

Achieving uniform thread load distribution in bolted joints using different pitch values

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Abstract. Bolted joints are a critical component of machines and structures under cyclic loading. Bolt fatigue failure usually takes place in the first engaged thread for being the most loaded one. In this sense, a uniform thread load distribution improves its mechanical response and consequently the reliability of the joint. For this purpose, different thread pitch values can be used in the bolt and the nut, and there is an optimum value that leads to the most uniform load distribution for each particular joint configuration (geometry, preload level, materials and boundary conditions). Sopwith developed an analytical model to calculate this value, but no validation was carried out. This work presents a semianalytical model to estimate the load distribution for any differential thread spacing, which can also be used to calculate the optimum value. The results of this model are nearly coincident with Finite Element results and more accurate than those obtained from Sopwith model.

Keywords: Bolted joint / uniform load / pitch difference / differential thread spacing / thread stripping

1 Introduction

Bolted joints are widely used in industry due to their ease of assembly/disassembly operations, which allows performing maintenance operations with no need of skilled labour [1–4]. Despite these advantages, they are usually the critical component of machines that work under variable loads, so the reliability of a machine with bolted joints is usually determined by the fatigue reliability of the joint itself [5]. According to literature, 15% of bolt failures take place in the head-shank transition section, the 20% in the first thread and the 65% in the first engaged thread [6–9]. In effect, in a preloaded bolt the first threads in contact withstand most of the axial load [10], and consequently the failure generally takes place in the first thread turn in contact between the bolt and nut [7]. In this sense, a uniform load distribution amongst all the threads improves the behaviour of the bolted joint not only in terms of thread stripping, but also regarding fatigue failure [11]. In fact, a uniform load distribution not only decreases the load in the first engaged thread, but also alleviates to some extent the stress concentration in the root of the thread.

Different solutions can be adopted for this purpose. If the nut material is more flexible than the bolt, the force

distribution in the threads becomes more uniform, and experimental tests prove that the fatigue resistance of the bolt significantly increases (between 35 and 60%, depending on the materials) [8,12,13]. Non-conventional nut designs such as tapered threads or tension nuts are also available, which have shown a better response to cyclic loading than standard nuts [7,8,13,14]. Finally, differential thread spacing (different thread pitch values in the bolt and the nut) may also be used to obtain a more uniform load distribution [6,15,16].

The issue of the load distribution among the threads is a classical problem in bolted joint research. Several analytical models have been proposed when bolt and nut threads have the same pitch value. Joukovsky developed a model where threads only withstand shear stress and bolt and nut bodies deform in the longitudinal direction [5,17]. Later, Jacket considered bending stress of the threads, but assumed rigid nut [5,18], and Maduschka extended this model by implementing the axial and radial deformations of the nut and considering a finite number of threads [9,19]. The model developed by Birger can be considered more generalist, as it considers nut dimensions, thread pitch and profile, and bolt and nut materials, among other parameters [5,20,21]. Finally, Sopwith published a widely referenced analytical model for the thread load distribution and generalized it to consider different pitch values [15], but experimental

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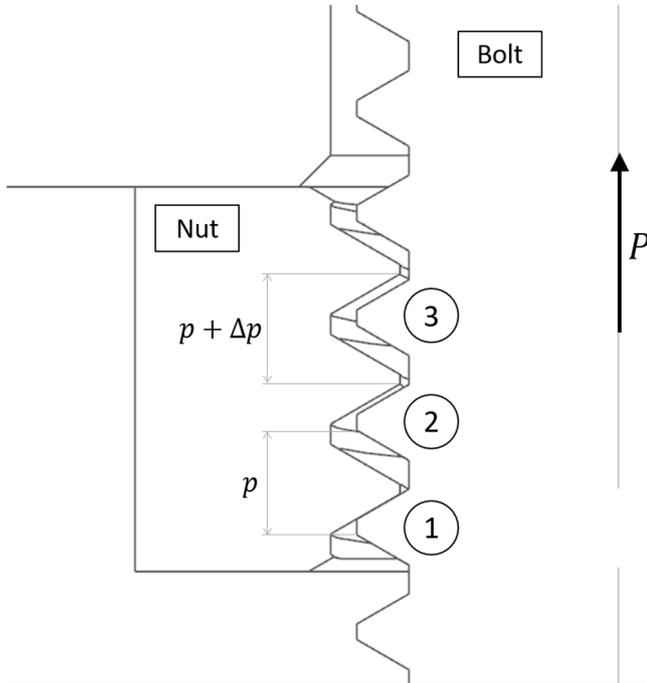


Fig. 1. Illustrative example of a bolted joint with three threads in contact.

validation was only provided for cases with no differential thread spacing.

In this sense, this work develops a semianalytical model to estimate the thread load distribution under different thread pitch values for the bolt and the nut. The values for the parameters of the model are obtained from a Finite Element model of the bolted joint. Thus, the optimum differential thread spacing that provides the desired thread load distribution can be worked out. The results of the model are validated via FEA and compared with those predicted by the classical formulation of Sopwith [15].

2 Materials and methods

A bolted joint consisting of three threads (see Fig. 1) will be analyzed to explain the model. The pitch in the bolt and the nut are p and $p + \Delta p$, respectively. Due to the pitch difference, initially only the thread number 1 is in contact, carrying the whole preload. However, as the preload increases during the bolt tightening operation, the other threads come into contact, and therefore the preload is distributed among them. Finally, when every thread is in contact, the remaining load (until the target preload P_T is reached) is distributed between all of them.

The proposed model assumes the behaviour schematized in Figure 2. The horizontal axis of the graph shows the preload (which increases as the bolt is being tightened) and the vertical axis shows how this preload is distributed among the threads. P_{I_j} corresponds to the preload value when thread j comes into contact, P_{M_j} is the preload value when thread j achieves its maximum percentage of load, and $M_{k,j}$ is the percentage of preload that thread k is

carrying at the preload value P_{M_j} . As mentioned, due to the pitch difference, initially thread number 1 (lower thread) supports the whole preload, but when other threads get in contact, the preload is distributed among them. If the dashed line (thread 2) is observed, it can be divided into three different steps. First, when the preload reaches P_{I_2} , the line gradually rises from zero to the maximum load percentage of thread 2, $M_{2,2}$, which takes place when the preload is P_{M_2} ; at this point, thread 2 is completely in contact. Then, the line decreases as the next threads start sharing the load. Thus, as the percentage of load in thread 3 increases (dotted line), threads 1 and 2 decrease their corresponding percentages. When the preload reaches P_{M_3} , that is, when thread 3 comes into full contact, the load distribution gets stabilized (the percentages remains unaltered) until the tightening operation gets completed at P_T because there are no more threads in this simple illustrative case.

Finite Element analyses were carried out with different configurations (bolt sizes and thread spacings), and it was observed that the response was well represented by the behaviour proposed in Figure 2. Besides, it was observed that, for any joint configuration, the P_{I_j} and P_{M_j} values were always proportional to the pitch difference Δp , while the $M_{k,j}$ values were the same for any Δp .

Finally, two controversial aspects of the linear behaviour assumed in Figure 2 must be discussed. In first place, it could be argued that the maximum percentage of load in a thread should take place just when the next thread comes into contact, but Finite Element results have shown that this is only true for the thread number 1 in Figure 1 (for that reason, in Figure 2 $P_{M_1} = P_{I_2}$ but $P_{M_2} > P_{I_3}$). Second, and as a consequence of the first aspect, it can be observed that in the $P_{I_3} - P_{M_2}$ interval the total load percentage (i.e. the summation of the three percentages) exceeds one, which obviously is not possible. These aspects could probably be overcome by assuming a behaviour more complex than the one presented in Figure 2. Nevertheless, as it will be demonstrated in the Results and discussion section, in spite of these aspects, the model that has been developed from Figure 2 provides very accurate results. Thus, there was no need for refining the model given its simplicity and efficiency as an engineering tool.

From Figure 2, it is deduced that the total load that any thread is carrying at a given value of the preload, is equal to the area below its corresponding line in the interval between null preload and the preload under study. As illustrated in Figure 3, the load increment that a thread will experiment when the preload increases from P_1 to P_2 can be easily calculated as:

$$Load = \int_{P_1}^{P_2} M \cdot dP = \frac{(M_2 + M_1)}{2} \cdot (P_2 - P_1) \quad (1)$$

According to equation (1) and Figure 2, the total load F_j in thread j under preload P_T can be calculated as the sum of three terms:

$$F_j = F_j^1 + F_j^2 + F_j^3 \quad (2)$$

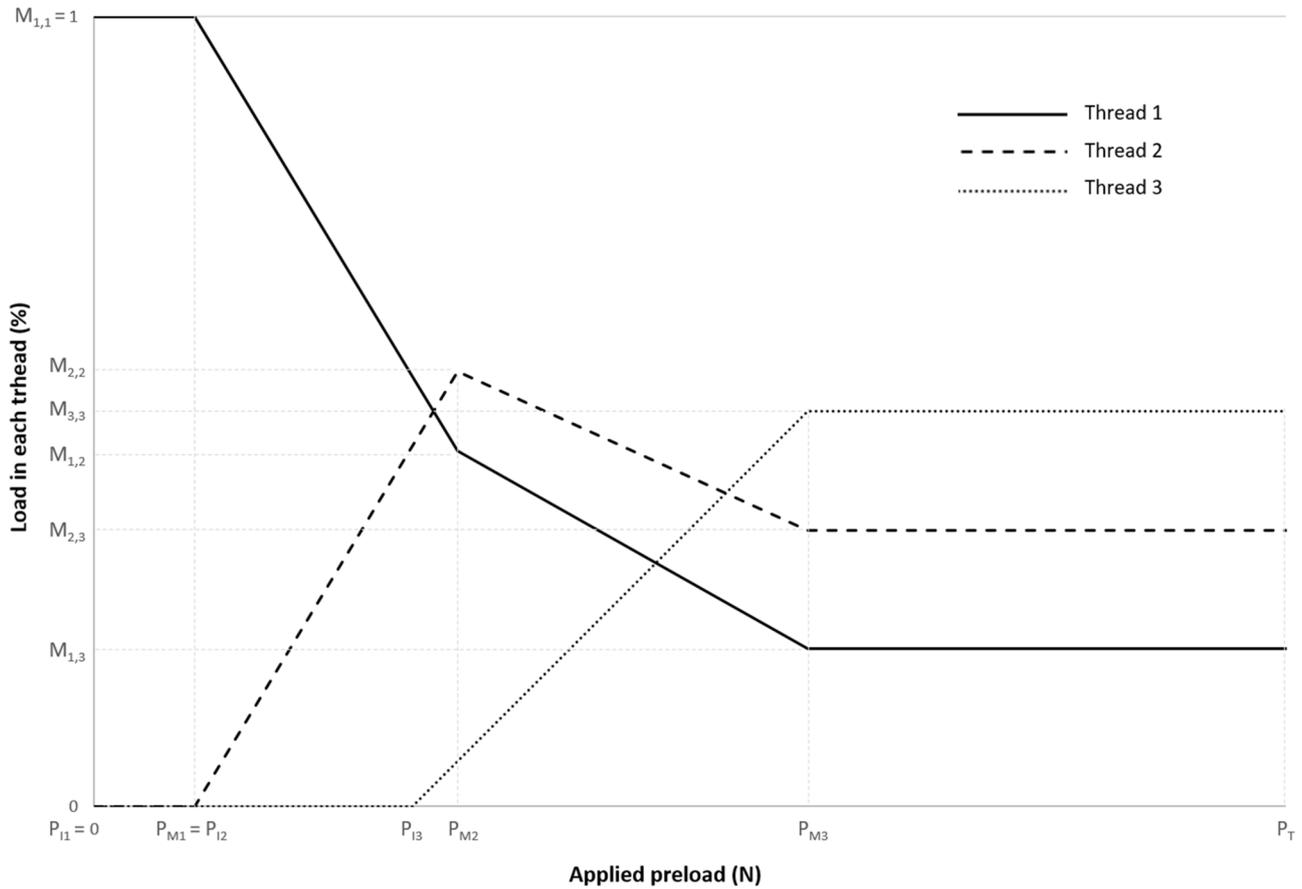


Fig. 2. Preload distribution among threads during preload operation: P_{I_j} =preload when thread j gets in contact; P_{M_j} = preload when thread j achieves maximum load percentage; $M_{k,j}$ =preload percentage in thread k at preload P_{M_j} .

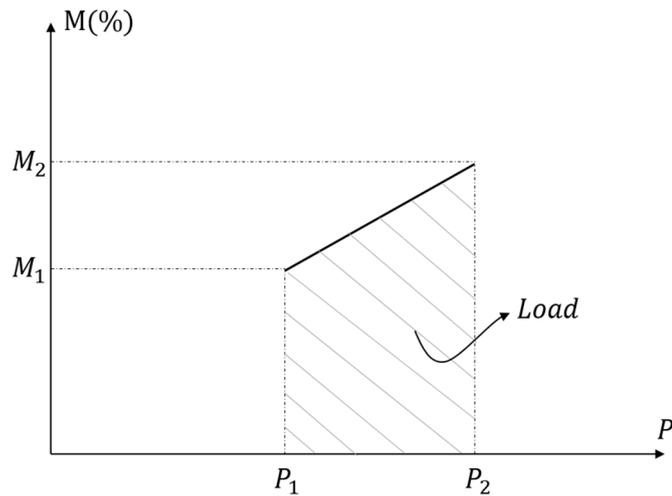


Fig. 3. Calculation of the load increment in a thread under a preload increment.

The first term F_j^1 is the load in thread j when that thread is completely in contact, and therefore achieves its maximum load percentage:

$$\begin{aligned} \text{if } j = 1: F_j^1 &= P_{M_1} \\ \text{if } j > 1: F_j^1 &= \frac{M_{j,j}}{2} \cdot (P_{M_j} - P_{I_j}) \end{aligned} \quad (3)$$

The second term F_j^2 is the additional load that thread j carries as the rest of the threads achieve complete contact:

$$F_j^2 = \sum_{k=j+1}^N \frac{(M_{j,k} + M_{j,k-1})}{2} \cdot (P_{M_k} - P_{M_{k-1}}) \quad (4)$$

where N is the total number of threads in the bolted joint ($N=3$ in the example of Fig. 2). Finally, F_j^3 is the additional load that thread j carries from the point where all the threads are in complete contact up to the target preload value P_T :

$$F_j^3 = M_{j,N} \cdot (P_T - P_{M_N}) \quad (5)$$

In order to obtain the values of P_{I_j} , P_{M_j} and $M_{k,j}$ for a particular bolted joint with a given pitch difference value Δp , a Finite Element analysis of the bolted joint with an arbitrary pitch difference Δp^{FE} must be performed. In that model, the bolt preload is gradually increased. As a result of that analysis, the preload values $P_{I_j}^{EF}$ at which each thread j enters into contact, the values $P_{M_j}^{EF}$ at which it reaches its maximum load percentage and the values $M_{k,j}^{FE}$ with the percentage of preload that each thread k is carrying at the preload value $P_{M_j}^{EF}$ are obtained for that Δp^{FE} . As previously mentioned, the values of P_{I_j} and P_{M_j} are proportional to the pitch difference Δp , whereas $M_{k,j}$ remains the same for any Δp . Thus, P_{I_j} , P_{M_j} and $M_{k,j}$ for any other pitch difference Δp are calculated as:

$$\begin{aligned} P_{I_j} &= \frac{\Delta p}{\Delta p^{FE}} \cdot \Delta P_{I_j}^{FE} \\ P_{M_j} &= \frac{\Delta p}{\Delta p^{FE}} \cdot \Delta P_{M_j}^{FE} \\ M_{k,j} &= M_{k,j}^{FE} \end{aligned} \quad (6)$$

Thus, assuming that every thread is in contact at the end of the assembly ($P_T > P_{M_N}$), equations (3)–(5) can be rewritten as:

$$\begin{aligned} \text{if } j = 1: F_j^1 &= P_{M_1} \\ \text{if } j > 1: F_j^1 &= \frac{M_{j,j}}{2} \cdot (P_{M_j}^{FE} - P_{I_j}^{FE}) \cdot \frac{\Delta p}{\Delta p^{FE}} \\ F_j^2 &= \sum_{k=j+1}^N \frac{(M_{j,k} + M_{j,k-1})}{2} \cdot (P_{M_k}^{FE} - P_{M_{k-1}}^{FE}) \cdot \frac{\Delta p}{\Delta p^{FE}} \\ F_j^3 &= M_{j,N} \cdot \left(P_T - P_{M_N}^{FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) \end{aligned} \quad (7)$$

Thus, the load in each thread j for any given pitch difference value Δp under a preload P_T can be calculated by replacing equations (7) in equation (2). This equation can further be used to assess the optimum pitch difference $\Delta p_{optimum}$ that will provide a uniform load distribution among all the threads. Achieving a completely uniform load distribution is not possible because the pitch difference would have to be different for each thread. Thus, the best strategy is to establish that the first and last threads must have the same load, as proposed by Sopwith [15]:

$$F_1 = F_N \quad (8)$$

Developing equation (8), and substituting the terms of equation (7):

$$\begin{aligned} F_1^1 + F_1^2 + F_1^3 &= F_N^1 + F_N^2 + F_N^3 \\ &\times \left(P_{M_1}^{FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) + \left(F_1^{2FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) \\ &+ \left(M_{1,N} \cdot P_T - M_{1,N} \cdot P_{M_N}^{FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) \\ &= \left(F_N^{1FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) + (0) \\ &+ \left(M_{N,N} \cdot P_T - M_{N,N} \cdot P_{M_N}^{FE} \cdot \frac{\Delta p}{\Delta p^{FE}} \right) \end{aligned}$$

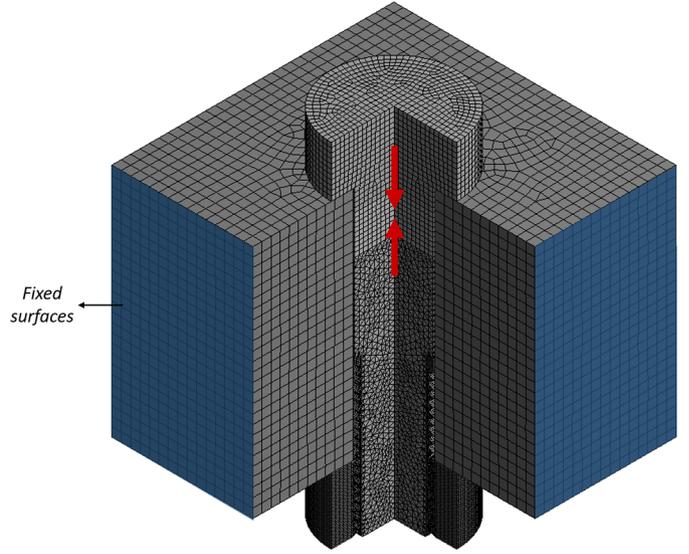


Fig. 4. Finite Element model of the bolted joint.

where, F_1^{2FE} and F_N^{1FE} are the values of F_1^2 and F_N^1 if $\Delta p = \Delta p^{FE}$. From this equation, the differential thread spacing $\Delta p_{optimum}$ that gives a nearly uniform load distribution along threads can be obtained:

$$p_{optimum} = \frac{P_T \cdot \Delta p^{FE} \cdot (M_{N,N} - M_{1,N})}{P_{M_1}^{FE} + F_1^{2FE} - F_N^{1FE} + P_{M_N}^{FE} \cdot (M_{N,N} - M_{1,N})} \quad (9)$$

The optimum pitch difference is expressed as a function of the bolt preload. If the tightening process is not preload controlled (by using bolt tensioners, ultrasonic devices, strain gauges or similar) but torque controlled (with a torque wrench), P_T in equation (9) could be substituted by any classical formula that expresses it in terms of the applied tightening torque, such as the classical one by Motosh [7].

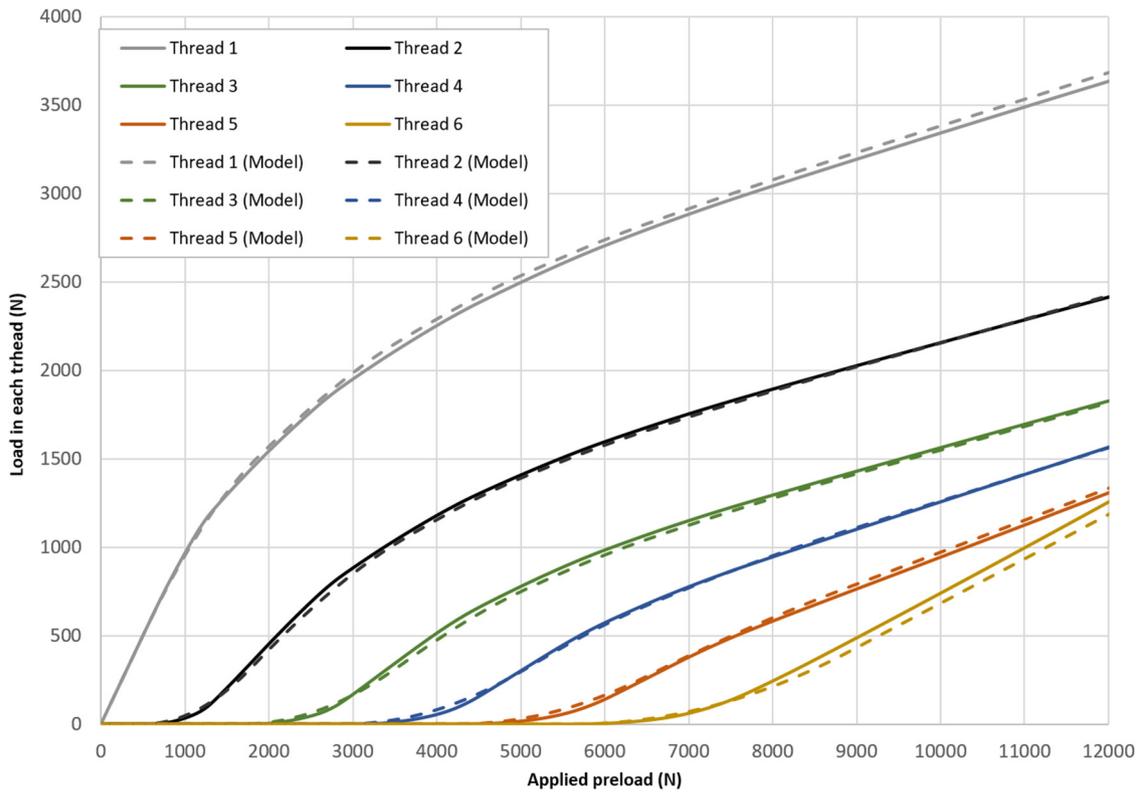
3 Results and discussion

A bolted joint with M10 × 1 bolt under a preload of 12 kN was studied. Figure 4 shows the Finite Element model with its components and fixed boundary conditions. The location of the fixed surfaces, that is, how far they are from the bolt, will affect the stiffness of the whole assembly, and consequently it will play an important role on the load distribution among the threads under bolt preload; in this sense, the size of the material surrounding the bolt must be carefully defined in order to reproduce the stiffness and therefore the behaviour of the bolted joint in a realistic way. The components of the bolted joint were made of steel with $E = 210 \text{ GPa}$ and $\nu = 0.3$, and the friction coefficient between bolt and nut threads was $\mu = 0.2$. The preload was applied by means of a pretension section. High order elements were used, resulting in 1.8M DoF.

As mentioned, an arbitrary pitch difference is analysed as a starting point (Δp^{FE}): in this case, $\Delta p^{FE} = 1.5 \mu\text{m}$ was used, that is, the nut pitch was defined as 1.0015 mm. Thus, preload is increasingly applied to the bolt of the FE model,

Table 1. $P_{I_j}^{FE}$, $P_{M_j}^{FE}$ and $M_{k,j}$ values for the case under study (threads numbered as in Fig. 1).

| | Thread number j | | | | | |
|-------------------|-------------------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| $P_{I_j}^{FE}(N)$ | 0 | 506 | 1671 | 2909 | 4243 | 5613 |
| $P_{M_j}^{FE}(N)$ | 506 | 1827 | 3594 | 5072 | 6467 | 8834 |
| $M_{j,1}$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $M_{j,2}$ | 0.49 | 0.51 | 0 | 0 | 0 | 0 |
| $M_{j,3}$ | 0.29 | 0.29 | 0.35 | 0 | 0 | 0 |
| $M_{j,4}$ | 0.22 | 0.2 | 0.23 | 0.3 | 0 | 0 |
| $M_{j,5}$ | 0.18 | 0.16 | 0.17 | 0.21 | 0.24 | 0 |
| $M_{j,6}$ | 0.15 | 0.13 | 0.13 | 0.15 | 0.18 | 0.25 |

**Fig. 5.** Load distribution during preload process: FE vs proposed model (threads numbered as in Fig. 1).

obtaining the preload values $P_{I_j}^{EF}$ at which each thread j enters into contact, the values $P_{M_j}^{EF}$ at which it reaches its maximum load percentage and the values $M_{k,j}$ with the percentage of preload that each thread k is carrying at the preload value P_{M_j} . Table 1 lists the obtained values of these parameters $P_{I_j}^{EF}$, $P_{M_j}^{EF}$ and $M_{k,j}$, to be introduced in the expressions of the previous section, and Figure 5 illustrate the load distribution obtained in the threads. As it can be appreciated, the behaviour is accurately estimated by the proposed model, the average error being 1.5%. In the table and figure, the threads are numbered as in Figure 1, from the bottom to the top of the bolt shank.

As explained, equation (9) can then be used to calculate the optimum pitch difference, which for the studied case

(bolted joint and target load) provides a value of $0.49 \mu\text{m}$. In contrast, using the formula proposed by Sopwith [15], a value of $0.63 \mu\text{m}$ is obtained.

$$\begin{aligned} \Delta p_{optimum} &= \frac{12000 \cdot 1.5 \cdot (0.25 - 0.15)}{506 + 2719 - 402 + 8834 \cdot (0.25 - 0.15)} \\ &= 0.49 \mu\text{m} \\ \Delta p_{Sopwith} &= \frac{P_T \cdot p}{2 \cdot E} \cdot \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \cdot 10^3 = \frac{12000 \cdot 1}{2 \cdot 210000} \\ &\cdot \left(\frac{1}{70.9} + \frac{1}{125.6} \right) \cdot 10^3 = 0.63 \mu\text{m} \end{aligned} \quad (10)$$

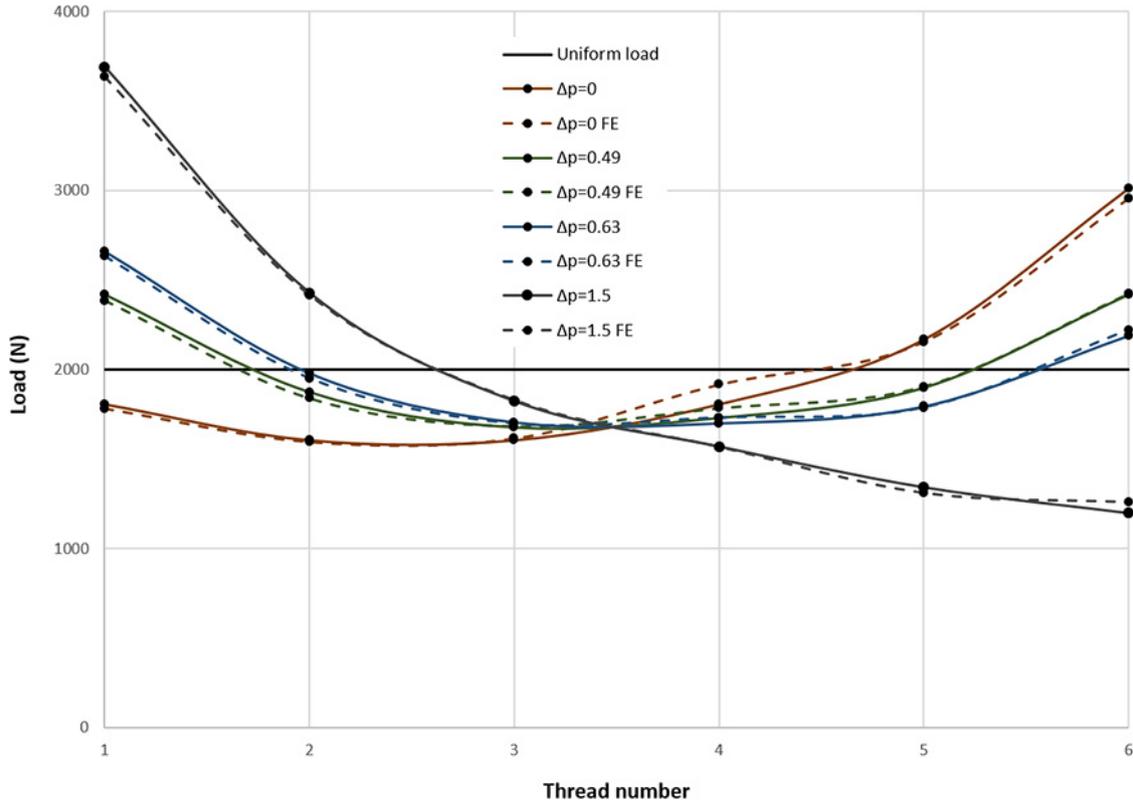


Fig. 6. Load distribution along threads at the end of the tightening operation (threads numbered as in Fig. 1).

Where A_1 and A_2 are the cross sectional areas of bolt and nut respectively. As it can be observed, a difference of 30% can be found between both values. Figure 6 shows the thread load distribution obtained by means of the proposed model and the Finite Element model for these two values of the optimum pitch difference, together with the results for 0 and 1.5 μm . The results show that with the value of 0.49 μm the load in the first and last thread are the same (as imposed in equation (8), from which equation (9) was derived), according both to the proposed model and the Finite Element analysis. Using 0.63 μm this condition is not so accurately fulfilled. Besides, and what is more important, using the optimum pitch difference of equation (9) the load distribution is more uniform than using the value provided by Sopwith in (10), being the standard deviations 290 N and 330 N, respectively. The model by Sopwith is fully analytical, based on some simplifying hypotheses. Despite these simplifications, the model is very useful for its efficiency, because it provides good results with a simple formula. The methodology developed in the present work is semi-analytical, meaning that the values of the parameters that are used to reproduce the behavior of the bolted joint are tuned with the results of a Finite element model of the joint with an arbitrary thread spacing value. In this sense, it gives more accurate results than Sopwith's model, but at the expense of a higher cost.

With this uniform load distribution, the thread stripping behaviour is much improved. In terms of fatigue response, as mentioned in the Introduction section, the

stress concentration in the first engaged thread (i.e. the upper one, not to be confused with thread number 1 in Fig. 1) will presumably be reduced by a more uniform load distribution among threads. As it can be seen in Figure 7, the peak von Mises stress value in the root of the first engaged thread is 1145 MPa without differential thread spacing (0 μm) and 957 MPa with the optimum value of 0.489 μm . This means that the stress concentration factor k_t is reduced almost a 20%, and consequently the fatigue response will be improved. Finally, it should be mentioned that these tolerance magnitudes are not feasible in general applications and steel bolts, but they may be possible in special applications with high precision bolts and light metal alloys.

4 Conclusions

A new model to estimate the thread load distribution in preloaded bolted joints with differential thread spacing between bolt and nut has been presented. Based on a preliminary Finite Element analysis and some simple analytical expressions, the load distribution for any differential thread spacing can be worked out. Furthermore, a formula to calculate the optimum pitch difference that leads to a uniform load distribution is derived, which improves the mechanical behaviour of the joint. The model results are almost coincident with Finite Element results, thus improving the accuracy of classical formulation.

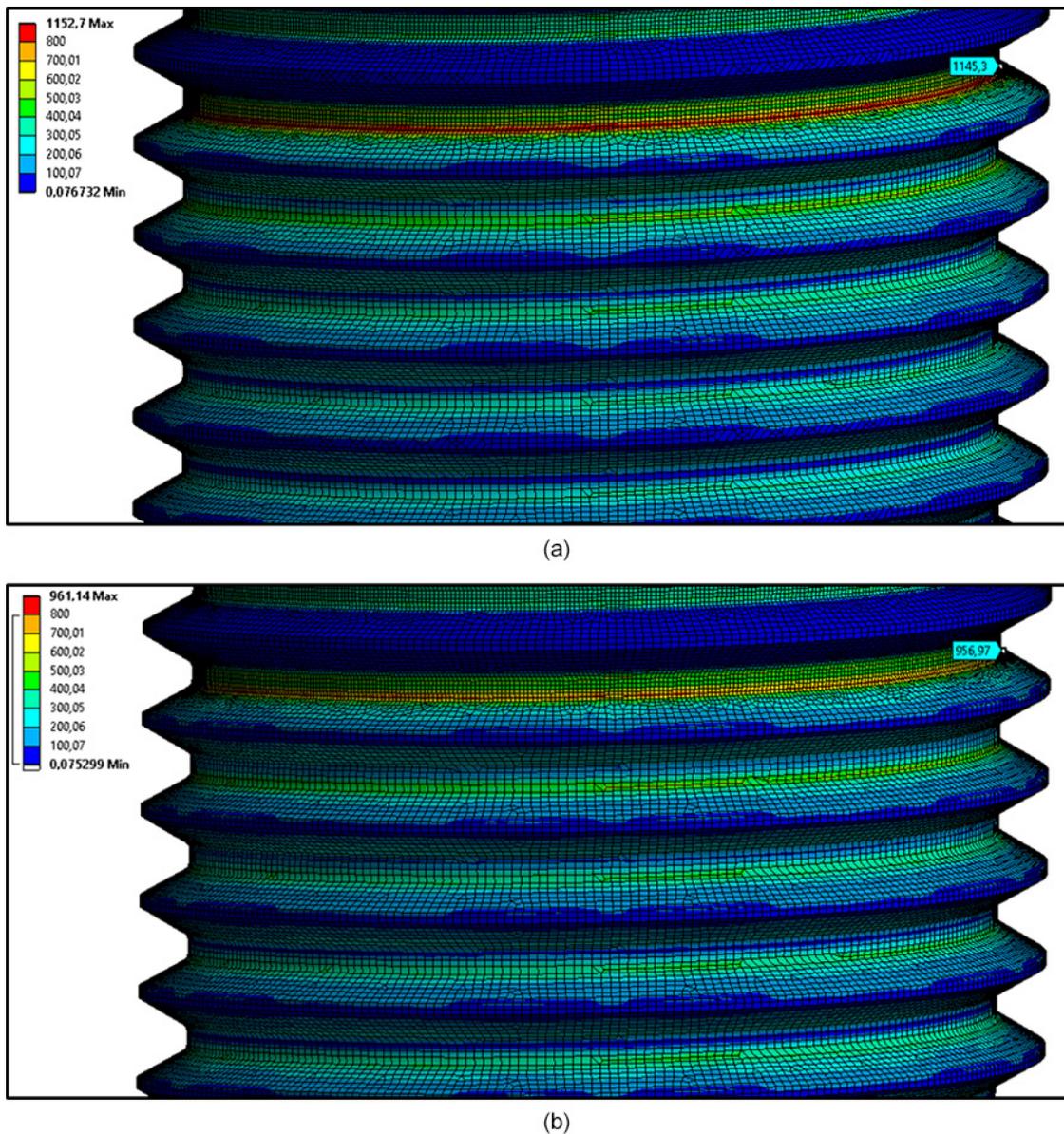


Fig. 7. Von Mises stress in the first engaged thread (upper thread) under a preload of 12 kN with a differential thread spacing of: (a) 0 μm , (b) 0.489 μm .

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