

# Research on bearing capacity of cross-type truss boom with variable cross-section of Crawler cranes

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**Abstract.** The web crossed truss boom is one of the commonly used truss boom structures of crawler cranes. However, the existing calculations fail to consider the limiting effect of the web members' bending resistance on the chord members, and cannot give full play to the load-bearing capacity of the existing structure. This paper takes the top section of the Crawler crane truss boom as the research object. The single-span truss theoretical model is established according to Timoshenko's elastic stability theory. And the theoretical critical load of the variable cross-section boom is obtained with full consideration of the limitation of the web member's bending resistance on the chord members. The finite element method simulation model is compared and verified. Compared with a large number of simulation experiments and theoretical calculations, it can be concluded that the theoretical calculations in this article are highly consistent with the simulation results, verified the assumptions that the web members' bending resistance help to improve the bending resistance of the chord members, and this will provide certain reference to the engineering designers.

**Keywords:** Elastic stability theory / bending resistance of web members / variable cross-section / critical load

## 1 Introduction

Lattice column with variable cross-section is applied widely in engineering such as transmission tower and hoisting machinery etc., because of its reasonable loading and material saving. For the stability of the crane boom and the lattice column with variable cross-section, a lot of research has been done by scholars at home and abroad. Drazumeric [1] and others [2,3] successively proposed to use the moment of inertia conversion method to equivalent lattice members to solid web members, and verified them respectively from two aspects of double limbs and four limbs lattice members. It is proved that when the slenderness ratio is greater than 30, the result error is within 5%, which is in line with engineering application. Hai [4] assumed the displacement mode, carried out variational operation on the displacement function, obtained the buckling critical load of simply supported variable section columns at both ends by using the minimum potential energy principle of elastic theory, and pointed out that this method is suitable for different materials and different boundary constraints. Jubran [5] deduced a method for deriving closed-form expressions for

the components of the stiffness matrix and fixed-end forces and moments for tapered members is presented. The necessary fixed-end forces and moments are also derived. The procedure of the proposed method is explained through a practical class of tapered members. Friedman and Kosmatka [6] developed the exact bending stiffness matrix for an arbitrary nonuniform beam including shear deformation. Selmić et al. [7] discuss the problem of optimization of triangular cross-section in lattice mechanical structures. Lagrange's multipliers method and optimization are used in the lattice construction of the tower crane boom. Erfei [8] studied the buckling load of chord of symmetrical cross truss boom and point-to-point truss boom, obtained its buckling law, and proposed the calculation equivalent length coefficient formula which considering the bending action of web member. Yuan et al. [9] proposed to simplify the governing equations for the free vibration of Timoshenko beams with both geometrical nonuniformity and material inhomogeneity along the beam axis. This method enables us to solve exact analytical solutions for Timoshenko beams. Dou et al. [10] investigates the sectional rigidities of trusses and the out-of-plane buckling loads of pin-ended circular steel tubular truss arches in uniform axial compression and in uniform bending. Boel [12] studied the stability of single limb of lattice beam chord, carried out detailed analysis by using

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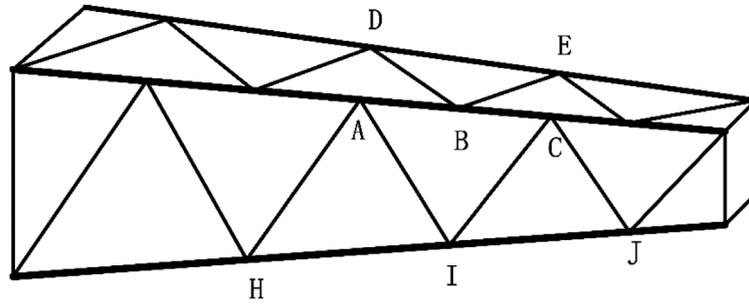


Fig. 1. Cross-type truss boom with variable cross-section web member.

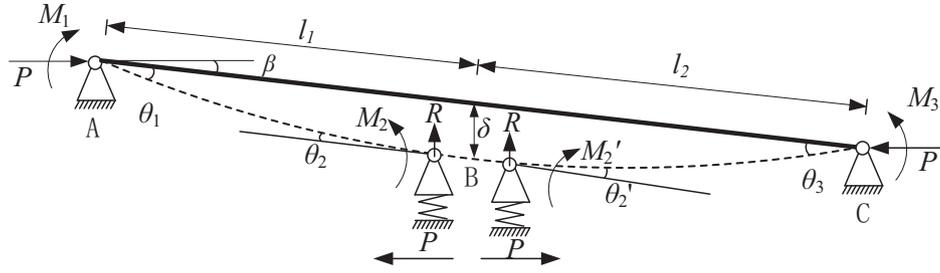


Fig. 2. Chord member simplified model and loading diagram.

finite element software, and obtained the effective length coefficient under various conditions. Rahai [13] studied the stability of variable cross-section hybrid beam and bar structure composed of multiple equal cross-section beams, and calculated the critical force of the structure by using the modified vibration mode and energy method.

Based on the existing theories and literatures, a detailed research has done on the stability of truss boom. However, according to the existing theory, the stiffness of the web member is smaller than that of the chord member, and the limiting effect of the web member on the chord at the node is weak. In order to simplify the calculation, it is generally considered that the node is approximately equal to a hinge point, and the internode length of the chord is its calculated length. However, in practice, the web member and the chord member are rigid connected, and the web member will have certain rotation restrictions on the bending of the chord member. Previous theories ignored the limiting effect of web members on chord member, so that the structural performance could not be made full use. Therefore, this paper takes the top section of crane truss boom as the research object, fully considers the limiting effect of web member bending on chord member, establishes the calculation model through Timoshenko elastic stability theory, analyzes the variable section of crane truss boom, compares it with several groups of simulation experiments, and studies the bearing capacity of variable section boom considering web member bending.

## 2 Establishment and theoretical derivation of mechanical model

### 2.1 Mechanical analysis and model establishment

The schematic diagram of cross-type truss boom with variable cross-section web member is shown in Figure 1.

Chord member ABC has web members AI and CI in AIC plane to constraint the chord member ABC, and web members DB and EB in DBE plane to constraint chord member DE. Chord member ABC may destabilize in both web plane AIC and web plane DBE, but web member DB and EB have strong support function in web plane DBE, so chord member ABC is difficult to destabilize in web plane DBE. Therefore, chord member ABC is most likely to destabilize in web plane AIC.

When the chord member ABC is bent in the web surface AIC, it will be constrained by the rotation of the web member AI around the axis AD and CI around the axis CE. And the constrained effect of DB and EB planar movement along the axis BI, which is simplified as a planar spring. Therefore, the web member plane AIC is simplified as shown in Figures 2 and 3:

Symbols description of Figure 1 in Table 1.

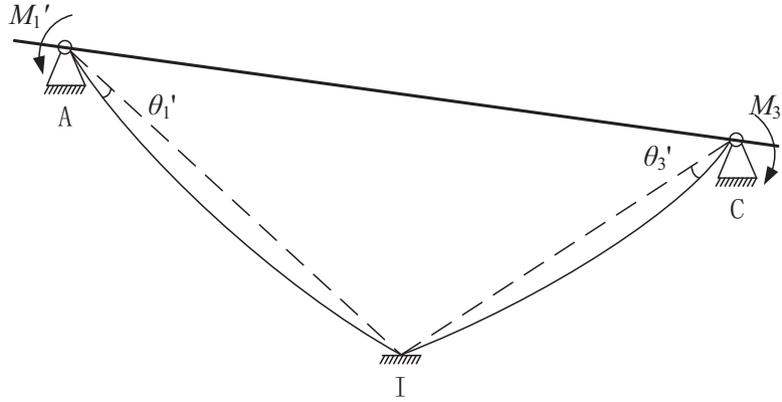
### 2.2 Establishment of the balance equation of every node

According to the Timoshenko theory [14], the assumption of beam-column bending caused by couple are as follows:

$$\varphi(u_i) = \frac{3}{u_i} \left( \frac{1}{\sin 2u_i} - \frac{1}{2u_i} \right), \quad (1)$$

$$\psi(u_i) = \frac{3}{2u_i} \left( \frac{1}{2u_i} - \frac{1}{\tan 2u_i} \right), \quad (2)$$

$$u_i = \frac{l_i}{2} \sqrt{\frac{P}{EI}} \quad (i = 1, 2). \quad (3)$$



**Fig. 3.** Loading of the web member.

**Table 1.** Symbol description.

Symbol	Description	Symbol	Description
$l_1, l_2$	Length of chord member AB and BC	$P$	Axial force on chord member AB and BC
$\beta$	Inclination of chord member in variable cross-section	$M_1$	Bending moment of chord member AB at point A
$M'_1$	Bending moment of web member AI at point A	$M_2$	Bending moment of chord member AB at point B
$M'_2$	Bending moment of web member BC at point B	$M_3$	Bending moment of chord member BC at point C
$M'_3$	Bending moment of web member CI at point C	$\theta_1$	Angle of chord member AB at point A
$\theta'_1$	Angle of web member AI at point A	$\theta_2$	Angle of chord member AB at point B
$\theta'_2$	Angle of chord member BC at point B	$\theta_3$	Angle of chord member BC at point C
$\theta'_3$	Angle of web member CI at point C	$\delta$	Downward deflection of chord member ABC at point B

Next, the rotation angle of points A, B and C is analyzed according to Timoshenko type beam-column theory under elastic constraints. After simplification, the model supports of the variable cross-section boom are not located on the same horizontal line. When analyzing the end turn-angle, the support offset should be taken into consideration. In the compressed and bent rod member with relative displacement, it is necessary to use the end turn-angle as  $\theta - \beta$  replace such as Simitses and Hodges illustrate [15].

The analysis of the turn-angle of point A is as follows:

$$\theta_1 = \frac{M_1 l_1}{3EI} \psi(u_1) + \frac{M_2 l_1}{6EI} \varphi(u_1) + \frac{\delta}{l_1}. \quad (4)$$

The analysis of the turn-angle of left side of point B is as follows:

$$\theta_2 = \frac{M_2 l_1}{3EI} \psi(u_1) + \frac{M_1 l_1}{6EI} \varphi(u_1) - \frac{\delta}{l_1}. \quad (5)$$

The analysis of the turn-angle of right side of point B is as follows:

$$\theta'_2 = \frac{M_2 l_2}{3EI} \psi(u_2) + \frac{M_3 l_2}{6EI} \varphi(u_2) - \frac{\delta}{l_2}. \quad (6)$$

The analysis of the turn-angle of point C is as follows:

$$\theta_3 = \frac{M_3 l_2}{3EI} \psi(u_2) + \frac{M_2 l_2}{6EI} \varphi(u_2) + \frac{\delta}{l_2} \quad (7)$$

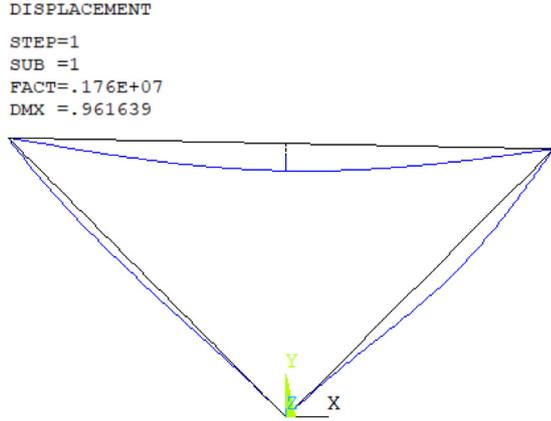
where  $I$  – the inertia moment of the chord member.

## 2.3 Establishment of equilibrium equation for each point

### 2.3.1 Analysis of equilibrium equation for point A

Bending moment equilibrium equation for point A:

$$-M'_1 + M_1 = 0. \quad (8)$$



**Fig. 4.** Buckling results of single-span mechanical model in ANSYS.

Point A is regarded as hinged, from the displacement condition, it can be obtained as follow.

$$\theta_1 = \theta'_1. \quad (9)$$

Web member AI is connected to point I of chord member HIJ in web member plane ACI, and chord member HIJ does not deform in web member plane ACI, so it can be regarded as that one end of web member AI is fixed to point I of chord member HIJ. Since point A is regarded as hinged support, the web member AI is a statically indeterminate beam with one end fixed and the other end hinged in the web member plane ACI, then the turn-angle  $\theta'_1$  of the hinged point A of web member AI can be obtained as follow.

$$\theta'_1 = \frac{-M'_1 b_i}{4EI_Z} \quad (10)$$

where  $b_i$  = length of web member AI;  $I_Z$  = inertia moment of AI Section of web member.

The constraint coefficient of web member AZ to chord member ABC is  $\alpha_1$

$$\alpha_1 = \frac{4EI_Z}{b_i}. \quad (11)$$

Then by formula (4), (10) and (11), equilibrium equation for point A can be obtained as follows:

$$\left(2\psi(u_1) + \frac{6EI}{\alpha_1 l_1}\right) M_1 + \varphi(u_1) M_2 + \frac{6EI(l_1 + 2l_2)}{l_1^2 l_2} \delta = 0. \quad (12)$$

### 2.3.2 Analysis of equilibrium equation for point B

Point B is regarded as hinge support, according to displacement condition

$$\theta_2 + \theta'_2 = 0. \quad (13)$$

Combined with equations (5) and (6), the turn-angle equilibrium equation of point B can be obtained as follow

$$\frac{\varphi(u_1)}{l_2} M_1 + \frac{2(\psi(u_1)l_1 + \psi(u_2)l_2)}{l_1 l_2} M_2 + \frac{\varphi(u_2)}{l_1} M_3 - \frac{6EI(l_1 + l_2)}{(l_1 l_2)^2} \delta = 0. \quad (14)$$

In the web member plane DBE, the chord member ABC is constrained by BD and EB at point B. Similarly, in the web member plane DBE, the chord member DE does not deform, then BD and BE at point D and E are regarded as fixed end constraints. In the web member plane AIC, point B moves up and down along the axis BI, so point B is regarded as a free end, namely, both BD and BE are cantilever beams with one end fixed and the other end free. Assuming that the transverse force  $R$  effects on point B, the displacement of point B at the free end can be obtained from the calculation formula of cantilever beam displacement:

$$\delta = \frac{R}{2} * \frac{b_j^3}{3EI_Z} \quad (15)$$

where  $\delta$  = displacement of the free end,  $b_j$  = length of web member BD and BE.

DBE plane web member has little effect on chord member ABC. The length of two is close, so the average of the two is taken as approximate length.

Assuming the stiffness coefficients of web member BD and BE are  $\lambda$ , there are:

$$\lambda = \frac{6EI_Z}{b_j^3}. \quad (16)$$

Reaction force  $R$  of point B is obtained by taking moments of point A and point C in section AB and section BC

$$R = \left(\frac{P}{l_1} + \frac{P}{l_2}\right) \delta + \frac{M_1}{l_1} - \frac{l_1 + l_2}{l_1 l_2} M_2 + \frac{M_3}{l_2}. \quad (17)$$

Then by formula (15), (16) and (17), displacement equilibrium equation for point can be obtained as follow

$$l_2 M_1 - (l_1 + l_2) M_2 + l_1 M_3 + [(l_1 + l_2)P - l_1 l_2 \lambda] \delta = 0. \quad (18)$$

### 2.3.3 Analysis of equilibrium equation for point C

Bending moment equilibrium equation for point C:

$$-M'_3 + M_3 = 0. \quad (19)$$

Point C is regarded as hinge support, according to displacement condition

$$\theta_3 = \theta'_3. \quad (20)$$

Similar to point A, the web member CI can be regarded as a statically indeterminate beam with one end fixed and the other end hinged. Then the turn-angle  $\theta_3'$  of the web member CI at the hinged end can be obtained as follow.

$$\theta_3' = \frac{-M_1' b_k}{4EI_Z} \tag{21}$$

$b_k$ =length of web member CI.

The constraint coefficient of web member CI to chord member ABC is  $\alpha_2$

$$\alpha_2 = \frac{4EI_Z}{b_k}. \tag{22}$$

Then by formula (7), (21) and (22), equilibrium equation for point C can be obtained as follows:

$$\varphi(u_2)M_2 + \left(2\psi(u_2) + \frac{6EI}{\alpha_2 l_2}\right)M_3 + \frac{6EI(2l_1 + l_2)}{l_1 l_2^2} \delta = 0. \tag{23}$$

### 2.4 Establishment of equilibrium equations

Combined formula (12), (14), (18) and (23), equations about  $M_1$ ,  $M_2$ ,  $M_3$  and  $\delta$  can be obtained. It is a homogeneous equation about the  $M_1$ ,  $M_2$ ,  $M_3$  and  $\delta$ .

See Equation (24) below

According to Cramer's rule to make the bending equilibrium form of formula (24) have non-zero solutions, only if the determinant of formula (24) equal to zero which is show as follow.

See Equation (25) below

**Table 2.** The parameters of the cross-type truss boom with variable cross-section.

Bottom length and width of the top section	2 m × 2 m
Top length and width of the top section	1.56 m × 1.56 m
Whole length	10.65 m
Specification of main chord member	140 × 8 mm
Specification of web member	65 × 3 mm
Inertia moment of chord member	$7.521 \times 10^{-6} \text{ m}^4$
Inertia moment of web member	$2.8143 \times 10^{-7} \text{ m}^4$
Inclination angle of the web	45°

The homogeneous equation of  $P$  is obtained through formula (25), which mean that critical load  $P$  can be obtained.

### 3 Verification of examples

Taking a type of crawler crane truss boom as the research object, the relevant parameters are shown in Table 2.

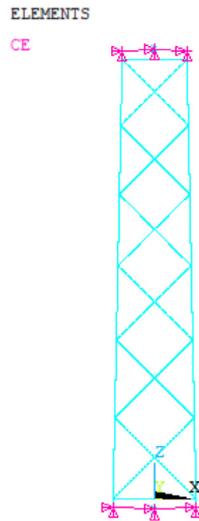
Bringing the parameters of Table 2 into formula (25), and using Matlab software to get the critical load of theoretical calculation, the result is  $P = 1.7736 \times 10^6 \text{ N}$ .

#### 3.1 Buckling results of single-span mechanical model

Finite element method model is established in ANSYS17.0 as shown in Figures 2 and 3. The constraints added to points A and C are defined as hinges, point I is defined as fixed constraints, and point B is defined as a planar spring. The spring stiffness coefficient is calculated as  $\lambda = \frac{6EI_Z}{b_j^3}$ .

$$\begin{cases} \left(2\psi(u_1) + \frac{6EI}{\alpha_1 l_1}\right)M_1 + \varphi(u_1)M_2 + \frac{6EI(l_1 + 2l_2)}{l_1^2 l_2} \delta = 0 \\ \frac{\varphi(u_1)}{l_2} M_1 + \frac{2(\psi(u_1)l_1 + \psi(u_2)l_2)}{l_1 l_2} M_2 + \frac{\varphi(u_2)}{l_1} M_3 - \frac{6EI(l_1 + l_2)}{(l_1 l_2)^2} \delta = 0. \\ l_2 M_1 - (l_1 + l_2) M_2 + l_1 M_3 + [(l_1 + l_2)P - l_1 l_2 \lambda] \delta = 0 \\ \varphi(u_2)M_2 + \left(2\psi(u_2) + \frac{6EI}{\alpha_2 l_2}\right)M_3 + \frac{6EI(2l_1 + l_2)}{l_1 l_2^2} \delta = 0 \end{cases} \tag{24}$$

$$\begin{vmatrix} \left(2\psi(u_1) + \frac{6EI}{\alpha_1 l_1}\right) & \varphi(u_1) & 0 & \frac{6EI(l_1 + 2l_2)}{l_1^2 l_2} \\ \frac{\varphi(u_1)}{l_2} & \frac{2(\psi(u_1)l_1 + \psi(u_2)l_2)}{l_1 l_2} & \frac{\varphi(u_2)}{l_1} & -\frac{6EI(l_1 + l_2)}{(l_1 l_2)^2} \\ l_2 & -(l_1 + l_2) & l_1 & [(l_1 + l_2)P - l_1 l_2 \lambda] \\ 0 & \varphi(u_2) & \left(2\psi(u_2) + \frac{6EI}{\alpha_2 l_2}\right) & \frac{6EI(2l_1 + l_2)}{l_1 l_2^2} \end{vmatrix} = 0. \tag{25}$$



**Fig. 5.** Model establishment of variable cross-section top section and rigid coupling.

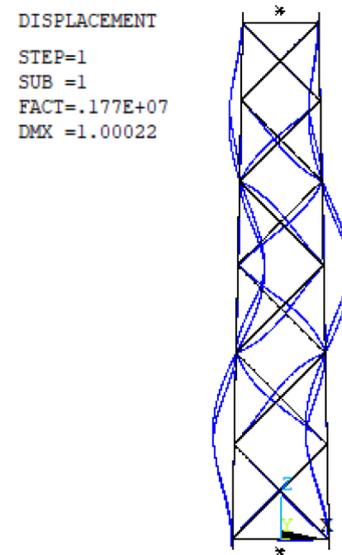
The spring of B point is simulated by COMBIN14 element in ANSYS, where the spring stiffness is  $\lambda$ . A unit load is applied at C point which along the negative direction of  $x$  axis. The buckling results of the mechanical model in this paper are obtained through the buckling analysis in ANSYS, with the critical load  $P = 1.76 \times 10^6$  N.

### 3.2 Buckling results of whole section with variable cross-section

According to the crane design standard, the working plane of the crane truss boom is divided into luffing plane and slewing plane. The constraint conditions of the boom in these two working planes are different and cannot be lumped together. The boom is generally regarded as hinged at both ends in the luffing plane, and it is generally considered as one end fixed and other end free in the slewing plane. In this paper, the working condition of the boom in luffing plane is mainly considered. That is, the constraint of the top joint of the variable cross-section boom is regarded as the hinge support at both ends.

ANSYS node modeling method is used for the top section of variable cross-section boom. Two rigid nodes are established through Mass21 element in both the center of bottom and top, respectively. Each end node of the four chords member is coupled with rigid nodes through CERIG command on both top and bottom, respectively. Finally, the hinge constraint is established for the two rigid nodes at both ends. As shown in Figure 5.

In order to further verify the correctness of the theoretical model and the results of the single-span mechanical model, the finite element model of the variable cross-section truss boom shown in Figure 1 is established as below. The specific parameters of the variable cross-section truss boom are shown in Table 2, where the inclination angle of the web member is  $45^\circ$ . The overall length of the variable cross-section truss boom is 10.65 m and the span number arranged by the web member is 3, which is consistent with the mechanical model of this paper. A unit



**Fig. 6.** ANSYS buckling results of variable cross-section (3 span).

load is applied on each chord to analyze the buckling of the truss boom. Buckling of variable cross-section truss boom is shown in Figure 6.

The buckling analysis of variable cross-section truss boom have been carried out when the span number varied from 0.5 to 3, and the critical load is shown in Figure 7. It can be seen from the data in the figure that the critical load of the boom is large when the span number is small, which means that the straight web members at both ends have strong constraint on the chord members. With the increase of span number, the critical load tends to be stable and close to the theoretical model and theoretical mechanical model analysis results. It shows that the hypothesis is correct and the theoretical mechanical model is reasonable.

### 3.3 Results comparison

The analysis results of the theoretical model, single-span mechanical model and the actual variable cross-section truss boom are summarized in Table 3.

It can be seen from Table 3 that the Timoshenko theory model, single-span mechanical model and actual variable cross-section truss boom model in this paper are very close to the actual buckling results of the variable cross-section top section, and the relative error meets the actual use of the project. It shows that the hypothesis is correct and verifies the accuracy of the theoretical model.

## 4 Influence of geometric model change

In order to verify the universality of the theoretical model, the following analysis is further verified by changing the inclination angle of the web member and the parameters of the truss boom.

### 4.1 Change the inclination angle of web member

The web member mainly bears the shear force between spans. The arrangement of web member should make the

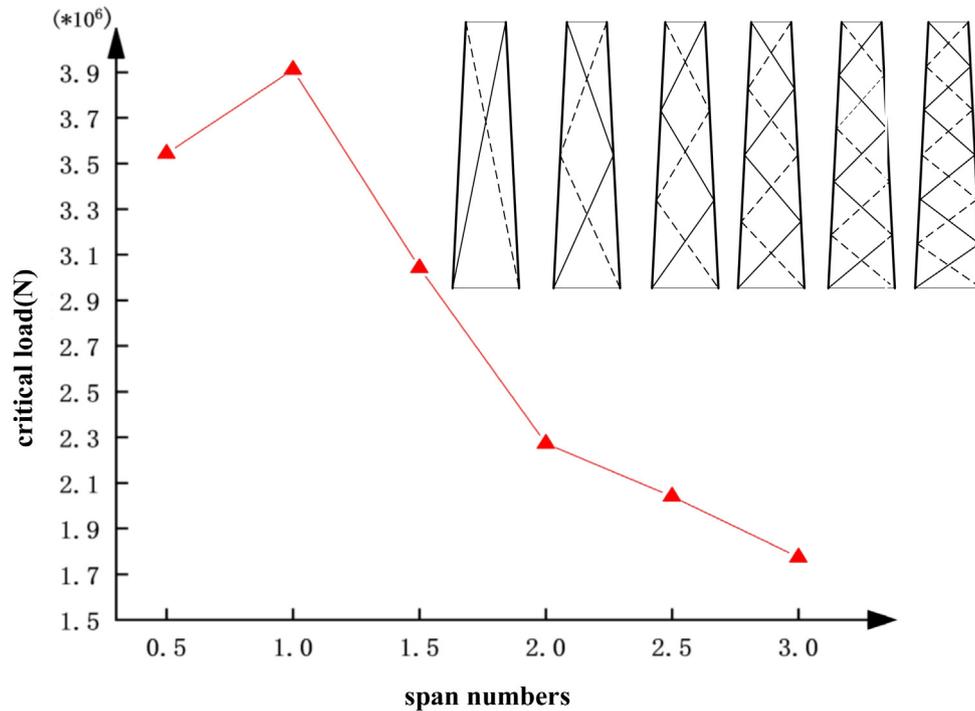


Fig. 7. Critical load of actual variable cross-section truss boom (span number 0.5–3).

Table 3. Comparison of analysis results of three calculation models.

Different model	Critical load ( $\times 10^6 N$ )	Relative error
Timoshenko theory	1.7775	–
Single-span mechanical model (FEM)	1.757	1.15%
Whole section of actual variable cross-section truss boom (FEM)	1.7736	0.21%

Table 4. Comparison of critical load calculation for changing the web member inclination angle.

Inclination angle ( $^\circ$ )	Timoshenko theory ( $\times 10^6 N$ )	Single-span ( $\times 10^6 N$ )	Relative error	Whole boom section ( $\times 10^6 N$ )	Relative error
35	3.3869	3.2023	5.45%	3.2046	5.38%
37.5	2.804	2.6715	4.72%	2.654	5.35%
40	2.392	2.365	1.13%	2.3886	0.14%
42.5	2.024	1.969	2.71%	2.065	–2.02%
45	1.7775	1.757	1.15%	1.7736	0.22%
47.5	1.5255	1.5122	0.87%	1.5846	–3.87%
50	1.2752	1.2828	–0.59%	1.3847	–8.5%

member stress reasonable. The inclination angle of web member has great influence on the internal force, which should be between  $35^\circ$  and  $55^\circ$ , and  $45^\circ$  is the most reasonable angle.

The inclination angle of the web member is changed from  $35^\circ$  to  $50^\circ$ . When the inclination angle of the web member is greater than  $50^\circ$ , the number of the web member is less than  $45^\circ$ . And the span is larger, the restriction of the web member on the chord member is significantly reduced, which will make the analysis results have a large deviation,

and the actual engineering application is less common. Therefore, the inclination angle of the web member from  $50^\circ$  to  $55^\circ$  are not the concern of this article.

Multiple groups of analysis are carried out by increasing the inclination angle of the web member by  $2.5^\circ$  every time from  $35^\circ$  to  $50^\circ$ , as shown in Table 4 and Figure 8. And the diameter remained unchanged as 65 mm.

It is can be seen from Table 4 and Figure 8 that Timoshenko theoretical model, single-span model and the whole section of actual variable cross-section truss boom in

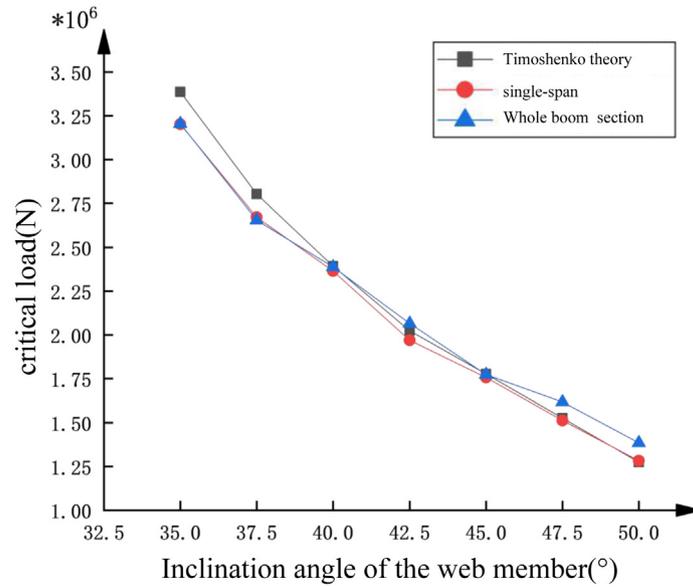


Fig. 8. Critical force variation trend with the inclination angle change.

Table 5. Comparison under different chord member sizes.

Diameter of the chord member (mm)	Timoshenko theory (×10 <sup>6</sup> N)	Single-span (×10 <sup>6</sup> N)	Relative error	Whole boom section (×10 <sup>6</sup> N)	Relative error
120	1.2065	1.1676	3.22%	1.1451	5.08%
125	1.3327	1.3	2.45%	1.2841	3.64%
130	1.4699	1.4431	1.82%	1.435	2.37%
135	1.6185	1.5972	1.31%	1.598	1.27%
140	1.7775	1.757	1.15%	1.7736	0.22%
145	1.9524	1.9386	0.71%	1.9624	-0.51%
150	2.1385	2.1258	0.59%	2.1646	1.22%
155	2.3382	2.3233	0.64%	2.3806	1.81%
160	2.5518	2.5302	0.85%	2.6109	-2.31%

this paper are still highly consistent under different inclination angles of the web member, which further verifies the correctness of the theoretical model in this paper. It can be seen that the critical load of the variable cross-section boom decreases gradually with the increase of the inclination angle of the web member. This is because with the increase of the inclination angle of the web member, the number of web members decrease and the constraint effect of web member on chord member reduce. Additionally, increased gradually with the span, the length of the chord member of each span will more prone to instability.

#### 4.2 Change the dimension of the chord member

The lattice boom is composed of chord members and web members. The chord member is arranged in the four focal points of the rectangular section to bear the axial load which are the main force component in the boom. The theory of this paper is verified by changing the chord

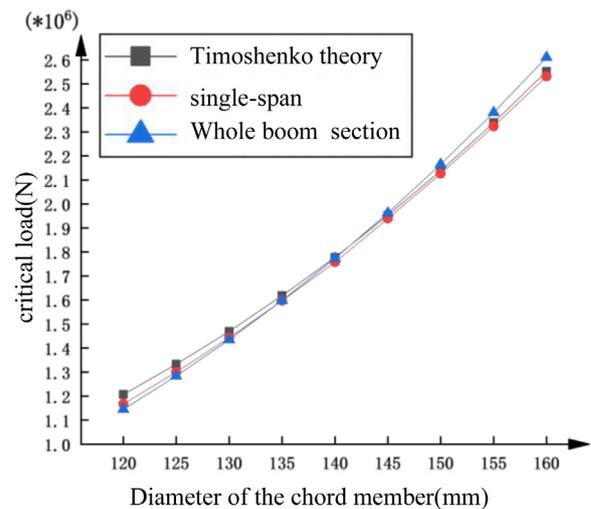
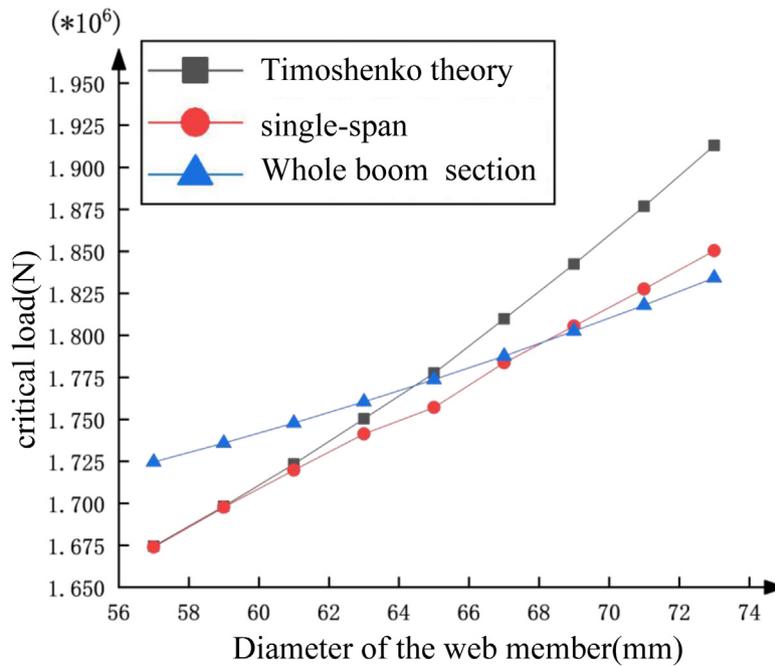


Fig. 9. Critical force variation trend with the chord member diameter change.

**Table 6.** Comparison under different web member sizes.

Diameter of the web member (mm)	Timoshenko theory ( $\times 10^6 N$ )	Single-span ( $\times 10^6 N$ )	Relative error	Whole boom section ( $\times 10^6 N$ )	Relative error
57	1.6745	1.6739	0.04%	1.7247	-2.99%
59	1.6981	1.6977	0.023%	1.7359	-2.22%
61	1.7234	1.7198	0.21%	1.7478	-1.41%
63	1.7504	1.7412	0.52%	1.7604	-0.57%
65	1.7775	1.757	1.15%	1.7736	0.22%
67	1.8098	1.7836	1.44%	1.7876	1.22
69	1.8423	1.8053	2%	1.8024	2.16%
71	1.8767	1.8275	2.62%	1.8179	3.13%
73	1.9130	1.8503	3.27%	1.8341	4.12%

**Fig. 10.** Critical force variation trend with the web member diameter change.

member dimension. Multiple groups of analysis are carried out by increasing the diameter of the chord member by 5 mm every time from 120 mm to 160 mm, as shown in Table 5 and Figure 9.

It can be seen from Table 5 and Figure 9 that the results of the three calculation models are still highly consistent under different chord member diameters, which further verifies the correctness of the theoretical model in this paper. It also can be seen that with the increase of the diameter of the chord member, the critical load of the variable cross-section boom increases gradually, which is due to the increase of the diameter of the chord member. The stiffness of the truss chord increases, and the ability of the chord to resist deformation is stronger, so that the critical load of the boom increases.

### 4.3 Change the dimension of the web member

The web member can prevent excessive deflection between the chord members and increase the overall stiffness of the truss, so that the force can be shared by the chord member of the truss boom. The theory of this paper is verified by changing the web member dimension. Multiple groups of analysis are carried out by increasing the diameter of the web member by 2 mm every time from 57 mm to 73 mm, as shown in Table 6 and Figure 10. And the inclination angle remained unchanged as  $45^\circ$ .

It can be seen from Table 6 and Figure 10 that the relative error of the three calculation models under different diameters of web member is very small, which further verifies the correctness of the theoretical model in

this paper. It also can be seen that the critical load of the variable cross-section boom increases with the increase of the diameter of the web member, which is due to the diameter increase of the web member, the constraint effect of the web member on the chord member is gradually enhanced, and the critical load of the boom is increased. Furthermore, it can be concluded from [Tables 5](#) and [6](#) that the influence of chord member size on the bearing capacity of truss boom is much greater than that of web member.

## 5 Conclusion

In this paper, the top section of the truss boom of the crawler crane is taken as the research object, and the single-span truss theoretical model is established by Timoshenko elastic stability theory. The theoretical critical load of the variable cross-section boom is solved under the constraint of the bending of the web member on the chord member. The results are compared with the finite element single-span simulation and the Whole section of actual variable cross-section truss boom model, and the relative error of the results is controlled within 5%, which verifies the hypothesis and theoretical calculation in this paper. It is indicated that the limit effect of bending on chord member should not be ignored in practical calculation. In order to verify the universality of the theoretical model, a variety of models are formed by changing the dimension of the truss boom. A large number of theoretical calculations and simulation experiments show that the theoretical calculation in this paper is highly consistent with the simulation results, which verifies the hypothesis, indicating that the calculation results obtained by the existing theoretical calculation are conservative, which limits the performance of the material. Therefore, in the actual production calculation, it is necessary to consider the restraint effect of the bending of the web member on the chord member. It also shows that smaller inclination angle of the web member restrict the chord member stronger. And the the bearing capacity of the boom is better. However, too small inclination angle of the web member will lead to too many web member and too much weight of the truss boom. The actual inclination angle of the web member is  $45^\circ$ . On the other hand, the influence of chord member size on the bearing capacity of truss boom is much greater than that of web size.

## Declaration of conflicting interests

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