1]. With the rapid development of automated and unmanned ships, the operating environment of ships has been elevated to a more advanced level [2]. Therefore, research of propulsion shafting dynamics in ship collisions is a key link to a comprehensive study on the reliability of a ship with regard to vitality and battle effectiveness. It is also an indispensable technology for special types of ships such as unmanned ships, among other [3].

The ship collision loads include the total load on the hull structure, and hull stiffness is the ability of the hull to resist the total bending deformation [4]. In practical engineering applications, exploring and resolving the impact of hull stiffness on the shafting vibration is particularly essential. Most researchers have established a simplified model of hull stiffness from the perspective of static loads. However, this model of hull stiffness is too simple and the calculated results of shafting vibration usually do not meet the actual requirements [5]. Therefore, in this study, based on a total consideration of the hull stiffness, a coupled model of ship collision is established and the theory of ship collision is applied to study its effect on

the hull. In addition, a test rig is designed to simulate the actual working conditions of propulsion shafting under different collision conditions [6]. The study can play a guiding role in solving practical engineering problems such as ship design and stranded ships.

2 Model of ship collision

According to the construction features of a propulsion system, the hull of a struck ship can be considered to be a continuous girder structure of length L, width B, thickness H, elastic modulus of E, and density ρ, and the shafting can be considered to be a single disk system whose ends are supported on a sliding bearing. During the collision, the striking ship is similar to a mass block with a single degree of freedom connected by a linear spring to the struck ship at the point of impact. The spring stiffness k_b is the stiffness of the side of the ship when it is in the elastic stage. The impact speed of the ship is v_b and the mass of the ship is m_b; the spring stiffness k_a is the stiffness of the collided hull in the elastic stage, m_1 and m_2 are the structural masses of the fore and aft sterns, respectively, m_3 is the mass of the disk, and c_1 is the damping coefficient of the stern structure. The influence of the hull on the stern shaft system transfers the stiffness of the spring through the support of the stern bearing. The mechanical model is as shown in Figure 1.

To consider the hull of a struck ship in collision as a beam element with bending deformation, shear deformation, and moment of inertia [7], the effect of gravity on the ship's equivalent mass block on the beam element is taken into account. The product of the axial force N and the
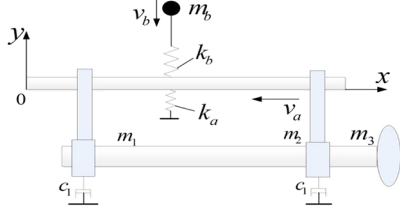
* e-mail: 1358469631@qq.com

Fig. 1. Mechanical model of ship collision.

(a)   (b)

Fig. 2. Schematic diagram of the force on the hull micro beam section.

curvature $\frac{\partial^2 y}{\partial x^2}$ result in additional loads acting perpendicular to the beam axis. The isolation body of the beam element is shown in Figure 2 and the parameters are $\frac{\partial y}{\partial x}$ — the corners of the axis; θ − the bending angle; γ − the shear angle $\gamma = \frac{\partial y}{\partial x} - \theta$; M − the cross-sectional bending moment; Q − the shear force; f_1 − the distributed inertial forces; $f_{1\theta}$ − The rotational moment of inertia of beam cross section mass distribution relative to neutral axis generated by angular acceleration.

2.1 Theoretical solution of simplified mechanical model of ship collision

During the collision, the hull of the struck ship and the striking ship are regarded as a whole system and the collision problem is transformed into a lateral vibration problem of the system [8]. The dynamic response of the simplified mechanical model of the ship collision can be obtained by solving the problem [9]. The hull girder has the characteristics of a distributed parameter system in which the mass, elasticity, and damping are equally distributed. In Figure 2, I is the moment of inertia of the section, EI is the bending stiffness changing along the direction of the length, and m is the mass per unit length. Based on Figure 2, the characteristics may be expressed by the following equation:

$$kAG\left(\frac{\partial\theta}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right) + m\frac{\partial^2 y}{\partial t^2} + N\frac{\partial^2 y}{\partial x^2} = 0 \qquad (1)$$

A is the cross-sectional area of the hull girder, G is the shear modulus, and k is the constant factor, which is related to the shape of the cross section and the non-uniform distribution of the shear stress on the cross section.

Equation (1) can be written as

$$\frac{\partial\theta}{\partial x} = \frac{\partial^2 y}{\partial x^2} + \frac{1}{kAG}\left(-m\frac{\partial^2 y}{\partial t^2} - N\frac{\partial^2 y}{\partial x^2}\right). \qquad (2)$$

According to the moment balance condition, the moment is calculated for the intersection of the right section and axis, omitting dx^2:

$$EI\frac{\partial^2\theta}{\partial x^2} = -kAG\left(\frac{\partial y}{\partial x} - \theta\right) + \rho I\frac{\partial^2\theta(x,t)}{\partial t^2}. \qquad (3)$$

Taking the derivative of equation (3) to x, we obtain

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial\theta}{\partial x}\right) = -m\frac{\partial^2 y}{\partial t^2} - N\frac{\partial^2 y}{\partial x^2} + \rho I\frac{\partial^3\theta(x,t)}{\partial t^2}. \qquad (4)$$
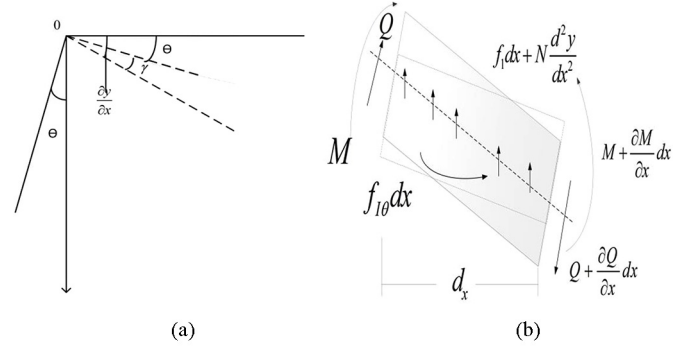
By substituting equation (2) in equation (4), the partial differential equation of the pier element can be expressed as equation (5) below:

$$EI\left(1 - \frac{N}{kAG}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + N\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$+ \rho I\left(\frac{N}{kAG} - \frac{E}{kG} - 1\right)\frac{\partial^4 y(x,t)}{\partial x^2\partial t^2} + m\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$+ \frac{\rho^2 I}{kG}\frac{\partial^4 y(x,t)}{\partial t^4} = 0. \qquad (5)$$

Equation (5) can be simplified as:

$$\frac{\partial^4 y(x,t)}{\partial x^4} + a_1\frac{\partial^2 y(x,t)}{\partial x^2} + a_2\frac{\partial^4 y(x,t)}{\partial x^2\partial t^2} + a_3\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$+ a_4\frac{\partial^4 y(x,t)}{\partial t^4} = 0 \qquad (6)$$

where a_1, a_2, a_3, a_4 are the coefficients of the second, third, fourth, and fifth terms in equation (5) divided by the coefficient of the first term.

2.2 Partial differential equation solution in frequency domain

Because $y|_{t=0} = 0, \frac{\partial y}{\partial t}|_{t=0} = 0, \frac{\partial^2 y}{\partial t^2}|_{t=0} = 0, \frac{\partial^3 y}{\partial t^3}|_{t=0} = 0, \frac{\partial^4 y}{\partial t^4}|_{t=0} = 0,$ equation (7) can be obtained by Laplace transform of equation (6) with respect to t

$$\frac{\partial^4\tilde{y}}{\partial x^4} + \left(a_1 p^2 + a_2 p^2\right)\frac{\partial^2\tilde{y}}{\partial x^2} + \left(a_3 p^2 + a_4 p^4\right)\tilde{y} = 0. \qquad (7)$$

The impact condition at the spring joint is

$$\begin{cases} m_b\frac{\partial^2 y}{\partial t^2} + k_b[u(t) - y] = 0 \\ k_b[u(t) - y] = Q_1 - Q_2 \end{cases} \qquad (8)$$
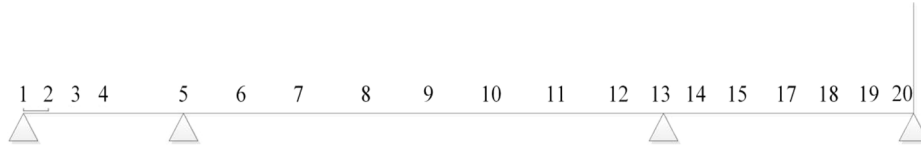
Fig. 3. Node model of shafting and node numbers.

where $u(t)$ is the displacement against the side of the ship, Q_1 and Q_2 are the shear forces before and after the impact respectively, and the initial condition for striking the ship is given by:

$$u(0) = 0; \quad \frac{du}{dt}\big|_{t=0} = v.$$

The dynamics equation of the shafting-oil film-stern structure system is expressed as follows:

$$\overline{M}\ddot{Z} + (P + J)\dot{Z} + SZ = F. \tag{9}$$

In equation (9), the parameters are as defined below: $\overline{M}$ − the mass matrix; Z − the matrix of center coordinates; J − the matrix of the gyro; P − the damping matrix; S − the stiffness matrix; F − the load matrix.

The above equations are coupled to each other in which the coupling term in the stiffness matrix is called elastic coupling. It is difficult to decouple them directly [10], therefore, iterative steps for solving equations (7), (8), and (9) are described below:

– The partial differential equation of the impacted ship motion, the dynamics equation of the shafting-oil film-stern structure system and the contact conditions of the impact point are considered, and programmed for calculation.
– The initial values of the displacements, velocities, and accelerations are set.
– The deformation of the hull girder caused by the collision load after t seconds is calculated. After this step, the displacement of the stern bearing in the vertical direction is obtained.
– The radius clearance of stern bearing by the coordinate-difference between the stern bearings and the tail journals in the vertical direction is calculated.
– Then the impact force of the tail journals is obtained by solving equation (9) by the Euler method.

In equation (9), the initial parameters are set as: $m_1 = 100$ kg, $m_2 = 80$ kg, $m_3 = 200$ kg, $p_1 = 5000$ N S/M, $p_2 = 4000$ N S/M, $p_3 = 6000$ N S/M, $s_1 = 5 \times 10^6$ N/m, $s_2 = 4 \times 10^6$ N/m, and $s_3 = 6 \times 10^6$ N/m. The initial values of the system kinetics parameters, such as $\dot{X}_1$, $\ddot{X}_1$, $\dot{X}_2$, $\ddot{X}_2$, $\dot{X}_5$, $\ddot{X}_5$, $\dot{Y}_1$, $\ddot{Y}_1$, $\dot{Y}_2$, $\dot{Y}_2$, $\ddot{Y}_3$, $\dot{Y}_4$, $\dot{Y}_5$, $\dot{Y}_5$, Y_5 are taken as zero. The results of the sensibility analysis show that the change in the initial values had no effect on the accuracy of the calculations.

3 Model of struck ship based on dynamic stiffness of hull

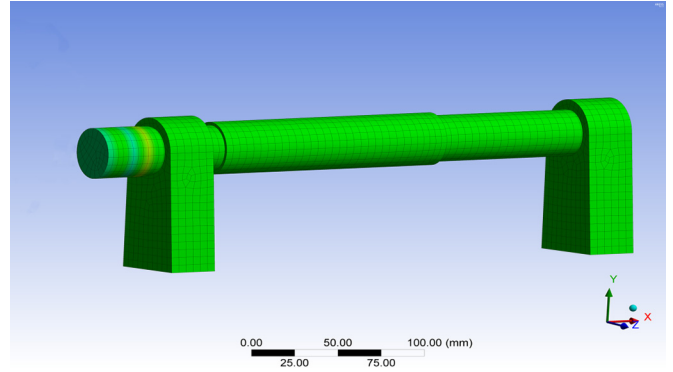The ability to resist deformation under a static load is known as static stiffness and it is generally measured by the deformation of the structure und a static load [11]. The ability to resist deformation under a dynamic load is called dynamic stiffness, which is the dynamic force required to cause unit amplitude. When a dynamic force acts upon a structure, the smaller the vibration amplitude, the larger the dynamic stiffness and vice versa. As the collision of a ship is a complicated process, to approach the actual situation, both dynamic and static forces must be considered [12].

3.1 Finite-element model of shafting in colliding ships

During the collision, the impact force changes with the collision penetration. Therefore, the shafting dynamic characteristics are calculated based on the relationship between the impact force and the penetration [13]. The most practical method currently used for collision analysis and simulation is the numerical simulation method. In this study, a 5000 DWT ship impact is considered as the research object, assuming that the hull of the struck ship is stationary and the peed of the striking ship is 10 m/s. The numerical simulation is conducted using ANSYS.

Based on the shafting structure, a node model of the shafting is established as shown in Figure 3. The joint where the shaft connects with the hull is supported with a bearing and the bearing is simulated by the spring element in the mode [14]. The bearing is set to COMBINEI4 and the model mesh size is set to 0.2 m*0.2 m. To facilitate the calculation, the shafting in the ship is simplified into a continuous beam with multiple supports, ignoring insignificant features, such as the chamfered rounded convex platform. The model of shafting is established as shown in Figure 4.

Fig. 4. Model of shafting.

Table 1. Parameters of the shafting.

Parameters	Symbol	Value
Length of the shaft	l	15 230 mm
Shaft diameter	d	230 mm
Intermediate shaft diameter	D	180 mm
Bearing maximum clearance	$\dot{c}$	2 mm
Bearing radius	r	67 mm
Bearing width	μ	60 mm
Bearing damping	$\overline{D}$	$2.1e^3$ N S/M
Sealing gap	$\ddot{c}$	3 mm
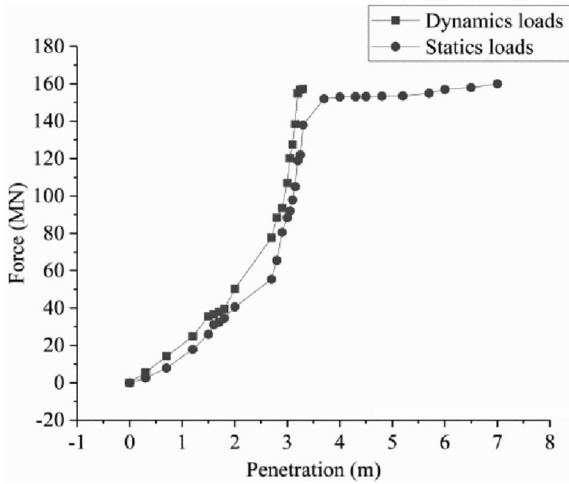
Fig. 5. Comparison of force-penetration curves between static loads and dynamic loads.

The values of the parameters of the shafting are shown in Table 1.

Setting the number of time course steps as 500, force-penetration curves between static and dynamic loads are obtained within the 1s time range.

Figure 5 shows that the first half of the dynamic and static impact force-penetration curve is consistent, but in the actual project, it is not possible to obtain the latter half of the curve. This is because at the end of the static process, the contact force between the ship and the rigid wall is maximized and balanced and the ship collapses to a peak. At the end of the power process, the ship and the rigid wall are quickly disengaged, and the ship components have a small amount of deformation recovery and the collision penetration reduces by a small amount.

3.2 Performance calculation of colliding ships based on stiffness

According to the change law of the collision force as shown in Figure 5, the curve to be fitted is divided into two segments, each with an asymptote. Therefore, the hull deformation function is expressed in the form of a piecewise function: the first half of the piecewise function is an exponential function and the second half is a polynomial function.

– Equation (10) describes the exponential function as given below:

$$y = c_2 e^{wx}, \ w > 0. \tag{10}$$

The logarithm is taken on both sides of the equation, and the function is derived from the linearly processed equation.

$$y_1 = ax + b \tag{11}$$

where $y_1 = \ln y$, $a = w$, $b = \ln c_2$.

a and b are obtained by the least squares method for any given coordinate point. The specific steps are as follows: (1) define the function $\text{pline}(x, \ y)$ under MATLAB; (2) input the coordinate to the function; (3) call the function $\text{pline}[x, \ \lg(y)]$, solve for $a = 0.0012$, $b = 14.5087$, and then get $w = 0.0012$ and $c_2 = e^{14.5087} = 2 \times 10^6$. The fitting function thus obtained is given below:

$$y = 2 \times 10^6 \times e^{0.0012x} \tag{12}$$

– The polynomial function is obtained by one order function fitting as shown in equation (13):

$$y = 5901.04x + 1.11 \times 10^8. \tag{13}$$

Therefore, the simplified model of the static stiffness of the ship is expressed as follows:

$$f = \begin{cases} \alpha_1 \times 2 \times 10^6 \times e^{0.0012x} & (0 \le x \le X') \\ 5901.04x + 1.11 \times 10^8 & (X' \le x \le X) \end{cases}. \tag{14}$$

In equation (14), $X' = 3.380$ m is the intersection of the piecewise fitting curves and α_1 is the dynamic amplification factor.

Assuming that the ship speed is 9 m/s and the shafting speed is 110 r/min, the shaft nodes 1 and nodes 8 were selected. Numerical simulation was conducted based on the dynamic stiffness and static stiffness models of ship collisions the amplitude-time curves for node 1 and node 8 were obtained as shown in Figures 6 and 7.

Keeping the speed of the ship unchanged, the maximum amplitude for different shafting speeds is obtained for node 1 and node 8 through simulation as shown in Figures 8 and 9.

The following conclusions are drawn:
– When two ships collide, the impact force is transferred to the shafting through the hull structure. Under the shock load, the displacement response and stress response, mainly including the shaft section and bearings, have a direct connection with the dynamic stiffness of the hull [15]. After amending the dynamic stiffness of the hull, the mechanical model can explain the results of the ANSYS numerical simulation better. The mechanical model with the dynamic stiffness of the hull is more consistent with the ANSYS numerical simulation results than that without the dynamic stiffness.
– Comparing the impact of the collision in the dynamic stiffness and static stiffness models as shown in Figures 6–9, it is seen that the vibration amplitude in
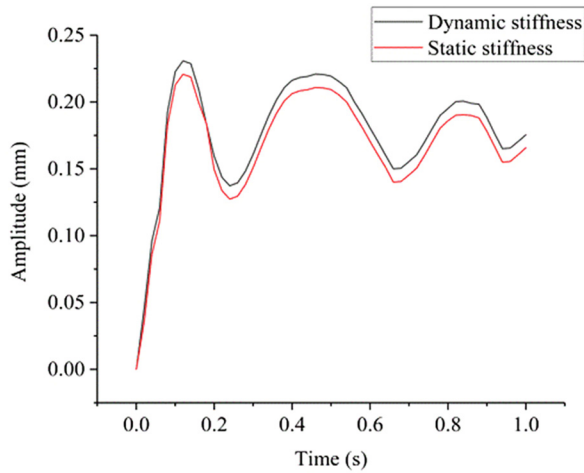
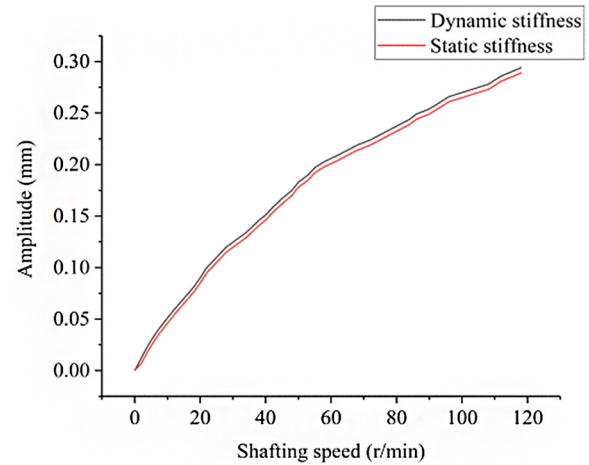**Fig. 6.** Amplitude-time curves for node 1.

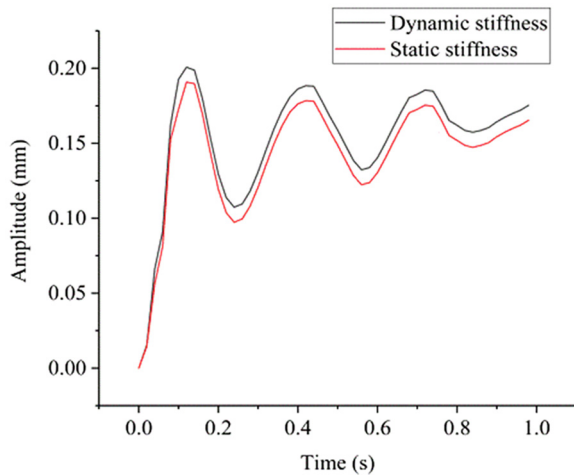**Fig. 8.** Amplitude-speed curves for node 1.

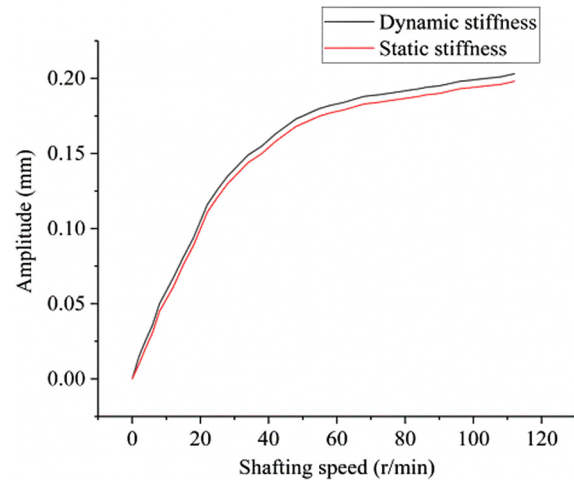
Fig. 7. Amplitude-time curves for node 8.

Fig. 9. Amplitude-speed curves for node 8.

the dynamic stiffness model is larger than that in the static stiffness model in the stress accumulation stage. In the deformation stage, the dynamic stiffness of the hull leads to maximum amplitude (in the fracture stage) earlier and the system enters the unstable range sooner (in the deformation stage). Thus, the influence of dynamic stiffness introduces unstable factors and earlier instabilities. It is obvious that the required estimate is insufficient, hence more attention must be paid to it in the calculation of the ship collision performance.

3.3 Numerical simulation of ship collision based on dynamic stiffness

As established above, the vibration amplitude in the dynamic stiffness model is larger than that in the static stiffness model in the stress accumulation stage; hence,

the dynamic stiffness model is mainly considered. The value obtained based on the mathematical model of dynamic stiffness is the theoretical value, and that obtained by numerical simulation based on the dynamic stiffness is the simulation value. Setting the initial status of the struck ship in collision rested, the simulation based on ANSYS was conducted in accordance with the above conditions. Setting $v_b = 10$ m/s, $t = 1s$, for the sake of discussion, and selecting the same nodes 1 and 8, the amplitude-time curve and maximum amplitude-speed curve for dynamic stiffness are obtained as shown in Figures 10–13.

According to the simulation results, the graphs are as shown in Figures 10–13. It is seen that the numerical simulation results and theoretical values are not very different. The consistency between the theoretical and simulated values indicates that the established dynamic stiffness model has a certain reliability, and the maximum
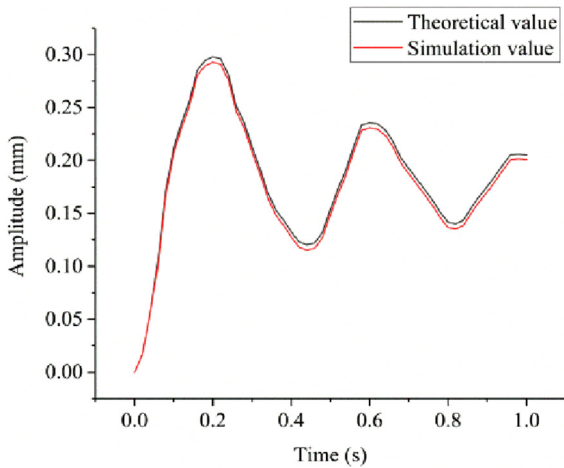
Fig. 10. Amplitude-time curves for node 1 based on theoretical and simulation values.

Fig. 12. Amplitude-time curves for node 8 based on theoretical and simulation values.

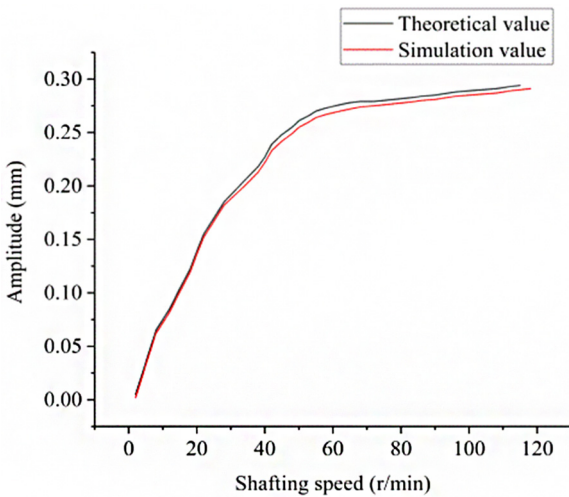**Fig. 11.** Amplitude-speed curves for node 1 based on theoretical and simulation values.
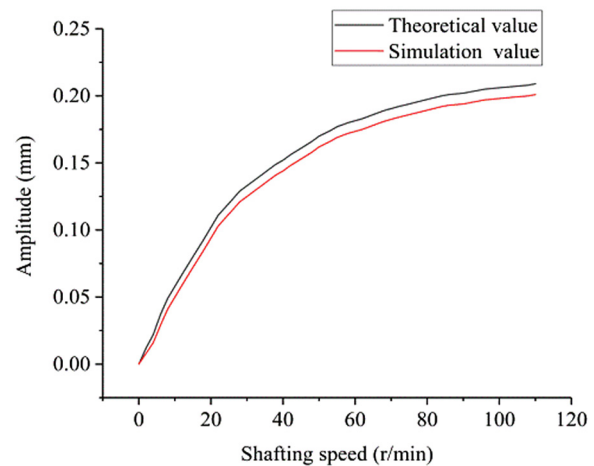
Fig. 13. Amplitude-speed curves for node 8 based on theoretical and simulation values.

amplitude of node 1 is larger than that of node 8, indicating that the damage to the tail shaft is more serious than that to the intermediate axis during the collision.

4 Experimental analyzers of shaft of the ship

4.1 Test rig design on ship collision based on dynamic stiffness

To analyze the influence of ship collision on propulsion shafting, an auxiliary shaft is used to simulate the effect of hull deformation and hull impact on propulsion shafting [16]. According to the mechanical model of the ship collision, a test rig with an impact load system is prepared as shown in Figure 14. In the test rig, the impact force and collision position during the ship collision test can be changed with the transformation of the bearing rod and
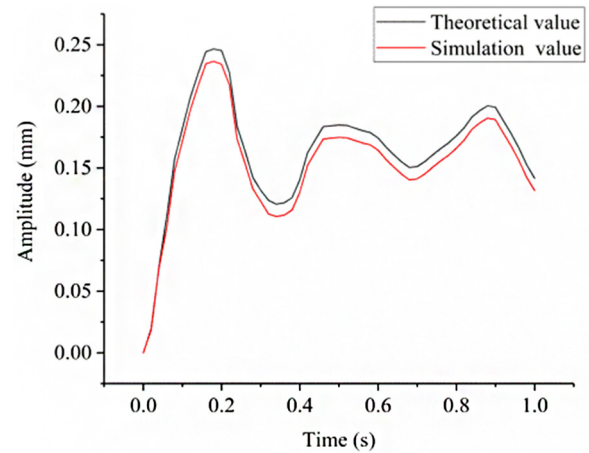
supporting rod. Thus, dynamic stiffness of the auxiliary shaft under various working conditions is generated, and various loading effects of hull collision on shafting in actual sea conditions are effectively simulated [17].

The experimental procedure based on the test rig is described as follows:

– Based on the location of the collided ship and the impact force, choose the solenoid valves and set the supply current which can simulate the collision process.
– Energizing the selected solenoid valve in turn, the drop bar will strike the impact block [17]. The collision force from the impact block is transmitted to the transition shaft through the support bar and then to the shafting through the front and rear bearing brackets.
– When the impact force reaches a certain value, the sliding sleeve moves and the support rod turns quickly which leads to the brace stiffness of transitional shaft change rapidly.
– In the ship collision test, the amplitude response diagram under collision loads can be taken at the front of the tail shaft. After completion of the experiment, the test rig is
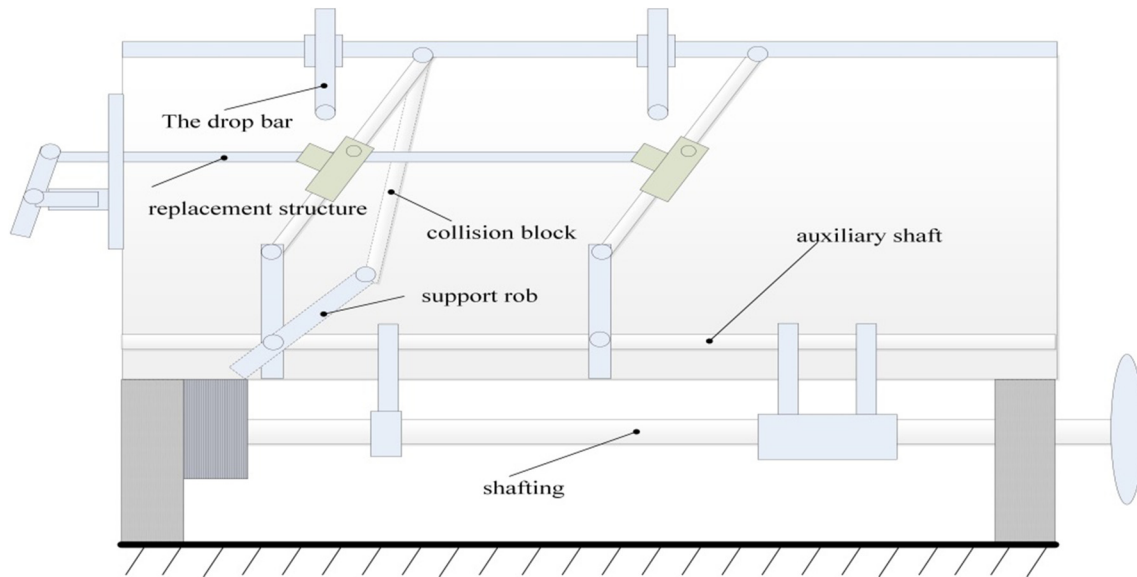
Fig. 14. Structure drawing of shafting test rig.

reset by operating the reset bar. The maximum displacement and maximum deformation value during the collision of the ship can be measured through repeated experiments.

4.2 Test of collision force response on shafting

From the above analysis, it is known that the damage to the tail shaft is more serious than that to the intermediate shaft during the collision of the ship. Therefore, it is important to analyze the tail shaft part and select node 1. Using the established test rig, the experimental values are obtained by measuring the maximum amplitude at different collision speeds. A comparison of the experimental values, simulated values, and theoretical values is shown in Figure 15.

It is seen that the theoretical value is the largest, the experimental value is the smallest, and the simulation value lies between the two. The good agreement between the three within the tolerance of error proves the correctness of the dynamic stiffness model. At the same time, when the speed changes, the degree of coincidence as well as the amplitude fluctuation law are different, indicating that the speed of the ship has a greater impact on the ship collision.

5 Study on the diagram of safe operation of shafting

Based on the previous comparison between the simulated, theoretical, and experimental values of the amplitude, it is inferred found that the error in the result is small. Therefore, the dynamic stiffness model of the ship established in this study can provide a reference for engineering applications. From a practical perspective, this study created a map similar to an operating guide. To

evaluate the collision condition of the ship systematically, it is necessary to compare the rules of change of amplitude under different collisions condition generated by two ships [18]. The parameters studied are the shafting speed and collision speed of the striking ship [19]. In particular, the shafting speed of the striking ship in case of collision must be selected accurately to minimize the damage to the shafting caused by the collision.In this study, for different collision speeds, the shafting speeds of the striking ship are chosen as 100, 150, 200, 250, 300, 350, 400, 450, 500, 600, 700, and 800 r/min, and the corresponding simulations are implemented for vibration characteristics of shafting using the above mechanical model. The maximum amplitude of the struck ship at different shafting speeds and collision speeds is calculated and shown in Table 2.

According to the characteristics of data of Table 2, when the shafting speed is certain, the change rules of impact amplitudes with collision speed are fitted into a smooth curve. In the same way, taking the shafting speeds and collision speeds as the control parameters, the fitting curves at impact amplitudes of 0.86, 0.76, 0.66, and 0.56 are established as shown in Figure 16. This figure can serve as a standard map to examine collision hazards.

As shown in the above graph, under different shafting speeds and collision speeds, the results show that ship collisions affect the propulsion system in the following manner:

– For the same collision speed, when the shafting speed of the shaft system is different, the impact amplitude exhibits different variation characteristics. As the collision speed increases, the amplitude of the shock caused by it increases, but the amplitude growth rate is different under different speed conditions.
– By analyzing the evaluation map of ship collision we can reach a critical value. Only when the collision strength is below that value, the function intensity decreases with
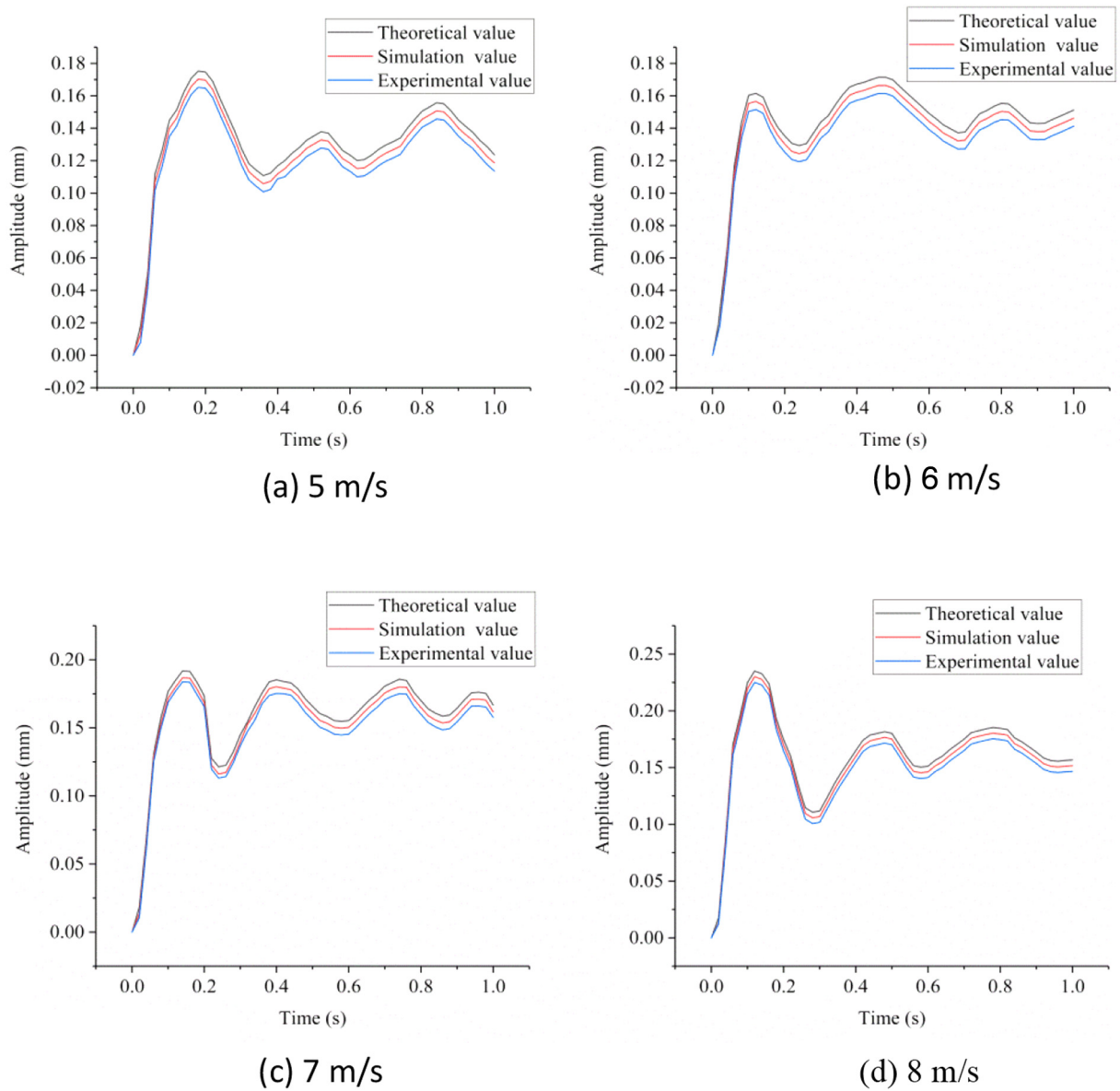
Fig. 15. Amplitude-time curves at different speeds for node 1.

Table 2. Impact amplitudes at different shafting speeds and collision speeds.

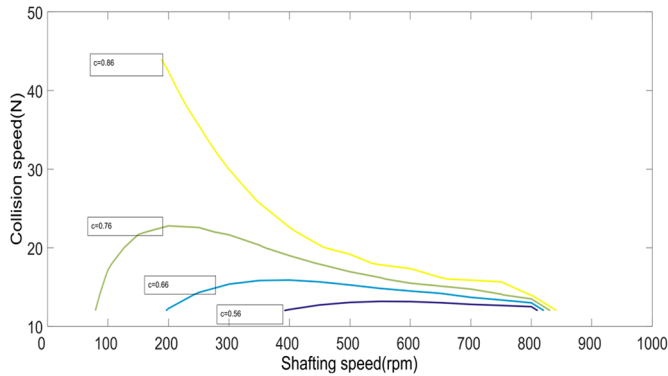| Shafting speed (r min^{-1})	Collision (m/s)							
	15	20	25	30	35	40	45	50
150	0.470	0.482	0.503	0.531	0.565	0.607	0.664	0.750
250	0.827	0.805	0.712	0.647	0.640	0.656	0.690	0.757
350	0.850	0.850	0.838	0.804	0.715	0.698	0.713	0.764
450	0.831	0.836	0.857	0.842	0.791	0.734	0.734	0.770
550	0.862	0.842	0.839	0.862	0.824	0.795	0.750	0.776
650	0.876	0.853	0.821	0.860	0.840	0.792	0.762	0.782
750	0.874	0.851	0.838	0.842	0.851	0.810	0.774	0.788

Fig. 16. Evaluation map of the impact amplitude.

increase in the shafting speed; otherwise the function intensity will decreases with an increase in the shafting speed. Therefore, after a minor crash, the ship can choose the shafting speed outside the operating region $c = 0.56$, and try to keep it in the operating region between $c = 0.66$ and $c = 0.76$. If the ship is seriously damaged, it is best to choose the shafting speed in the operating region $c = 0.86$ that has a better linearity in a wide speed range. Otherwise, the vibration of the ship shafting changes rapidly and may cause the shafting to become jammed, which must be considered in ship design.

6 Conclusion

The research object in this study is, propulsion shafting in the course of a ship and the model of dynamic stiffness of the hull is used to analyze the influence of ship collision on propulsion shafting. The main conclusions are as follows:

– When ships collide, the hull is damaged by a large external force, which is transmitted to the propulsion shafting through the hull structure changing its kinematic characteristics. In the simulation, the simplified mechanical model of ship collision simulated and validated the influence of ship collision on propulsion shafting quickly and effectively. It is recommended that the dynamic stiffness of the hull be applied in simulations based on actual situations.
– After the collision, the rigidity of the ship's hull is changed, and the dynamic stiffness influences the impact energy of the propulsion shafting, which affects the motion characteristics of the shafting and increases the maximum stress and displacement. Therefore, the influence of dynamic stiffness on the propulsion shafting must be taken into account in the design of a ship.
– During the collision, the collision speed and speed of the shafting have an important influence on the motion characteristics of shafting. To reduce the shafting

damage caused by the loading striking ship, the safety speed map of the ship shafting must be prepared in advance for selecting the running speed of the propulsion shafting reasonably.

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