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On the anti-missile interception technique of unpowered phase based on data-driven theory

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Abstract. The anti-missile interception technique of unpowered phase is of much importance in the military field, which depends on the prediction of the missile trajectory and the establishment of the missile model. With rapid development of data science field and large amounts of available data observed, there are more and more powerful data-driven methods proposed recently in discovering governing equations of complex systems. In this work, we introduce an anti-missile interception technique via a data-driven method based on Koopman operator theory. More specifically, we describe the dynamical model of the missile established by classical mechanics to generate the trajectorial data. Then we perform the data-driven method based on Koopman operator to identify the governing equations for the position and velocity of the missile. Numerical experiments show that the trajectories of the learned model agree well with the ones of the true model. The effectiveness and accuracy of this technique suggest that it will be realized in practical applications of anti-missile interception.

Keywords: Anti-missile interception / data-driven modelling / machine learning / Koopman operator

1 Introduction

The anti-missile interception technique plays a crucial role in military field to prevent missile attack. Usually, the interception in unpowered phase of the missile is more important since it has barely any thrust or control during this process [1]. The basic idea is to predict the trajectory of the missile based on the positions of the missile observed by the radar and then an anti-missile interceptor will be launched to capture the missile. Therefore, it is desirable and essential to establish the missile model and to further predict its moving trajectory.

The traditional missile modeling methods mainly depend on its dynamical analysis via classical mechanics [2,3]. With all sorts of forces and fluctuations emerging in practical case, it is sometimes difficult to take all these factors into consideration to establish a sufficiently accurate model. Fortunately, there are more and more available observable, experimental or simulated data in the missile system with the rapid development of the scientific tools and simulation capabilities. Consequently, how to infer the governing equations of the missile from data is of great significance in developing anti-missile interception technique.

With new progresses achieved in data science field recently, many data-driven methods are proposed to extract dynamical models from data. For example, the Koopman operator theory can be used to identify the deterministic and stochastic differential equations from time-series data [4–6]. Some researchers designed the Sparse Identification of Nonlinear Dynamics method to discover the deterministic ordinary [7,8] or partial [9–11] differential equations from data. Boninsegna et al. [12] generalized this approach to infer Itô stochastic differential equations based on Kramers-Moyal formulas from data. Then Li and Duan [13,14] made further investigation to propose nonlocal Kramers-Moyal formulas and accordingly devised a novel data-driven method to extract stochastic dynamical systems with (Gaussian) Brownian motion and (non-Gaussian) Lévy motion from sample path data. Additionally, there are also some data-driven methods based on neural networks to learn dynamical models from time-series data [15–18].

In this research, we aim to realize the anti-missile interception technique via a data-driven method based on Koopman operator theory. In order to demonstrate the efficacy of this framework, we use an existing dynamical model of the missile established by classical mechanics to generate data and take it as ground truth to examine our data-driven model.

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This work is arranged as follows. In Section 2, we briefly describe the model of the missile established by classical mechanics and assign it as ground truth for subsequent learned model. In Section 3, we introduce a data-driven method based on Koopman operator theory to identify the dynamical system from trajectorial data. Then the numerical algorithm is performed in this missile model to show its effectiveness and accuracy in anti-missile interception technique in Section 4. Finally, the conclusions are presented in Section 5.

2 Model

The flight process of a missile is divided into powered phase and unpowered phase. During the powered phase, the high-temperature and high-pressure gas via the burning propellant is continuously sprayed outside the body to provide thrust for the missile. After that, the missile enters the unpowered phase and flies freely without thrust until it lands. In this work, we consider the anti-missile interception technique in the unpowered phase. In order to verify our results, we employ other method to establish the dynamical model of the missile in this section to generate the simulated data and compare the trajectories between the learned and true systems in later sections.

For the sake of simplification, we make the following assumptions in the missile modeling. First, assume that the missile moves in the vertical plane, and the velocity vector and all forces always lie in this plane. Then consider the missile as a rigid body without elastic deformation. We also ignore the influence of the earth's rotation and the weather. Assume that the missile flies without control and autorotation.

Under these assumptions, the equations of the missile motion can be derived via Newton's second law

$$\begin{aligned} \frac{dx}{dt} &= V\cos\theta, \\ \frac{dy}{dt} &= V\sin\theta, \\ m\frac{dV}{dt} &= -X - mg\sin\theta, \\ mV\frac{d\theta}{dt} &= Y - mg\cos\theta, \\ J_z\frac{d\omega_z}{dt} &= M_z, \\ \frac{d\vartheta}{dt} &= \omega_z, \\ \alpha &= \vartheta - \theta. \end{aligned} \tag{1}$$

Here O-xyz denotes the fixed ground coordinate system, where the missile moves in the vertical plane O-xy and the axis O-z is orthogonal to this plane. The variables x and y indicate the horizontal position and vertical height of the missile, respectively. The velocity vector is represented by its magnitude V and the angle θ with the horizontal direction. The variable ϑ denotes the angle between the axis-direction of the missile and the horizontal direction, and then α is the Attack Angle. The variable ω_z is the angular velocity of ϑ.

The aerodynamic forces and moments in equation (1) have the following expressions

$$\begin{aligned} X &= c_x\frac{1}{2}\rho V^2 S, \\ Y &= c_y\frac{1}{2}\rho V^2 S, \\ M_z &= M_z^{\alpha} + M_z^{\bar{\omega}_z}\bar{\omega}_z = \left(m_z^{\alpha}\alpha + m_z^{\bar{\omega}_z}\bar{\omega}_z\right)\frac{1}{2}\rho V^2 SL, \end{aligned} \tag{2}$$

where

$$c_x = C_{x0} + C_x^{\alpha^2}\alpha^2,\ c_y = C_y^{\alpha}\alpha,\ \bar{\omega}_z = \omega_z d/V. \tag{3}$$

The five aerodynamic parameters C_{x0}, $C_x^{\alpha^2}$, C_y^{α}, $m_z^{\bar{\omega}_z}$ and m_z^{α} in equations (2) and (3) depend on the Mach number of the missile, which are shown in Table 1. The Mach number is defined as the ratio between the speed and the sound velocity $V_s = 20.046 \times \sqrt{288.34 - 5.86 \times 10^{-3}y}$ m/s. When the Mach number is greater than 1.1 (or less than 0.6), the values of these parameters are fixed as the ones at 1.1 (or 0.6). If $\mathrm{Ma} \in [0.6, 1.1]$, then these parameters can be calculated by linear interpolation. Some other parameters are given by the earth radius $R = 6371000$m, the air density $\rho = 1.225 \times (1.0 - 2.0323 \times 10^{-5}y)$kg/m^3 and the gravitational acceleration $g = 9.806 \times (1.0 - 2y/R)$. In addition, the structure parameters of the missile body are listed in Table 2. The parameters S, m, L, d and J_z denote characteristic area, mass, length, diameter and the moment of inertia of the missile body, respectively.

For the convenience of system identification subsequently, we perform a scale transformation to unify the magnitude of the variables. Choosing $V_0 = 500\,\mathrm{m/s}$, $s_0 = 5000\,\mathrm{m}$ and taking $x_1 = x/s_0$, $x_2 = y/s_0$, $x_3 = V/V_0$, $x_4 = \theta$, $x_5 = \omega_z$, $x_6 = \vartheta$, the missile motion equations are transformed into

$$\begin{aligned} \dot{x_1} &= V_0/s_0 \cdot x_3\cos x_4, \\ \dot{x_2} &= V_0/s_0 \cdot x_3\sin x_4, \\ \dot{x_3} &= -(X + mg\sin x_4)/(mV_0), \\ \dot{x_4} &= (Y - mg\cos x_4)/(mV_0x_3), \\ \dot{x_5} &= M_z/J_z, \\ \dot{x_6} &= x_5. \end{aligned} \tag{4}$$

In the practical situation, we can only observe the position x and y of the missile by the radar when the missile is flying with a high speed. The velocity vector V and θ can be computed via the difference of the position data. Therefore, how to predict the moving trajectory of the missile based on the initial value of x, y, V, θ (or x_1, x_2, x_3, x_4) is a key problem in the anti-missile interception technique.

Table 1. Aerodynamic parameters.

Ma	0.6	0.8	0.9	1	1.1
C_{x0}	0.1423	0.1556	0.1687	0.2771	0.3365
$C_x^{\alpha^2}$	0.00178	0.00189	0.00191	0.00198	0.00207
C_y^{α}	0.06896	0.06932	0.06959	0.07099	0.07395
$m_z^{\bar{\omega}_z}$	−13.1058	−12.9125	−13.9052	−14.5589	-15.3402
m_z^{α}	−0.1143	−0.1129	−0.1095	−0.1219	−0.1297

Table 2. Structure parameters of the missile body.

Parameters	Characteristic area S/m^2	Mass m/kg	Length L/m	Diameter d/m	$J_z/\mathrm{kg\,m}^2$
Value	0.0606987	250	2.8	0.278	78.45

3 Theory and method

Note that in order to predict the trajectory of the missile, we need to extract the dynamical system of the missile that can best approximate equation (4) from data. In this section, we use an effective data-driven method based on Koopman operator to achieve this goal [4–6].

3.1 Koopman operator

Consider a dynamical system with the form of ordinary differential equation

$$\dot{x}(t) = b(x(t)), \tag{5}$$

where the phase space of the state vector x is $\mathbb{R}^n$ and the vector field $b : \mathbb{R}^n \to \mathbb{R}^n$. The associated Koopman semigroup of operators $\mathcal{K}^t$ is defined as

$$(\mathcal{K}^t f)(x) = f(\Phi^t(x)), \tag{6}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a real-valued measurable function, indicating the observation of the system. The flow map $\Phi^t(x)$ is a solution of the system (5) starting from the initial point $x(0) = x$, in the sense that $\Phi^t(x) = x(t)$. The infinitesimal generator $\mathcal{L}$ of this Koopman semigroup is defined as the derivative of $\mathcal{K}^t$ at $t = 0$,

$$\mathcal{L}f = \lim_{t\to 0} \frac{1}{t}(\mathcal{K}^t f - f), \tag{7}$$

which is given by

$$\mathcal{L}f = \frac{d}{dt} f = b \cdot \nabla_x f = \sum_{i=1}^{n} b_i \frac{\partial f}{\partial x_i}. \tag{8}$$

Therefore, if f is continuously differentiable, then $u(t, x) := \mathcal{K}^t f(x)$ satisfies the first-order partial differential equation

$$\frac{\partial u}{\partial t} = \mathcal{L}u. \tag{9}$$

3.2 Numerical algorithms

The extended dynamic mode decomposition is an effective method to numerically approximate the Koopman operator by a matrix [4–6]. We briefly review this method in this subsection. Then the vector field of the dynamical system can be identified by the approximate Koopman operator.

Assume that there exists a pair of data sets for the solution process $x(t)$ of equation (5) containing M elements, respectively,

$$\begin{aligned} \mathbb{X} &= [x_1, x_2, \ldots, x_M], \\ \mathbb{Y} &= [y_1, y_2, \ldots, y_M], \end{aligned} \tag{10}$$

where each y_i is the image point of x_i after a small evolution time h for $i = 1, 2, \ldots, M$, i.e., $y_i = \Phi^h(x_i)$. In other words, the equation (5) is integrated by numerical integral method such as Runge-Kutta method from initial point x_i to get y_i in time h. It is also need to choose a dictionary of observable functions $\Psi(x) = [\psi_1(x), \psi_2(x), \ldots, \psi_K(x)]$ to approximate the vector field. The results will be better if we seek more rich type of the observable functions, while the amount of work is immense and polynomial functions are sufficiently accurate for most cases. Thus we select polynomial functions as the dictionary in this research.

Assume that a function f can be written as $f = \Psi B$, where B is a weight vector. Then

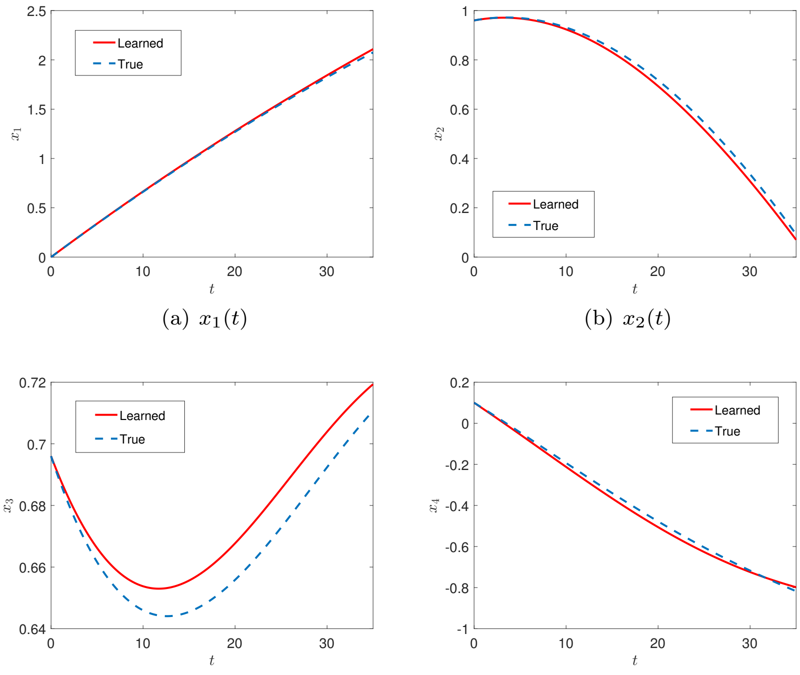
$$\begin{aligned} (\mathcal{K}^h f)(x) &= f \circ \Phi^h(x) \\ &= \Psi \circ \Phi^h(x) B \\ &= \Psi(x)(KB) + r(x). \end{aligned} \tag{11}$$

Via minimizing the residual term $r(x)$, we obtain

$$\begin{aligned} K &= G^+ A, \\ G &= \frac{1}{M} \sum_{m=1}^{M} \Psi(x_m)^T \Psi(x_m), \\ A &= \frac{1}{M} \sum_{m=1}^{M} \Psi(x_m)^T \Psi(y_m), \end{aligned} \tag{12}$$

Table 3. The algorithm for identifying the vector field from data.

Algorithm: Identifying the vector field $b(x)$ from data
Input: Dictionary $\Psi(x)$, data sets X and Y, threshold λ.
1. Compute the matrices G, A, K, L and every vector B^i in equations (12) and (13) for $i = 1, \ldots, n$.
2. Calculate $\widetilde{c}_i = LB_i$ via equation (13).
3. Delete the coefficients smaller than λ and corresponding basis functions in the dictionary.
4. Iterate step 1-3 until no coeffcient is smaller than λ.
Output: The optimal sparsity solution $\widetilde{c}_i$ and corresponding approximate vector field $\widetilde{b}_i(x) = \Psi(x)\widetilde{c}_i$ for $i = 1, \ldots, n$.



(a) $x_1(t)$ (b) $x_2(t)$ (c) $x_3(t)$ (d) $x_4(t)$

Fig. 1. Comparison between the trajectories of learned model (14) and true model (4).

where "+" denotes the pseudoinverse of the matrix. Thus we have a finite-dimensional approximation K of the operator $\mathcal{K}^h$. Using the definition of the infinitesimal generator $\mathcal{L}$ we can also obtain its finite-dimensional approximation L.

According to equation (8), if we take $f_i(x) = x_i$, $i = 1, 2, \ldots, n$, then $\mathcal{L}f_i = b_i$. Assume that $f_i(x) = \Psi B_i$. Thus the vector field can be expressed in terms of the dictionary

$$b_i(x) = (\mathcal{L}f_i)(x) \approx \Psi(x)(LB_i),\ \ i = 1, 2, \ldots, n. \tag{13}$$

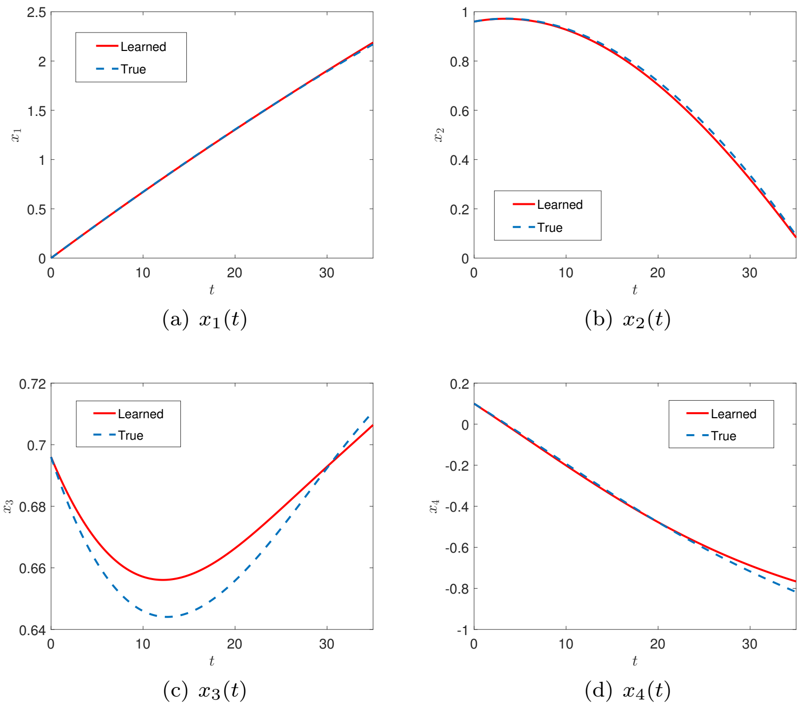
Therefore, the dynamical system (5) is identified from the trajectorial data. Via deleting the coefficients below a small pre-defined threshold parameter λ, we can also reduce the number of observable functions by sparse learning method to avoid overfitting [7,12,13]. The magnitude of λ is usually chosen as about 0.1–10% of the largest coeffcient in equation (13). The complete algorithm for identifying the vector field from data is concluded in Table 3.

4 Numerical experiments

In this section, we employ the data-driven method described in Section 3 to discover the dynamical system from simulated data. Then we can integrate the learned model to obtain its trajectory and compare it with the one integrated by equation (4) in Section 2 to show the effectiveness of the anti-missile interception technique.

Note that the data of x_5 and x_6 in equation (4) are hard to be measured by the radar since they represent the attitude of the missile. Additionally, the exact governing equations for x_1 and x_2 in equation (4) are already known such that we just need to extract the dynamical model in the following form

$$\begin{aligned} \dot{x_1} &= V_0/s_0 \cdot x_3 \cos x_4, \\ \dot{x_2} &= V_0/s_0 \cdot x_3 \sin x_4, \\ \dot{x_3} &= b_3(x2, x3, x4), \\ \dot{x_4} &= b_4(x2, x3, x4). \end{aligned} \tag{14}$$



(a) $x_1(t)$ (b) $x_2(t)$ (c) $x_3(t)$ (d) $x_4(t)$

Fig. 2. Comparison between the trajectories of learned model (14) and true model (4) with noise.

The components b_3 and b_4 do not depend on x_1 since the horizontal position of the missile does not affect its velocity.

First we need to generate the data sets $\mathbb{X}$ and $\mathbb{Y}$ with $M = 1000$ from equation (4). The initial points in $\mathbb{X}$ are randomly and uniformly chosen in the way $x_1 = 0$, $x_2 \in [0, 1.2]$, $x_3 \in [0.1, 1.5]$, $x_4 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $x_5 \in [-1, 1]$, $x_6 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. Then the Euler scheme is used to integrate equation (4) to get the data set $\mathbb{Y}$ with the time step $h = 0.001$. The dictionary of observable functions is selected as polynomial functions with the order up to 3, i.e.,

$$\begin{aligned}\Psi(x) =&[1, x_2, x_3, x_4, x_2^2, x_2x_3, x_2x_4, x_3^2, x_3x_4, x_4^2, x_2^3, x_2^2x_3,\\ &x_2^2x_4, x_2x_3^2, x_2x_3x_4, x_2x_4^2, x_3^3, x_3^2x_4, x_3x_4^2, x_4^3].\end{aligned} \tag{15}$$

Then we can obtain $b_3(x) = \Psi(x)c_3$ and $b_4(x) = \Psi(x)c_4$ with the coefficients

$$\begin{aligned}c_3 =&[-0.0020, 0, 0.0205, -0.0192, 0, 0.0019, 0, -0.0549, 0, 0,\\ &0, -0.0033, 0, 0.0116, 0, 0, 0.0114, 0, 0, 0.0028],\\ c_4 =&[-0.1684, 0.0425, 0.3928, 0, -0.0471, -0.0367, 0.0011,\\ &-0.3691, -0.0115, 0.0399, 0.0137, 0.0233, 0, 0.0055,\\ &0.0025, 0, 0.1218, 0.0042, -0.0297, 0].\end{aligned} \tag{16}$$

Thus the dynamical model of the missile as the form (14) is identified.

The learned model needs to be verified by comparing its trajectory with the one in Section 2. We choose the time interval length $T = 35$s and initial point $(0, 0.96, 0.6960, 0.1, 0.001, 0.00062)$. The initial condition of our learned model just needs the first four components of this point. Then the fourth-order Runge-Kutta method is used to integrate equations (4) and (14) to get their solution paths, respectively, as shown in Figure 1. It is seen that the position and velocity of the learned model are consistent with the ones of the true model.

In fact, there are many random fluctuations such as wind and rain that affect the motion of the missile during its flight process. Therefore, it is desirable to examine the robustness of the algorithm with existing noise. The noise intensity is usually weak and we choose it as about 5%. The concrete operation is to add $0.01B_t$ in the right hand side of equation (4) when we generate trajectorial data, where B_t is a standard Brownian motion.

When the data number $M = 1000$, the error is very large so that we choose $M = 1000000$ with other parameters fixed as before. Then we can learn the components of the vector field $b_3(x) = \Psi(x)c_3$ and $b_4(x) = \Psi(x)c_4$ with the coefficients

$$\begin{aligned}c_3 =&[-0.0018, -0.0054, 0.0266, -0.0183, 0.0019, 0.0061, -0.0020,\\ &-0.0677, 0.0025, 0, -0.0029, 0.0079, 0, 0.0025, 0.0029, 0,\\ &0.0190, -0.0024, 0, 0.0021],\\ c_4 =&[-0.1559, 0.0134, 0.3722, 0, -0.0316, 0.0039, -0.0018, -0.3626,\\ &-0.0097, 0.0391, 0.0159, 0.0018, 0.0010, -0.0030, 0.0022, 0,\\ &0.1232, 0.0024, -0.0295, 0.0012].\end{aligned} \tag{17}$$

The paths integrated by the learned and true model are compared in Figure 2. It is seen that the results also agree well except more data information, which implies that the algorithm is very robust against environmental noise. Since the trajectory $(x_1(t), x_2(t))$ of the missile is accurately predicted, then the anti-missile interception technique can be realized by launching an interceptor to destroy it.

5 Conclusion

In this work, we propose an anti-missile interception technique in unpowered phase of the missile via a data-driven method based on the Koopman operator theory. In particular, we first introduce the dynamical model of the missile established by classical mechanics to generate the trajectorial data and take this model as the ground truth to test the effectiveness of our data-driven modeling. Then the Koopman operator theory and associated data-driven method are described for identification of the vector field. Numerical experiments are performed to discover the governing equations for the position and velocity of the missile. Results show that the trajectories integrated by the learned and true model agree well, which implies that the missile's motion can be predicted well and thus it can be intercepted.

For the sake of simplification, the data in this research are generated by the simulation paths of a known model. Practically, we can measure and record the position and velocity of the missile and employ these real measurement data to discover the dynamical systems of the missile. This will lead to more accurate learned missile model and improve the ability of the anti-missile interception technique.

Competing interests

The authors declare that they have no conflict of interest.

Data availability statement

The data that support the findings of this study are openly available in GitHub https://github.com/liyangnuaa/On-the-anti-missile-interception-technique.

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