

Tolerance analysis of involute spur gear from the perspective of design

Hua Chen¹  and Xin Li^{2,*} 

¹ Sino-German College of Intelligent Manufacturing, Shenzhen Technology University, 3002 Lantian RD, Pingshan District, Shenzhen, Guangdong, China

² College of Electronics and Communication Engineering, Shenzhen Polytechnic, 7098 Liuxiandong RD, Nanshan District, Shenzhen, Guangdong, China

Received: 10 September 2021 / Accepted: 9 May 2022

Abstract. The existing gears' tolerance standards are formulated from the point of view of manufacturing and measurement. Tolerance categories are numerous. What is more, they have no correlation with each other. Tolerance analysis and optimization of gear and gear set are difficult. Module, pressure angle and center distance are the most important parameters for gears. Their variations directly affect the transmission precision of gears. Through careful research of the influence of deviations of module and pressure angle, a novel method of gear tolerance analysis is proposed in this paper from the perspective of design. For a single gear, tolerance specification and analysis only pay attention to these two tolerances. The influence of fluctuation of center distance for a gear set is also studied. Meanwhile, the calculation method of sensitivities and percent contributions about related parameters is introduced, which provides the basis for tolerance optimization. At last, a simple case is studied, which demonstrates the validity and novelty of the present work.

Keywords: Tolerance analysis / optimization / involute spur gear / tolerance design

1 Introduction

Tolerance design is important content for mechanical design and manufacturing because it affects performance and cost of products, which runs through the whole life cycle of products [1,2]. Tolerance design includes several categories, such as tolerance schemes, tolerance specification, tolerance analysis, tolerance synthesis. Among them, tolerance analysis and synthesis are received extensive attention because of their strong practicability in the fields of research and industry [3]. Tolerance analysis focuses on the feasibility and quality of assemblies or parts while tolerance synthesis pays attention to tolerance allocation and optimization [4,5]. Over the past few decades, too many models proposed by the literature carry out the tolerance analysis and synthesis. In chronological order, we enumerate some classic models [6,7]. Requicha [8] introduced a solid offset approach initially. Since then, Rivest et al. [9] represented and transferred tolerances by virtue of a kinematic formulation. Chase et al. [10] put forward a direct linearization method (DLM) which used vectors to represent dimensions and small kinematic adjustments. These models implement analysis in two dimensional (2D) space. Therefore, they are difficult to deal with geometric tolerance which variational range is three

dimensional (3D) space. 3D tolerance analysis is suitable for geometric tolerance and has many other advantages [11,12]. So tolerance analysis technology developed from 2D to 3D. Bourdet et al. [13] illustrated variation of a feature in relation to another feature by virtue of small displacement taylor (SDT), i.e., three translational vectors and three rotational vectors. Desrochers and Rivière [14] proposed a 4×4 matrix to represent tolerance zones and clearances. Gao et al. [15] brought the DLM into 3D situation. Davidson et al. [16] presented a T-Map model represent all possible variations of a target feature by means of hypothetical Euclidean volume. Schleich et al. [17] extracted discrete points from CAD features to represent variations constrained by tolerances, which is named as skin model.

Gears are the most commonly used precision transmission parts. Especially in the situation of high speed and precision and heavy load, gear system is almost the only option. Transmission accuracy directly affects the reliability and the assembly quality of the gear transmission system, determination and control of which at the design stage is very critical. However, research object of existing tolerance analysis and synthesis models is cylinder and plane. Few of research apply to gears [18,19]. Brauer [20] introduced a global-local FE approach to calculate transmission error in anti-backlash conical involute gear transmission. Lin et al. [21] presented a method for kinematic error analysis and tolerance design of cycloidal gear reducers. Based on Monte

* e-mail: lixin123@szpt.edu.cn

Carlo simulation, Bruyère et al. [22] provided a method for tolerance analysis of bevel gear by tooth contact with a statistical way. Armillotta [23] tried to solve assembly-level geometric errors of spur gears by analogy with an equivalent problem of force analysis. After comprehensive considering the machining and assembly errors of key components, Zhang et al. [24] proposed a mathematical model of the total transmission error and a sensitivity method of gear transmission accuracy.

The research about gears mentioned above is aimed at the various errors of gears, rather than tolerances. More specifically, they taken several or more errors or deviations of gears into consideration, such as form errors, position errors, backlash, gap, center distance errors, shaft misalignments. Then, transmission errors of gears were calculated by virtue of a mathematic model. The existing algorithm is very complex and does not consider the correlation between errors. Errors or deviations mean that the accuracy analysis is from the point of view of manufacture and measurement. The gear is a specific part which surface and parameters are peculiar. ISO [25] or ANSI/AGMA [26] define lots of parameters and their measurements, such as profile form deviation and total profile deviation, tooth-to-tooth tangential composite deviation and total tangential composite deviation, radial runout. At the design stage, there is not a referential scheme of tolerance specification and analysis for gears.

In this paper, we focus on tolerance design stage and deduce tolerance representation of spur gears constrained by tolerances of module and pressure angle. Tolerance analysis about the transmission precision of gears is carried out. In addition, the sensitivities and percent contributions are introduced, which are the basis of tolerance optimization.

2 Tolerance design of involute spur gear

As we know, gear has a history of more than two hundred years, which has evolved into many types. In line with the relative position and motion of two axes, there are spur gears, helical gear, worm and worm-wheel, bevel gear and so on. According to the contour curve, each type of gear has four different contour curves, i.e., involutes, cycloidal, planar, novikov and parabolic. Among them, involute spur gear is one of the most commonly used typical gears, which structure and parameters are simple. The analysis object discussed in the rest of paper is involute spur gear.

2.1 Tolerance specification of involute spur gear

Figure 1 shows a single spur gear (a) and a gear set consisting of two gears (b). In the stage of gears' design, three basic parameters need to be determined, i.e., the module m , the number of teeth z and the pressure angle α . Except for the width of gear which decided by service environment, other dimensional parameters shown in Figure 1a are decided by the three basic parameters. The equations of radius about reference circle and basic circle are list below.

$$r = m \cdot z / 2 \quad (1)$$

$$r_b = m \cdot z \cdot \cos \alpha / 2 \quad (2)$$

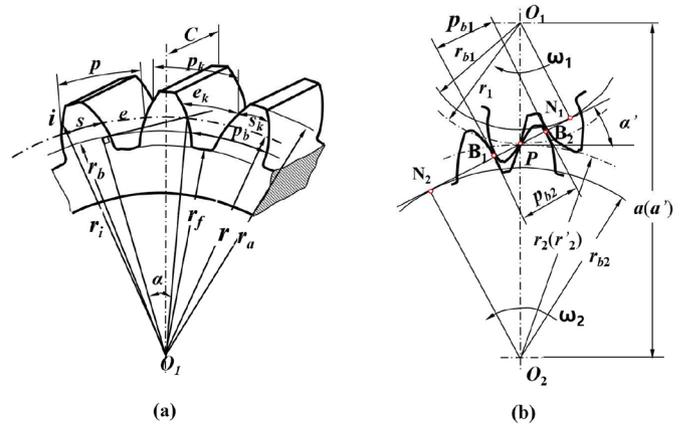


Fig. 1. Parameters of gear and gear set.

where a is the pressure angle which is defined and measured at the reference circle. Other pressure angle, such as point i in Figure 1a, can be expressed as:

$$\cos(\alpha_i) = r_b / r_i \quad (3)$$

The condition for two gears to mesh correctly is that the normal pitch must be equivalent, i.e., $p_{b1} = p_{b2}$, as shown in Figure 1b. Therefore, if the gear set is assembled standardly, the center distance a is equal to $r_1 + r_2$, and the pressure angle α is equal to the meshing angle α' . However, if it is not a standard assembly, many parameters will be changed and the two equations in the previous sentence do not exist.

It should be noted that the transmission ratio of involute gear set i_{12} in Figure 1b is constant theoretically, as shown in equation (4), regardless of standard or nonstandard assembly.

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{o_2 p}{o_1 p} = \frac{r_{b2}}{r_{b1}} = const \quad (4)$$

where point p is the relative instantaneous center of velocity of two gears.

Equation (4) is the tooth profile meshing law. It illustrates one of advantages of the involute gear. In other words, the gear ratio is not affected by the mounting distance which is illustrate as a or a' in Figure 1b.

Among the three basic parameters, z is an integer and has no fluctuation range. Therefore, we should pay attention to the influence of m and α on the transmission accuracy.

Although m and α are integer and predetermined, they have errors actually, which can be explained from gears' manufacturing process. As shown in Figure 2, because there are inevitable eccentricities of geometry and motion, the pitch and tooth thickness are uneven. Due to the geometric eccentric (e_g in Fig. 2) caused by assembly, the radius of reference circle is changed as the rotation of gear. As shown in equation (1), r is related to m . This result can be regarded as that there is a tolerance zone about m . The motion eccentric (e_m in Fig. 2) caused by transmission

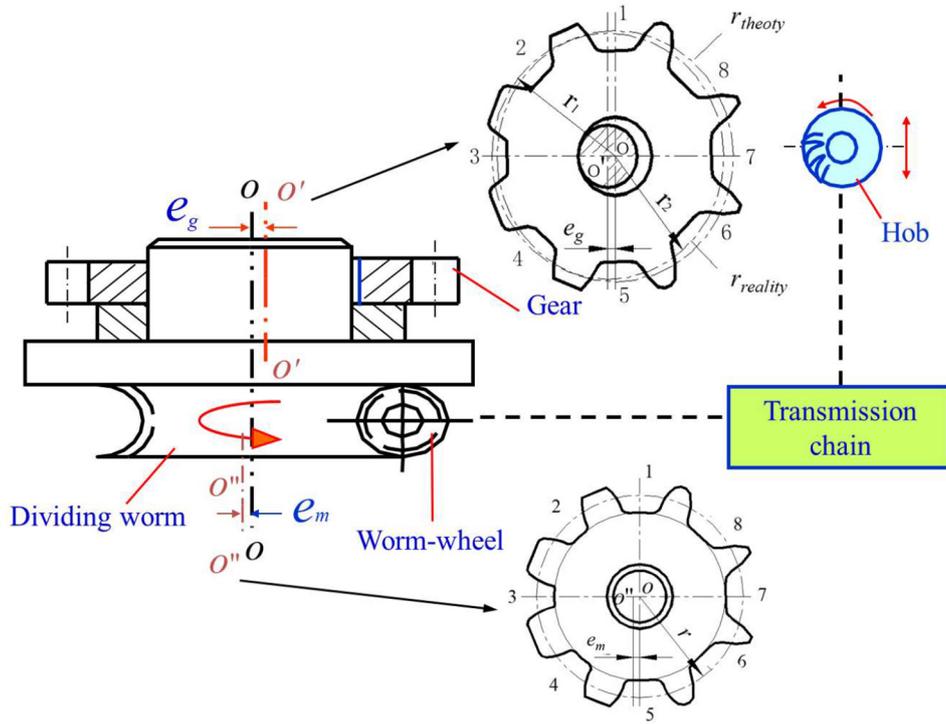


Fig. 2. Gears' errors in manufacturing process.

chain will lead to changes of the curvature of involute tooth surface. The curvature of involute is decided by r_b . Meanwhile, r_b is related to α , as shown in equation (2). This situation can be seen as that there is a tolerance zone about α .

This equivalent method can be explained by equation (4). If the point p (Fig. 1b) is fixed or r_b s are constant, i_{12} is constant, which means that there is no transmission errors. Actually, these changed parameters and point p caused by errors of m and α eventually lead to the transmission error.

According to detailed discussion above, errors of m and α are the main source of transmission errors. Therefore, both of them need to be specified by tolerance at the design stage.

Tolerance analysis includes two parts, i.e., tolerance representation and tolerance propagation, the former expresses deviations of a functional element (FE) within tolerance zone, while the latter transfers these deviations to the functional requirement (FR). In next section, tolerance representation of gears specified by tolerances of m and α is discussed.

2.2 Tolerance representation of involute spur gear

A tooth of a spur gear is shown in Figure 3, where tolerances of m and α are $\pm\Delta m$ and $\pm\Delta\alpha$ respectively. Firstly, we take the meshing point B at the reference circle as the study object. The mathematic expression of point B is written as below formulation.

$$\left. \begin{aligned} x_B &= r_b \sin(u_B) - r_b \cdot u_B \cos(u_B) \\ y_B &= r_b \cos(u_B) + r_b \cdot u_B \sin(u_B) \end{aligned} \right\} \quad (5)$$

where $u_B = \theta_B + \alpha_B$ which is called the roll angle. θ_B is the evolving angle, which expression is:

$$\theta_B = \text{tg } \alpha_B - \alpha_B \quad (6)$$

where α_B is equal to α , because point B is in the reference circle. Combined with equation (2) and equations (5) and (6), the above equation can be rewritten as follows:

$$\left. \begin{aligned} x_B &= m \cdot z \cdot \cos(\alpha) (\sin(\text{tg } \alpha) - (\text{tg } \alpha) \cdot \cos(\text{tg } \alpha)) / 2 \\ y_B &= m \cdot z \cdot \cos(\alpha) (\cos(\text{tg } \alpha) + (\text{tg } \alpha) \cdot \sin(\text{tg } \alpha)) / 2 \end{aligned} \right\} \quad (7)$$

m and α varied in tolerance zones, which lead to x and y of B varied within a range. As we all known, m mainly affects the size of the tooth and α mainly affects the profile of the tooth. Therefore, the tolerance zone of point B consists of four limiting points can be obtained from four sets of variables, which is shown Figure 3. The correspondence between limit points and deviations is shown as follows.

$$\left. \begin{aligned} (x_B, y_B)_{b_1} &\rightarrow (m_{|\pm\Delta m}, \alpha_{|\pm\Delta\alpha}) \\ (x_B, y_B)_{b_2} &\rightarrow (m_{|\pm\Delta m}, \alpha_{|-\Delta\alpha}) \\ (x_B, y_B)_{b_3} &\rightarrow (m_{|-\Delta m}, \alpha_{|-\Delta\alpha}) \\ (x_B, y_B)_{b_4} &\rightarrow (m_{|-\Delta m}, \alpha_{|\pm\Delta\alpha}) \end{aligned} \right\} \quad (8)$$

The tolerance zone of point B is very small, which can be seen as a tilted parallelogram approximately. The width and height of the tolerance zone are $x_{b4} - x_{b3}$ and $y_{b1} - y_{b4}$ respectively.

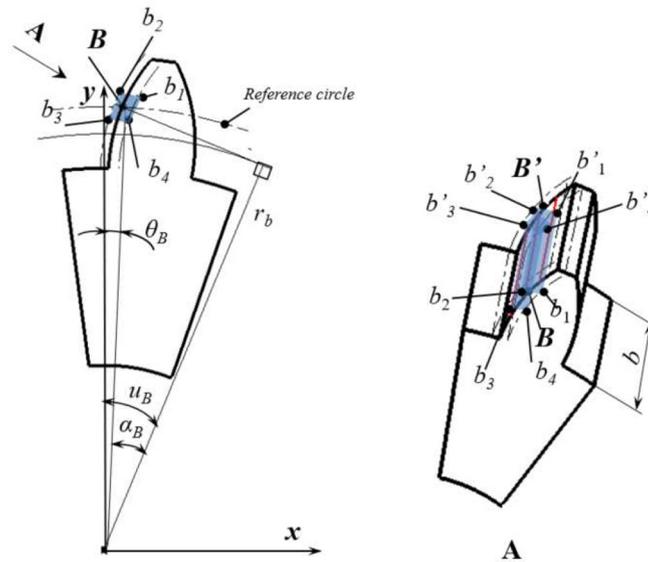


Fig. 3. Tolerance zone of tooth surface.

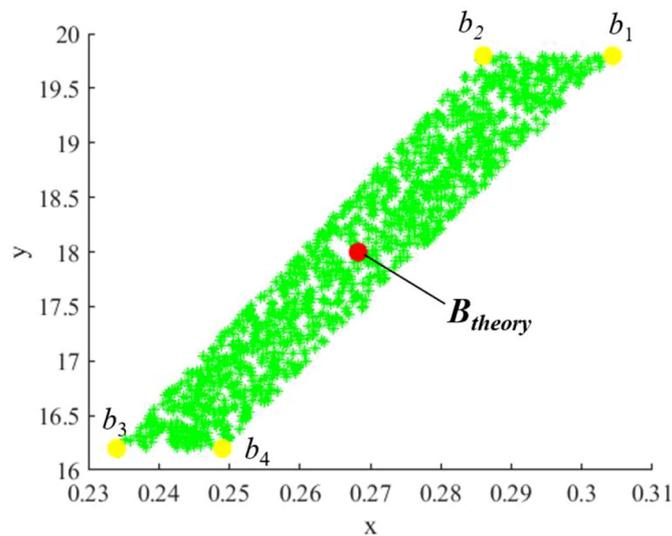


Fig. 4. Simulation results.

The tolerance zone of point B can also be tested by a specific case with another way, i.e., Monte Carlo simulation. The result is shown in Figure 4, where the number of simulations is set as 1200, $z=18$, $m=2 \pm 0.2$, $\alpha=20^\circ \pm 0.2^\circ$.

The result proves the derivation process of tolerance zone of point B is correct. Actually, Gear meshing is line contact, as illustrated by red lines in view A in Figure 3. That is to say, the contacting line $B-B'$ is constrained within the quadrangular prism consisting of points b_1 , b_2 , b_3 , b_4 , b'_1 , b'_2 , b'_3 , b'_4 .

Other contacting lines on involute of tooth can be expressed by the same way. Finally, the whole tolerance zone of the involute gear is obtained by adding all tolerance zones of contacting lines, as shown in view A of Figure 3. It is an offset zone from theoretical involute line, which is a comprehensive precision control zone and is equal to

multiple effect caused by the tangential and radial composite deviations and runout deviation in existing standards.

2.3 Tolerance analysis of gear set

The aim of tolerance analysis of gears' set is to analysis the influence of tolerances from single gear and assembly on transmission. So the FR is transmission tolerances, which includes two targets. The first target is transmission ratio which affects transmission precision. The second target is contact ratio which has impact on transmission stability.

Theoretically, tow gears can mesh correctly only if their pressure angles and modulus are equal. Two gears transmission ratio is shown in equation (4). Because of

tolerances of m and α , i_{12} is not a constant value, which can be rewritten as:

$$i_{12} = \frac{r_{b2}}{r_{b1}} = \frac{m_2 \cdot z_2 \cdot \cos(\alpha_2)}{m_1 \cdot z_1 \cdot \cos(\alpha_1)} \quad (9)$$

Taking the tolerance of m and α into consideration, the range of i_{12} is easily calculated.

$$\begin{aligned} \frac{(m_2 - \Delta m_2) \cdot z_2 \cdot \cos(\alpha_2 + \Delta \alpha_2)}{(m_1 + \Delta m_1) \cdot z_1 \cdot \cos(\alpha_1 - \Delta \alpha_1)} &\leq i_{12} \\ &\leq \frac{(m_2 + \Delta m_2) \cdot z_2 \cdot \cos(\alpha_2 - \Delta \alpha_2)}{(m_1 - \Delta m_1) \cdot z_1 \cdot \cos(\alpha_1 + \Delta \alpha_1)} \end{aligned} \quad (10)$$

The fluctuation of transmission ratio shown in above equation can also be understood as the change of point p shown in Figure 1b. Because of the tolerance zone of tooth surface produced by m and α , the contacting point B_i is floating in its tolerance zone, which leads to the fluctuation of point p .

Meanwhile, tow gears can mesh continuously and stably only if their contact ratio is greater than or equal to 1. The theoretical contact ratio ε is illustrated as follows.

$$\varepsilon = [z_1 \cdot (tg\alpha_{a1} - tg\alpha') + z_2 \cdot (tg\alpha_{a2} - tg\alpha')] / 2 \cdot \pi \geq 1 \quad (11)$$

where α_{a1} and α_{a2} are pressure angles of addendum circles. α' is the meshing angle, which is decided by center distance of two gears.

$$a \cdot \cos(\alpha) = a' \cdot \cos(\alpha') \quad (12)$$

where a is the theoretic center distance and a' is the real center distance. Obviously, $\alpha < \alpha'$.

It should be stressed that the real center distance is decided by the distance of bearing bores where gears are mounted.

The range of ε is:

See equation (13) below.

where ha^* is addendum coefficient, which is equal to 1 for standard gears. α'_{\min} and α'_{\max} are the maximum and minimum value of meshing angle, which depend on the center distance and tolerances of pressure angle. With equation (12), it can be written as follows:

See equation (14) below.

According to above two equations, we can see that the influence factor of ε is pressure angle and center distance. m has no effect on ε .

3 Tolerance optimization for involute spur gear

Tolerance optimization is a necessary step to improve the quality of tolerance design. Generally, tolerance analysis only tells us the result of tolerances accumulation. If the result is beyond the range of FR , there is no doubt that some tolerances of FEs need to be adjusted. If the result is within the range of FR , some tight tolerances also needed to be loosened to reduce the manufacturing cost. Well-designed tolerances can ensure that a product is produced with high quality at low cost.

Sensitivities and percent contributions are two main important references of tolerance optimization. The sensitivity is obtained by taking the derivative of the function of FR . Then, the percent contribution can be calculated by virtue of the sensitivity.

The transmission ratio shown in equation (9) has four sensitivities, i.e., $\partial i_{12} / \partial m_{1or2}$, $\partial i_{12} / \partial \alpha_{1or2}$, while the contact ratio shown in equation (13) has three sensitivities. Because the process of derivative is simple, their functions are not listed in here.

It should be noted that the first order derivative of the function of FR is adapt as the sensitivity is only suitable for simple transfer function. For intricate function, such as high order nonlinear functions, the calculation method of the sensitivity will be complex [27].

The percent contribution is calculated by the sensitivities and tolerances [28]. Taking the pressure angle as an example, the percent contribution of α_1 related to i_{12} is obtained by means of the following equation.

$$p_{\alpha_1} = \frac{|\partial i_{12} / \partial \alpha_1| \times T_{\alpha_1}}{T_{i_{12}}} \% \quad (15)$$

The sensitivities and percent contributions of contact ratio are calculated with the same way.

When all percent contributions of tolerances are obtained, we can arrange them by order. Then, tolerance optimization can be carried out according to the order.

$$\begin{aligned} \varepsilon_{\max} &= [z_1 \cdot (tg(\arccos(z_1 \cdot \cos((\alpha_1 + \Delta \alpha_1) / (2 \cdot ha^* + z_1)))) - tg(\alpha'_{\min})) \\ &\quad + z_2 \cdot (tg(\arccos(z_2 \cdot \cos((\alpha_2 + \Delta \alpha_2) / (2 \cdot ha^* + z_2)))) - tg(\alpha'_{\min}))] / 2 \cdot \pi \\ \varepsilon_{\min} &= [z_1 \cdot (tg(\arccos(z_1 \cdot \cos((\alpha_1 - \Delta \alpha_1) / (2 \cdot ha^* + z_1)))) - tg(\alpha'_{\max})) \\ &\quad + z_2 \cdot (tg(\arccos(z_2 \cdot \cos((\alpha_2 - \Delta \alpha_2) / (2 \cdot ha^* + z_2)))) - tg(\alpha'_{\max}))] / 2 \cdot \pi \end{aligned} \quad (13)$$

$$\begin{aligned} \alpha'_{\max} &= \text{Max}[\arccos(a \cdot \cos(\alpha_{1\max}) / a'_{\max}), \arccos(a \cdot \cos(\alpha_{2\max}) / a'_{\max})] \\ \alpha'_{\min} &= \text{Min}[\arccos(a \cdot \cos(\alpha_{1\min}) / a'_{\min}), \arccos(a \cdot \cos(\alpha_{2\min}) / a'_{\min})] \end{aligned} \quad (14)$$

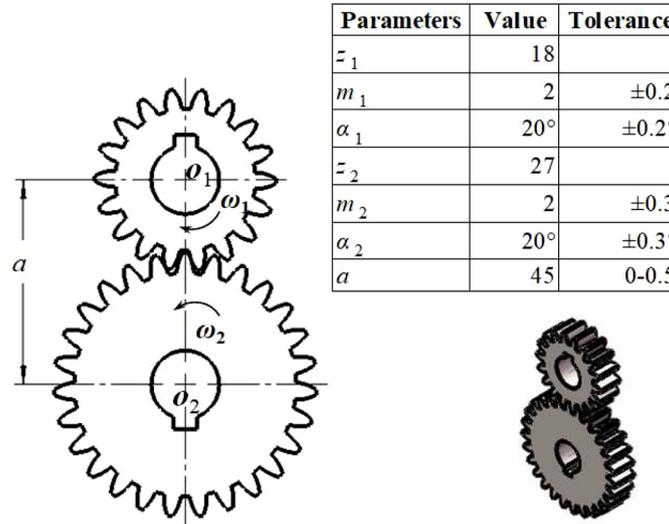


Fig. 5. Tolerance analysis of a gear set.

Table 1. Results of sensitivities and percent contributions.

	Sensitivities		Percent contributions		$P_{\text{Normalization}}$	
	i_{12}	ε	i_{12}	ε	i_{12}	ε
m_1	-0.75	<i>null</i>	42.86%	<i>null</i>	39.5%	<i>null</i>
α_1	+0.546	2.311	0.54%	4.25%	0.5%	5.34%
m_2	+0.75	<i>null</i>	64.29%	<i>null</i>	59.25%	<i>null</i>
α_2	-0.546	3.691	0.82%	10.17%	0.76%	12.78%
a'	<i>null</i>	-0.495	<i>null</i>	65.13%	<i>null</i>	81.88%

4 Case study

In order to validate the method discussed above, a gear set consisting of two gears are shown in Figure 5, where parameters and their tolerances are list.

With equations (9) and (10), the transmission ratio i_{12} is 1.5, and its tolerance zone is [1.1554, 1.9227]. It can be written as the standard expression, i.e., $i_{12} = 1.5_{-0.3}^{+0.4}$. if $\omega_1 = 60$ rpm, the angular velocity of slave gear ω_2 is fluctuating in the range 31.2–51.9 rpm, which means a low precision transmission under the existing tolerance design.

Similarly, the contact ratio and its tolerance is $\varepsilon = 1.58_{-0.31}^{+0.07}$. Although the minimum value is larger than 1, the fluctuation range of the contact ratio is big, which means that the stability of transmission is not well.

The sensitivities and percent contributions of tolerances are calculated and demonstrated in Table 1. The sensitivities' results show that the real center distance has no influence (*null*) on the transmission ratio, which is an advantage of involute gear transmission. But it affects the contact ratio, i.e., the larger the center distance, the smaller the contact ratio.

Because the units of variables are different, the sum of percent contribution in Table 1 is not 100%. The normalized percent contributions are shown in the last column.

The results in Table 1 are very useful for tolerance optimization of gears' design. For example, if the minimum requirement of contact ratio is 1.41, the existing tolerances are unable to meet the design requirement. According to Table 1, the sensitivity and percent contribution of the center distance is very high. Therefore, its tolerance is the first key optimization object. By numerical fitting method, the relation between the upper deviation of a and the minimum ε is shown in Figure 6. Therefore, the upper deviation of a need shrinking from 0.5 to 0.2 to meet the design requirement.

Above results are based on an assumption that the assembly clearances are zero. In actual gear set, the assembly clearances are also important components of the tolerance of real center distance. As shown in Figure 7, two gears in Figure 5 are assembled in a box. The maximum clearance between shaft and the hole of gear 1 or 2 is 0.036 (C_1). The maximum clearance between shaft and the hole of box is 0.041 (C_2). Therefore, the maximum real center distance is 45.577, which percent contribution for ε arrives at 83.9%.

The percent contribution of the real center distance can be continually divided to indicate to each component. By a simple calculating process, the percent contributions of upper deviation of a , C_1 and C_2 are 5.23%, 5.96% and 72.7% respectively.

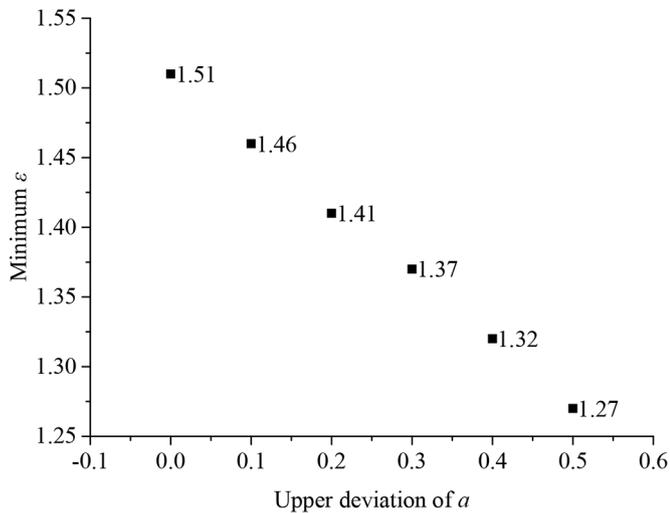


Fig. 6. Relation between the upper deviation of a and the minimum ε .

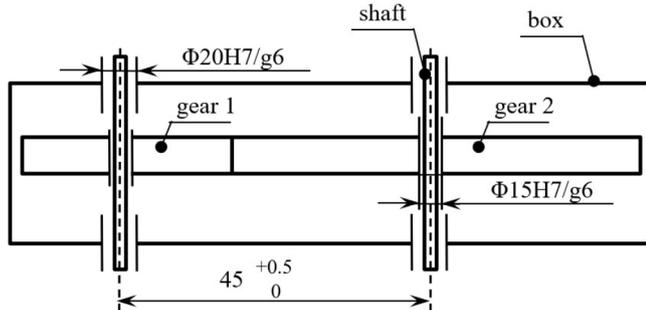


Fig. 7. Assembly condition of two gears with clearances.

5 Comparison and discussion

In the previous section, tolerance analysis and optimization of gear set are illustrated by a simple case. Compared with other tolerance analysis models, the method presented in this paper is simple and convenient. The interrelationship of three tolerances is also can be considered. Because of a novel tolerance analysis method, we can make a detailed comparison and discussion from following five points.

- The number of tolerances need to be specified is little. At the stage of design, the module, the pressure angle, the center distance and the number of teeth are four main design parameters. Among them, the tolerances for the first three parameters need to be specified according to the method shown in this paper. The number of tolerances is less than other methods and standards. Meanwhile, interrelations between these three tolerances are considered.
- Measurement of module and pressure angle is feasible. Generally, tolerance in drawing means that it should be measured after manufacturing. There are many direct or indirect methods or instruments for the measurement of module and pressure angle, let alone the center distance. Meanwhile, the number of tolerances is little means that the cost of quality control is relatively low.

- Theoretically, the number of teeth is also an important variable for transmission ratio and contact ratio. However, it has no influence on transmission precision because it is an integer without error or tolerance. Therefore, it is not considered in this paper.
- The center distance is decided by the distance of bearing bores where gears are mounted. Besides, the assembly clearances, such as the clearance between the shaft and the center hole of gears and the clearance between the shaft and the bearing, are also important components of the tolerance of real center distance.
- Backlash is another importance influence on gears' transmission, which is produced by tolerances of profile of tooth and the center distance. However, it is not discussed here, because its influence on transmission ratio and contact ratio belongs to the effects of module and pressure angle, as well as the center distance.

6 Conclusion

Based on the equation of involute, the attempt of the presented work was to propose a novel method of gear tolerance analysis from the perspective of design. For a single gear, tolerance specification and analysis only pay attention to tolerances of module and pressure angle. Besides, the tolerance of center distance of gear set is another important variable for tolerance analysis. Meanwhile, the calculation method of sensitivities and percent contributions about related parameters is introduced, which provides the basis for tolerance optimization. At last, a simple case is studied, which demonstrates the validity and novelty of the present work.

This work is a preliminary try of tolerance design for involute spur gear from the perspective of design. Future work should focus on other types of gears, such as helical gear and bevel gear. It is promising but challenging.

Acknowledgments. The work described in this paper is supported in part by grant from the teaching reform research project of Shenzhen Technology University (Grant No. 20211002).

References

- [1] S. Du, L. Xi, J. Ni, P. Ershun, C.R. Liu, Product lifecycle-oriented quality and productivity improvement based on stream of variation methodology, *Comput. Ind.* **59**, 180–192 (2008)
- [2] Y.S. Hong, T.C. Chang, A comprehensive review of tolerancing research, *Int. J. Prod. Res.* **40**, 2425–2459 (2002)
- [3] O.W. Salomons, F.J. Haalboom, H. Poerink, F. VanSlooten, F. VanHouten, H. Kals, A computer aided tolerancing tool II: tolerance analysis, *Comput. Ind.* **31**, 175–186 (1996)
- [4] H. Chen, S. Jin, Z. Li, X. Lai, A solution of partial parallel connections for the unified Jacobian-Torsor model, *Mech. Mach. Theory* **91**, 39–49 (2015)
- [5] Z. Shen, G. Ameta, J.J. Shah, J.K. Davidson, A comparative study of tolerance analysis methods, *J. Comput. Inf. Sci. Eng.* **5**, 247–256 (2005)
- [6] G. Ameta, S. Serge, M. Giordano, Comparison of spatial math models for tolerance analysis: tolerance-maps, deviation domain, and TTRS, *J. Comput. Inf. Sci. Eng.* **11**, 0210042 (2011)

- [7] C. Bo, Z. Yang, L. Wang, H. Chen, A comparison of tolerance analysis models for assembly, *Int. J. Adv. Manuf. Tech.* **68**, 739–754 (2013)
- [8] A. Requicha, *Toward a theory of geometric tolerancing* (MIT Press, Cambridge, MA, 1983), pp. 45–60
- [9] L. Rivest, C. Fortin, C. Morel, Tolerancing a solid model with a kinematic formulation, *Comput. Aided Des.* **26**, 465–476 (1994)
- [10] K.W. Chase, J. Gao, S.P. Magleby, General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments, *J. Des. Manuf.* **5**, 263–274 (1995)
- [11] H. Chen, S. Jin, Z. Li, X. Lai, A comprehensive study of three dimensional tolerance analysis methods, *Comput. Aided Des.* **53**, 1–13 (2014)
- [12] S. Du, X. Yao, D. Huang, M. Wang, Three-dimensional variation propagation modeling for multistage turning process of rotary workpieces, *Comput. Ind. Eng.* **82**, 41–53 (2015)
- [13] P. Bourdet, L. Mathieu, C. Lartigue, A. Ballu, The concept of the small displacements torsor in metrology, in *International Euroconference, Advanced Mathematical Tools in Metrology*, 1996
- [14] A. Desrochers, A. Riviere, A matrix approach to the representation of tolerance zones and clearances, *Int. J. Adv. Manuf. Tech.* **13**, 630–636 (1997)
- [15] J.S. Gao, K.W. Chase, S.P. Magleby, Generalized 3-D tolerance analysis of mechanical assemblies with small kinematic adjustments, *IIE Trans.* **30**, 367–377 (1998)
- [16] J.K. Davidson, A. Mujezinovic, J.J. Shah, A new mathematical model for geometric tolerances as applied to round faces, *J. Mech. Des.* **124**, 609–622 (2002)
- [17] B. Schleich, N. Anwer, L. Mathieu, S. Wartzack, Skin model shapes: a new paradigm shift for geometric variations modelling in mechanical engineering, *Comput. Aided Des.* **50**, 1–15 (2014)
- [18] M. Kyung, E. Sacks, Nonlinear kinematic tolerance analysis of planar mechanical systems, *Comput. Aided Des.* **35**, 901–911 (2003)
- [19] E. Sacks, L. Joskowicz, Parametric kinematic tolerance analysis of general planar systems, *Comput. Aided Des.* **30**, 707–714 (1998)
- [20] J. Brauer, Transmission error in anti-backlash conical involute gear transmissions: a global-local FE approach, *Finite Elem. Anal. Des.* **41**, 431–457 (2005)
- [21] K. Lin, K. Chan, J. Lee, Kinematic error analysis and tolerance allocation of cycloidal gear reducers, *Mech. Mach. Theory* **124**, 73–91 (2018)
- [22] J. Bruyère, J. Dantan, R. Bigot, P. Martin, Statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation, *Mech. Mach. Theory* **42**, 1326–1351 (2007)
- [23] A. Armillotta, Tolerance analysis of gear trains by static analogy, *Mech. Mach. Theory* **135**, 65–80 (2019)
- [24] M. Zhang, Z. Zhang, L. Shi, P. Gao, J. Zhang, W. Zhang, A new assembly error modeling and calculating method of complex multi-stage gear transmission system for a large space manipulator, *Mech. Mach. Theory* **153**, 1–23 (2020)
- [25] ISO 1328-2. Cylindrical gears-ISO system of flank tolerance classification-Part 2: Definitions and allowable values of double flank radial composite deviations (2020)
- [26] ANSI/AGMA 1-A01. Accuracy classification system-tangential measurements for cylindrical gears (2015)
- [27] C.G. Glancy, K.W. Chase, A second order method for assembly tolerance analysis, in *Proceedings of 1999 ASME Design Engineering Technical Conferences*, Las Vegas, Nevada, USA, 1999
- [28] H. Chen, X. Li, S. Jin, A statistical method of distinguishing and quantifying tolerances in assemblies, *Comput. Ind. Eng.* **156**, 107259 (2021)

Cite this article as: H. Chen, X. Li, Tolerance analysis of involute spur gear from the perspective of design, *Mechanics & Industry* **23**, 16 (2022)