

An efficient method for estimating the damping ratio of a vibration isolation system

Qiang Yu¹, Dengfeng Xu^{2,*}, Yu Zhu^{1,2}, and Gaofeng Guan¹

¹ School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu 611731, PR China

² Department of Mechanical Engineering, Tsinghua University, Beijing 100084, PR China

Received: 17 October 2018 / Accepted: 16 July 2019

Abstract. As the damping ratio determines the response of a vibration isolation system at resonance, it is very necessary to estimate the damping ratio quickly and economically for an evaluation of the effectiveness to adjust the damping in practical engineering applications. An efficient method named the “ $\zeta-T_r$ ” method with the characteristics of simple operation and a high accuracy is introduced to estimate the damping ratio in this paper. According to the transmissibility curve, the specific mathematical relationship in which the value of the resonance peak corresponds to the value of the damping ratio is analysed theoretically. In this case, the recognition of the resonance peak can be used to directly estimate the damping ratio without an approximation or simplification. The “ $\zeta-T_r$ ” method is faster, more accurate and less costly than other estimation methods. Finally, the correctness of the “ $\zeta-T_r$ ” method is verified by a simulation and an experiment.

Keywords: Vibration isolation / damping ratio / estimation / resonance peak / “ $\zeta-T_r$ ” method

1 Introduction

Vibration isolation systems developed to prevent the transfer of vibrations are necessary for implementation of building structures, ultra-precision manufacturing and measurement, aerospace science, vehicle engineering and other fields [1–3]. In general, the harmful vibration energy imported by the environment or ground is mainly consumed by viscous damping in these systems. To simplify the theoretical analysis and numerical calculation, the damping model can similarly be considered in linear viscous form. The damping ratio is one of the key parameters in this linear model, and the estimation of the damping ratio is very important for accurately predicting the response of a vibration isolation system. The estimation methods are commonly known as the half-power bandwidth method and the time-domain attenuation method [4,5].

The procedure of the half-power bandwidth method [6,7] is shown in Figure 1. The damping ratio can be estimated by locating the half-power points and calculating the half-power bandwidth based on the measured transmissibility curve. This method is widely used because of its simplicity, but it may be influenced by many factors, such

as the sampling frequency and the frequency resolution [8–10]. Moreover, this is a simplified and approximate method, so there are some errors between the estimated and actual results.

The procedure of the time-domain attenuation method [11,12] is shown in Figure 2. The damping ratio can be estimated by calculating the average attenuation rate based on measuring the time-domain attenuation curve by adding a single-frequency vibration excitation. The estimated results are very accurate, but the operational process is complex. In particular, the calculation of the average attenuation rate often needs to fit the envelope curve of the time-domain attenuation curve.

After measuring the transmissibility curve or time-domain attenuation curve, the above two methods still require further analysis or calculations before estimating the damping ratio. The half-power bandwidth method must locate the half-power points and calculate the half-power bandwidth shown in the “Step 3” in Figure 1. The time-domain attenuation method must calculate the average attenuation rate shown in the “Step 3” in Figure 2. During the course of the product design, product development, product installation and testing of any vibration isolators or isolation system, the damping ratio needs to be estimated rapidly multiple times to verify whether the design value meets the actual requirement. Furthermore, these further operations such as “Step 3” in

* e-mail: xudf@mails.tsinghua.edu.cn

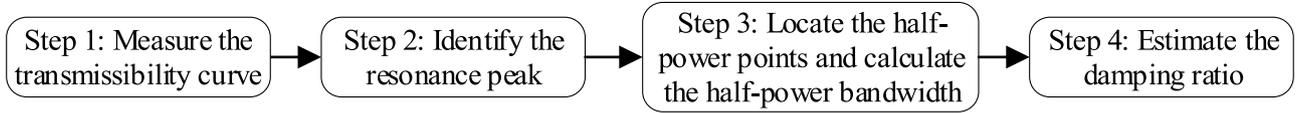


Fig. 1. Procedure of the half-power bandwidth method.

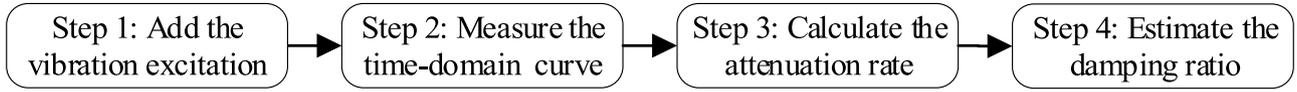


Fig. 2. Procedure of the time-domain attenuation method.

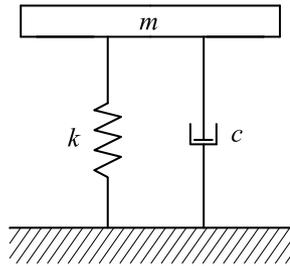


Fig. 3. Mass-spring-damper system.

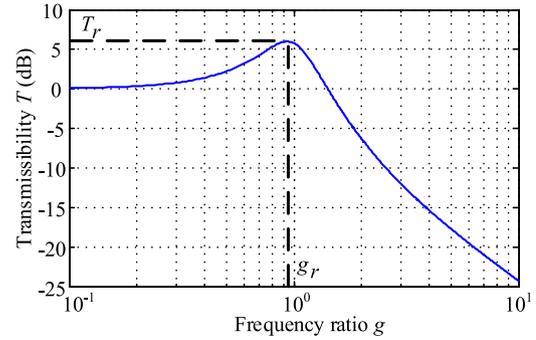


Fig. 4. Variation in $T(g)$ with g .

the above two methods require necessary professional knowledge or skills, and ordinary employees may not complete these tasks. Thus, more money must be spent in employing senior employees, and more time must be taken to finish the estimation of the damping ratio.

In this paper, an efficient method for estimating the damping ratio is proposed by analysing the relationship between the resonance peak of the transmissibility curve and the damping ratio. Compared with the above two methods, the proposed method only needs the transmissibility curve, and the damping ratio can be estimated directly depending on the value of the resonance peak without extra operations like “Step 3” in Figure 1 or 2.

2 “ ζ - T_r ” method for estimating the damping ratio

Typical passive vibration isolation systems can be reduced to a mass-spring-damper system as depicted in Figure 3, where m , k and c are the system mass, the system stiffness and the system damping, respectively.

The transmissibility curve, which is defined as the ratio of the output and the input, is often used to measure the performance of a mass-spring-damper system in the frequency domain. Through dynamic analysis, the motion equation of a passive vibration isolation system is derived, and then the transmissibility curve $T(g)$ can be expressed as [4,13]

$$T(g) = \sqrt{\frac{1 + 4\zeta^2 g^2}{(1 - g^2)^2 + 4\zeta^2 g^2}} \quad (1)$$

and

$$\zeta = \frac{c}{2\sqrt{km}}, \quad g = \frac{\omega}{\omega_n} \quad (2)$$

where ζ , ω , ω_n and g are the damping ratio, forcing frequency, natural frequency and frequency ratio, respectively.

The variation in the transmissibility $T(g)$ with the frequency ratio g is shown in Figure 4. It is obvious that $T(g)$ is a convex function. Set the first derivative of $T(g)$ to zero:

$$\frac{dT(g)}{dg} = 0. \quad (3)$$

Substitute equation (1) into equation (3) and further solve equation (3). The resonance frequency ratio g_r shown in Figure 4 can be obtained:

$$g_r = \frac{(\sqrt{1 + 8\zeta^2} - 1)^{1/2}}{2\zeta}. \quad (4)$$

Substituting equation (4) into equation (1), the resonance peak T_r shown in Figure 4 can be obtained:

$$T_r = \frac{4\zeta^2}{\sqrt{16\zeta^4 - (\sqrt{1 + 8\zeta^2} - 1)^2}} = f(\zeta). \quad (5)$$

As shown in equation (5), the function f expresses a one-to-one relationship between the resonance peak T_r and the damping ratio ζ . Conversely, if we can obtain the value of

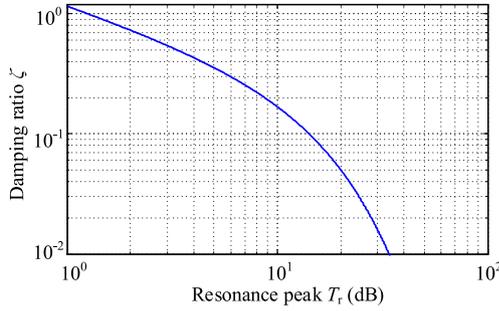


Fig. 5. Variation in ζ with T_r .

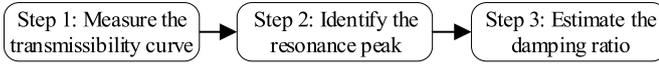


Fig. 6. Procedure of the “ ζ - T_r ” method.

the resonance peak T_r according to the measured transmissibility curve or the frequency response curve, the damping ratio ζ can be expressed as

$$\zeta = f^{-1}(T_r) = \sqrt{\frac{1}{2} \left(\frac{T_r^2}{T_r^2 - 1} - \frac{T_r}{\sqrt{T_r^2 - 1}} \right)}, \quad (6)$$

where the function f^{-1} is the inverse function of f . Note that T_r and ζ in equation (6) have no units. If T_r recognized from the transmissibility curve has units of “dB”, a simple transformation must be made before calculating the damping ratio ζ by substituting the resonance peak T_r into equation (6).

Furthermore, the variation in the damping ratio ζ with the resonance peak T_r is shown in Figure 5. Then, the damping ratio ζ can be estimated by a mathematical calculation method with equation (6) or diagramming method with Figure 5. These two methods are named the “ ζ - T_r ” method for short in the following parts. In this case, the “ ζ - T_r ” method can be directly used to estimate the damping ratio ζ based on the value of the resonance peak T_r , which can be obtained easily from the measured transmissibility curve. The procedure of the “ ζ - T_r ” method is shown in Figure 6, and the damping ratio can be estimated in only three steps. Both the half-power bandwidth method and the time-domain attenuation method require four steps and further work, such as locating the half-power points or calculating the average attenuation rate. Therefore, the “ ζ - T_r ” method is more efficient and has great importance in engineering practice for the estimation of the damping ratio ζ .

3 Comparison with the half-power bandwidth method

3.1 Simulation analysis

Through the above analysis, the estimation of the damping ratio using the “ ζ - T_r ” method is deduced directly without

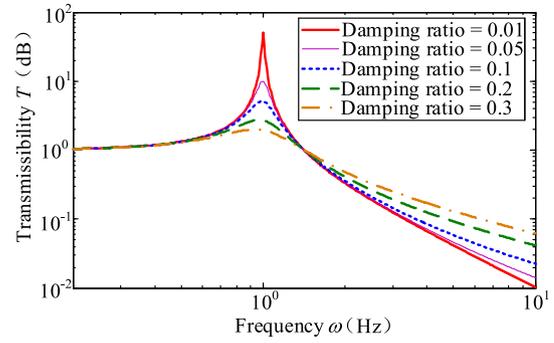


Fig. 7. Variation in T with ω for different damping ratios ζ .

an approximation or a simplification and the estimation results have less error. To clarify this point, we perform further simulation analysis and a comparison with the half-power bandwidth method, as shown below.

Without loss of generality, we assume that the natural frequency ω_n is 1 Hz. Then, we use MATLAB software to simulate the variation in the transmissibility T with the forcing frequency ω in the cases of a damping ratio of $\zeta = 0.01, 0.05, 0.1, 0.2$ or 0.3 , as shown in Figure 7. Because the natural frequency ω_n is very low, the sampling frequency is just set to 10 Hz. Finally, based on the simulation results of the transmissibility curve, different damping ratios can be calculated by the half-power bandwidth method [8,9] and the proposed method using the “ ζ - T_r ” method. By comparing the calculated results with the standard values of the different damping ratios, the errors of these two different methods can be analysed. To analyse whether the frequency resolution affects the accuracy of the estimation of the damping ratio, we choose four different values of the frequency resolution: 10^{-4} Hz, 10^{-3} Hz, 10^{-2} Hz, and 10^{-1} Hz.

The calculation of the damping ratio ζ by the half-power bandwidth method is shown in Table 1. While the frequency resolution is 10^{-4} Hz or 10^{-3} Hz, the calculation errors are small, within 6.83%. However, while the frequency resolution is 10^{-2} Hz or 10^{-1} Hz, some of the calculation errors are even 200% or 25%, which can be unacceptable. In some cases, the half-power bandwidth may be non-existent so that the damping ratio is incalculable. This also demonstrates that this method can be influenced by the frequency resolution and that the estimation results are incredible in some conditions.

The calculation of the damping ratio ζ by the “ ζ - T_r ” method is shown in Table 2. While the frequency resolution is 10^{-4} Hz, 10^{-3} Hz or 10^{-2} Hz, the calculation errors are very small. While the frequency resolution is 10^{-1} Hz, the calculation errors are still less than 2%. On one hand, the calculation errors of the proposed method are all less than those of the half-power bandwidth method. On the other hand, regardless of how the frequency resolution or the damping ratio changes, we can also complete the calculation of the damping ratio.

The damping ratio ζ is set to 0.01, 0.05, 0.1, 0.2 and 0.3 because the half-power bandwidth method can only be applied to the situation in which the damping ratio ζ

Table 1. Calculation of the damping ratio ζ by the half-power bandwidth method.

Standard values of ζ	Frequency resolution = 10^{-4} Hz		Frequency resolution = 10^{-3} Hz	
	Calculation	Error	Calculation	Error
0.01	0.01005	0.5%	0.0100	<0.1%
0.05	0.05010	0.2%	0.0500	<0.1%
0.1	0.10095	0.95%	0.1005	0.5%
0.2	0.20685	3.43%	0.2070	3.5%
0.3	0.32040	6.8%	0.3205	6.83%

Standard values of ζ	Frequency resolution = 10^{-2} Hz		Frequency resolution = 10^{-1} Hz	
	Calculation	Error	Calculation	Error
0.01	0.030	200%		Incalculable
0.05	0.050	<0.1%		Incalculable
0.1	0.105	5%		Incalculable
0.2	0.205	2.5%	0.25	25%
0.3	0.320	6.67%	0.30	<0.1%

Table 2. Calculation of the damping ratio ζ by the “ ζ - T_r ” method.

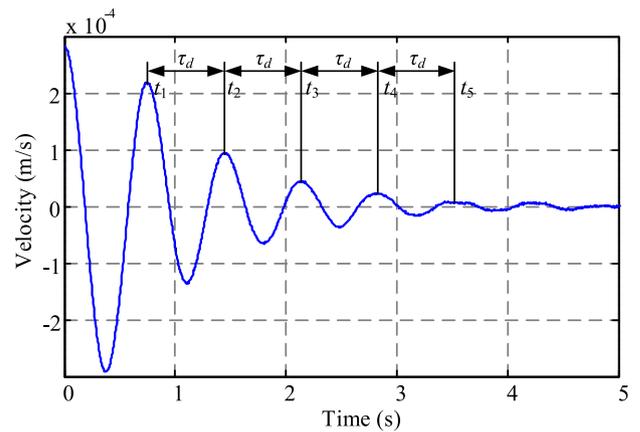
Standard values of ζ	Frequency resolution = 10^{-4} Hz, 10^{-3} Hz, 10^{-2} Hz		Frequency resolution = 10^{-1} Hz	
	Calculation	Error	Calculation	Error
0.01	0.01	<0.1%	0.010000	<0.1%
0.05	0.05	<0.1%	0.050062	0.124%
0.1	0.10	<0.1%	0.100489	0.489%
0.2	0.20	<0.1%	0.203675	1.84%
0.3	0.30	<0.1%	0.301994	0.67%

**Fig. 8.** Experiment platform supported by six air spring isolators.

must be less than $1/2\sqrt{2}$. However, the “ ζ - T_r ” method has no limit to the value of the damping ratio ζ . Therefore, the “ ζ - T_r ” method not only reduces the estimation error of the damping ratio ζ but also has a wider range of applications.

3.2 Experimental analysis

The experimental platform shown in Figure 8 is a passive vibration isolation system consisting of six air spring isolators. The testing devices are an ultralow-frequency vibration pickup sensor 991B produced by Institute of

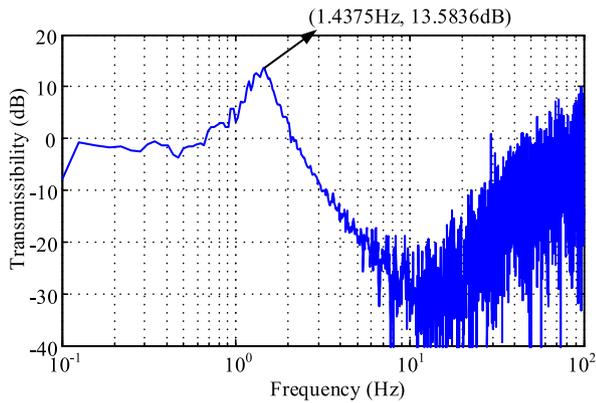
**Fig. 9.** Measured time-domain attenuation curve for 5 s.

Engineering Mechanics, China Earthquake Administration and a data acquisition instrument INV3060T produced by China Orient Institute of Noise & Vibration.

First, we beat on the load with a rubber hammer, and a time-domain attenuation curve in the vertical direction can be obtained. The measured curve for 5 s is shown in Figure 9, and the values of consecutive peaks are

Table 3. Values of 5 consecutive peaks.

Time (s)	t_1	t_2	t_3	t_4	t_5
Velocity (m/s)	2.10×10^{-4}	9.90×10^{-5}	5.19×10^{-5}	2.65×10^{-5}	1.35×10^{-5}
Attenuation period τ_d (s)	–	0.69	0.7	0.7	0.69
Attenuation rate δ	–	52.86%	47.58%	48.94%	49.06%

**Fig. 10.** Measured transmissibility curve in the vertical direction.**Table 4.** Three estimated values of the damping ratio.

Damping ratio	Value	Error
ζ_0 (Standard)	0.1084	–
ζ_1	0.1200	10.7%
ζ_2	0.1076	0.74%

shown in Table 3. The average attenuation rate can be calculated:

$$\bar{\delta} = 49.61\% = 0.4961. \quad (7)$$

Then, the damping ratio of the isolation system in the vertical direction can be estimated using equation (8) [11].

$$\zeta_0 = \sqrt{\frac{\ln^2(1 - \bar{\delta})}{4\pi^2 + \ln^2(1 - \bar{\delta})}} = 0.1084. \quad (8)$$

The above time-domain attenuation method is a very accurate approach. Therefore, the estimation result ζ_0 can be regarded as the standard value of the damping ratio.

Finally, the measured transmissibility curve in the vertical direction is shown in Figure 10. The resonance peak T_r is 13.58 dB, and then we can estimate the damping ratio ζ_1 and ζ_2 by the half-power bandwidth method [5,6] and the new method using equation (6), respectively. These three estimation results of the damping ratio are shown in

Table 4. It is obvious that the error of ζ_2 is significantly less than the error of ζ_1 .

4 Conclusions

This paper has presented an efficient approach called the “ ζ - T_r ” method to estimate the damping ratio of a vibration isolation system whose damping model is linear. According to an analysis of the relationship between the resonance peak of the transmissibility curve and the damping ratio, a mathematical expression for the one-to-one correspondence between the resonance peak of the transmissibility curve and the damping ratio can be obtained. Then, the damping ratio can be estimated on the basis of the recognition of the resonance peak from the measured transmissibility curve. In contrast to other methods, the “ ζ - T_r ” method has little approximations, simplifications or limitations and requires fewer steps. Consequently, the estimation results are more accurate, and the “ ζ - T_r ” method is more efficient and easier to use in engineering applications.

This work was supported by Major Scientific and Technological Project of China National Machinery Industry Corporation Ltd (SINOMAST-ZDZX-2017-05), National Science and Technology Major Project (2013ZX02104003) and the Natural Science Foundation of Hubei Province (2018CFC889).

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Cite this article as: Q. Yu, D. Xu, Y. Zhu, G. Guan, An efficient method for estimating the damping ratio of a vibration isolation system, *Mechanics & Industry* **21**, 103 (2020)