

A dynamic stiffness model for high-speed ball screw pair with the mass center of screw nut considered

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Abstract. The development of accurate dynamic stiffness models is a key aspect in the design process of ball screws. However, variations in the nut center of mass position can affect the dynamic stiffness, which makes it difficult to develop a stiffness model. To address this issue, this paper considers the variation of the nut center of mass position and the approach between the ball screw beam unit stiffness matrix and the nut interface element stiffness matrix. Then, based on Timoshenko beam theory and a concentrated mass parameter method, a dynamic stiffness model for a high-speed ball screw pair considering the variation of the nut center of mass position is developed. Finally, the feasibility of the dynamic stiffness matrix of the high-speed ball screw pairs is verified through the inherent frequencies obtained from experiments and simulations of the ball screw pair in the free state and compared with the classical finite element method to verify the importance of this method and to provide a theoretical basis for its dynamic design and structural optimisation.

Keywords: Ball screw / center of mass / Timoshenko beam theory / concentrated mass parameter method / dynamic stiffness

1 Introduction

With the widespread application of various new machining technologies [1], the performance requirements of mechanical equipment are becoming increasingly stringent. Due to the high precision, high stiffness, microfeed and high-speed feed characteristics of ball screws [2], they are widely used in feed systems. With the increasing speed of ball screw operation, its vibration problem becomes more and more prominent, which seriously affects its positioning accuracy and machining stability, so it is crucial to accurately establish the stiffness model of high-speed ball screw pair [3–5].

Stiffness is the most important performance indicator of ball screws [6,7]. The stiffness of ball screw pairs has been studied by many scholars. The first systematic study of ball screw pairs was reported by Nakashima [8] in 1988, which considered the effects of hertz contact values, thread deformation and load distribution on the stiffness of ball screws. Verl [9] studied the effect of preload force, nut and screw raceway geometry parameters on the axial stiffness

of ball screws from the structural characteristics of the nut and proposed a method for calculating the axial stiffness of ball screws. Okwudire [10] represented the screw as a beam and incorporated the effects of axial, torsional and lateral deformations into the contact stiffness model of the ball screw. Zhou [11] developed a stiffness model for a single-nut ball screw considering the helical lead angle and preload force by comparing the simulation results of two single-nut ball screws in finite element software. These studies above are not satisfactory for a good description of the stiffness characteristics of ball screws.

On the one hand, dynamic stiffness is a physical quantity different from static stiffness and should be given more attention in the practical application of ball screws. Chen [12] established a dynamic stiffness model for a single-nut ball screw pair and verified the accuracy of its dynamic stiffness model through experiments. On the other hand, during the operation of the feeding system, the dynamic characteristics of the ball screw will be changed due to the constant change of the nut position, which will directly affect the stiffness characteristics of the ball screw, which brings new challenges to the vibration damping research. Gao [13] reduced the vibration of the ball screw by designing a damping damper considering the position of the nut.

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Zou [14] developed a ball screw stiffness model for different screw-nut positions and used it to study the dynamic characteristics of the ball screw feeding system. Zhang [15] gave a method for calculating the stiffness and mass matrix of the system considering the variation of the nut bond position and investigated the effect of the rated dynamic load of the nut bond on the inherent frequency of the slender ball screw feeding system. Wang [16] considered the effect of feed speed and nut position of the ball screw feed system and derived an expression for the equivalent axial stiffness of a single moving union. However, in the current study, the dynamic characteristics of the nut center of mass variation and stiffness are not considered simultaneously.

As mentioned above, the development of a high precision dynamic stiffness model of the ball screw, taking into account the dynamic characteristics of the center of mass and stiffness of the screw nut, can be of great help in improving high speed feed systems. In this paper, the ball screw is divided into a number of axial beam segments based on Timoshenko beam theory [17]. The stiffness matrix of the ball screw beam elements is then formed by superimposing the stiffness matrix of each axis segment beam with the center of mass of the screw nut under consideration. Meanwhile, the stiffness matrix of the interface unit of the screw nut is obtained by modelling the screw nut assembly using the concentrated mass parameter method, by constructing the stiffness matrix of the ball screw beam element coupled to the stiffness matrix of the ball screw nut interface element to obtain the center of mass of the screw nut during the transfer motion. The dynamic stiffness of the high-speed ball screw pair can then be solved exactly as shown in Figure 1. Finally, the feasibility of the dynamic stiffness model of the high-speed ball screw pair is verified by experiments and simulations, and the accuracy and importance of the method in this paper is demonstrated by comparison with the classical method of neglecting the center of mass of the nut.

2 The dynamic stiffness model of ball screw pair

As shown in Figure 2, the feed system of CNC equipment such as the high-speed machining center is composed of motor, coupling, ball screw, nut, bearing and high-speed spindle unit. In actual operation, the motor drives the ball screw to rotate through the coupling, which converts the rotary motion of the ball screw into the reciprocating linear motion of the high-speed spindle unit along the axial direction of the screw. To solve the dynamic stiffness of the ball screw pair, this paper divides the shaft segments of the beam element according to the Timoshenko beam element. Then, the stiffness matrices of two shaft segments are obtained. Finally, this paper solves the dynamic stiffness model of the whole high-speed ball screw by considering the position of the screw nut.

2.1 The division of the shaft section

Segmental modelling is a prerequisite for stiffness modelling. As shown in Figure 3, the x and y directions are the radial directions of the ball screw axis and the z direction is

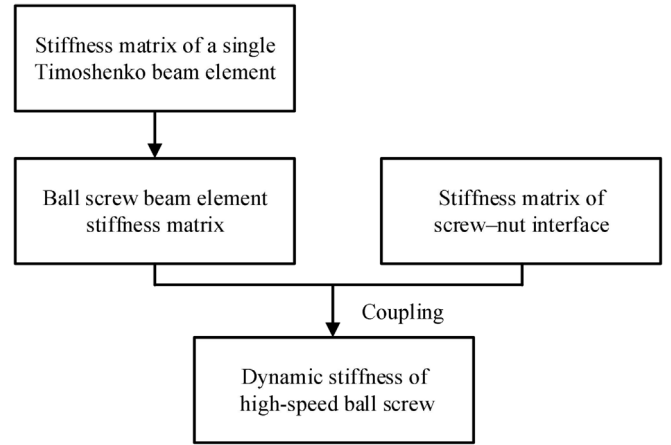


Fig. 1. The process of solving the dynamic stiffness of the high-speed ball screw.

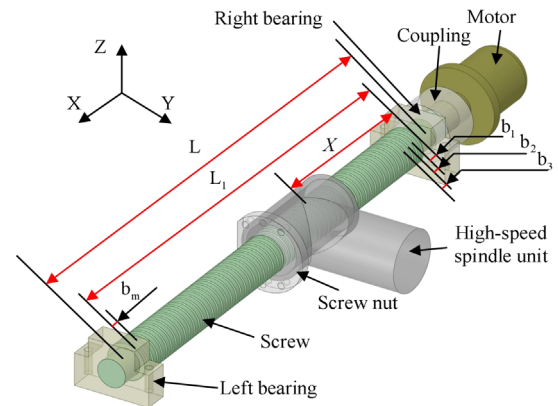


Fig. 2. Composition and Shaft Section Division of Z-direction high-speed ball screw pair.

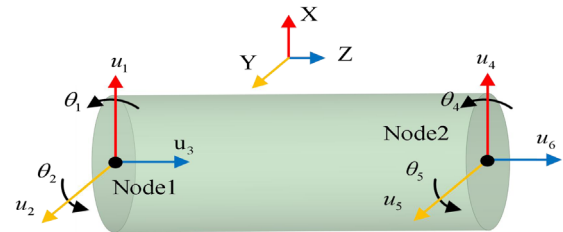


Fig. 3. Timoshenko beam element structure and degrees of freedom.

the axial direction of the ball screw axis. Each Timoshenko beam element contains nodes 1 and 2, which contain six translational degrees of freedom ($u_1, u_2, u_3, u_4, u_5, u_6$) along the x, y and z directions and four rotational degrees of freedom ($\theta_1, \theta_2, \theta_3, \theta_4$) around the x and y directions. In short, each beam element contains 10 degrees of freedom and the stiffness matrix and mass matrix of the beam element are given in reference [18].

In this paper, when dividing the Ball Screw shaft segment unit and node, the thread part of the Ball Screw is divided evenly due to the special structure of the thread. As

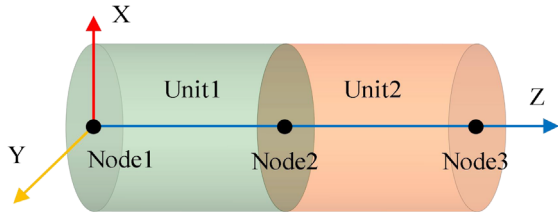


Fig. 4. Schematic diagram of two-element finite element shaft segment system.

shown in Figure 2, the effective travel of the Ball Screw L_1 is divided into m shaft segment units and $m + 1$ nodes. In this figure, L_1 is the support span of the ball screw; L_1 is the effective travel of the ball screw; b_m ($m = 1, 2, 3, \dots$) is the equally divided unit length of the effective travel around the ball screw; X is the location of the center of mass for the ball screw nut. The first segment b is called unit 1, the second segment b is called unit 2, ... Similarly, the m -axis segment b is the m -unit. The effective travel of the ball screw is then b_m ($m = 1, 2, 3, \dots$). In addition to the effective travel L of the ball screw, the other parts of the ball screw are divided according to the element division principle.

2.2 The stiffness matrix of two element shaft segment system

By dividing the shaft segments of the ball screw in the preliminary section, the stiffness matrix of each shaft segment can be obtained. Then, the beam element stiffness matrix of the ball screw is obtained by superposing each divided beam unit stiffness matrix in the order of the serial number. To explain the specific superposition process of the beam element stiffness matrix, two adjacent beam elements 1 and 2 in Section 2.1 are arbitrarily taken to form a finite element shaft system for explanation. As shown in Figure 4, the z direction is the axial direction of the high-speed ball screw. The No. 1 beam unit of the shaft segment system contains nodes 1 and 2, while the No. 2 beam unit contains nodes 2 and 3, and node 2 is a joint common to both beam units. The stiffness matrix expressions of two beam elements 1 and 2 can be taken as equations (1) and (2), respectively.

The stiffness matrices K_{e1} and K_{e2} of the two beam units are expanded to the same order as the stiffness matrix $K_{e'}$ of the axial segment system, and the elements of the stiffness matrices K_{e1} and K_{e2} are superimposed on the corresponding node locations to obtain the stiffness matrix $K_{e'}$ of the axial segment system, as shown in equation (3).

$$K_{e1} = \begin{bmatrix} k_{11}^1 & \dots & k_{15}^1 & k_{16}^1 & \dots & k_{110}^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{51}^1 & \dots & k_{55}^1 & k_{56}^1 & \dots & k_{510}^1 \\ k_{61}^1 & \dots & k_{65}^1 & k_{66}^1 & \dots & k_{610}^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{101}^1 & \dots & k_{105}^1 & k_{106}^1 & \dots & k_{1010}^1 \end{bmatrix}, \quad (1)$$

$$K_{e2} = \begin{bmatrix} k_{11}^2 & \dots & k_{15}^2 & k_{16}^2 & \dots & k_{110}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{51}^2 & \dots & k_{55}^2 & k_{56}^2 & \dots & k_{510}^2 \\ k_{61}^2 & \dots & k_{65}^2 & k_{66}^2 & \dots & k_{610}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{101}^2 & \dots & k_{105}^2 & k_{106}^2 & \dots & k_{1010}^2 \end{bmatrix}, \quad (2)$$

$$K_{e'} = \begin{bmatrix} k_{11}^1 & \dots & k_{15}^1 & k_{16}^1 & \dots & k_{110}^1 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{51}^1 & \dots & k_{55}^1 & k_{56}^1 & \dots & k_{510}^1 & 0 & \dots & 0 \\ k_{61}^1 & \dots & k_{65}^1 & k_{66}^1 + k_{11}^2 & \dots & k_{610}^1 + k_{15}^2 & k_{16}^2 & \dots & k_{110}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{101}^1 & \dots & k_{1010}^1 & k_{106}^1 + k_{51}^2 & \dots & k_{1010}^1 + k_{55}^2 & k_{56}^2 & \dots & k_{510}^2 \\ 0 & \dots & 0 & k_{61}^2 & \dots & k_{65}^2 & k_{66}^2 & \dots & k_{610}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & k_{101}^2 & \dots & k_{105}^2 & k_{106}^2 & \dots & k_{1010}^2 \end{bmatrix}, \quad (3)$$

where $k_{11}^1 \dots k_{1010}^1$, $k_{11}^2 \dots k_{1010}^2$ are the stiffness matrix parameters determined by the inherent characteristic of the ball screw. According to equation (3), the expanded unit stiffness matrix K_{e1} , K_{e2} can be considered as the composition of the two units to the overall structure of the shaft segment system. The contributions of each unit are superimposed to obtain the stiffness matrix of the shaft segment system $K_{e'}$, where $K_{e'}$ is a symmetric matrix of order 15×15 . By simplifying K , some of its element form $K_{e'5}$, as shown in equation (4). The elements corresponding to the matrix $K_{e'5}$ represent the composition of the common nodes 2 of two adjacent beam units 1 and 2 to the stiffness matrix of this axial segment system.

$$K_{e'5} = \begin{bmatrix} k_{66}^1 + k_{11}^2 & \dots & k_{610}^1 + k_{15}^2 \\ \vdots & \ddots & \vdots \\ k_{106}^1 + k_{51}^2 & \dots & k_{1010}^1 + k_{55}^2 \end{bmatrix}. \quad (4)$$

By describing the superposition process of the finite element axial segment system composed of two units 1 and 2 above, the stiffness matrix of the high-speed ball screw beam unit divided into n axial units and $n + 1$ nodes is derived.

2.3 Construction of screw nut interface unit stiffness matrix

To model the interface between the ball screw and the nut, the interface between the ball screw and the nut is constructed as a special nut interface unit stiffness matrix K_{SN} [19], as shown in equation (5). In this equation, in addition to the radial stiffness, axial stiffness and angular stiffness on the main diagonal, the cross-coupling terms that cause axial and radial displacement of the ball screw are included, which will affect the positioning accuracy of the table and the fatigue life of the high-speed ball screw [20].

$$K_{SN} = \begin{bmatrix} k_{x,x} & 0 & k_{xz} & k_{x,\theta x} & 0 & k_{x,\theta z} \\ 0 & k_{y,y} & 0 & 0 & k_{y,\theta y} & 0 \\ k_{x,z} & 0 & k_{zz} & k_{z,\theta x} & 0 & k_{z,\theta z} \\ k_{x,\theta x} & 0 & k_{z,\theta x} & k_{\theta x,\theta x} & 0 & k_{x,\theta z} \\ 0 & k_{y,\theta y} & 0 & 0 & k_{\theta y,\theta y} & 0 \\ k_{x,\theta z} & 0 & k_{z,\theta z} & k_{\theta x,\theta z} & 0 & k_{\theta z,\theta z} \end{bmatrix}, \quad (5)$$

where $k_{x,x}$, $k_{y,y}$ are the radial stiffness; $k_{z,z}$ is the axial stiffness; $k_{\theta x,\theta x}$, $k_{\theta y,\theta y}$, $k_{\theta z,\theta z}$ is the angular stiffness; $k_{x,z}$, $k_{x,\theta x}$, $k_{x,\theta z}$, $k_{z,\theta x}$, $k_{z,\theta z}$, $k_{\theta x,\theta z}$ are the cross-coupling terms of axial and radial displacements.

In this paper, the matrix K_{SN} of equation (2.17) is simplified to make it easier to couple the stiffness matrix of the screw-nut interface unit with the overall stiffness matrix of the high-speed ball screw in the form of ignoring the angular displacement z in the axial direction of the high-speed ball screw, thus obtaining an 5×5 screw-nut interface unit stiffness matrix K_{SN} , see equation (6). Since the cross-coupling term in the screw-nut interface unit stiffness matrix K_{SN} has taken into account the effect of the deformation of the ball screw, when the center of gravity of the screw nut is at any unit position of the effective stroke of the ball screw, then the Timoshenko beam unit stiffness matrix at any unit position is considered to be the screw-nut interface unit stiffness matrix K_{SN} .

$$K_{sn} = \begin{bmatrix} k_{x,x} & 0 & k_{xz} & k_{x,\theta x} & 0 \\ 0 & k_{y,y} & 0 & 0 & k_{y,\theta y} \\ k_{x,z} & 0 & k_{zz} & 0 & 0 \\ k_{x,\theta x} & 0 & k_{z,\theta x} & k_{\theta x,\theta x} & 0 \\ 0 & k_{y,\theta y} & 0 & 0 & k_{\theta y,\theta y} \end{bmatrix}. \quad (6)$$

2.4 Dynamic stiffness calculation of high-speed ball screw

After the previous modelling, the dynamic stiffness solution of the Ball Screw is presented below. The dynamic stiffness solution of the high-speed ball screw is mainly reflected in the change in the stiffness of the high-speed ball screw due to the arbitrary position of the screw nut during the feed motion of the effective travel. The arbitrary position of the nut causes a change in the stiffness of the high-speed ball screw. Therefore, the solution to the dynamic stiffness of the Ball Screw is determined by the position X of the screw nut. When the screw nut makes a feed motion along the ball screw, the interface between the ball screw and the screw nut is calculated using the stiffness matrix K_{SN} . By using the superposition method in Section 1.2, the expression form of the stiffness matrix K_e of the effective travel L_1 segment of the high-speed ball screw in Section 1.1 can be obtained, which is divided into m shaft segment elements and $m+1$ nodes.

The stiffness matrix K_e of the L_1 segment of the effective stroke of the high-speed ball screw is a square matrix of order $5(m+1) \times (m+1)$, and the square cell where any unit is located is regarded as the corresponding 10th-order beam unit stiffness matrix of that unit, and any unit contains two nodes, and the shaded part indicates the

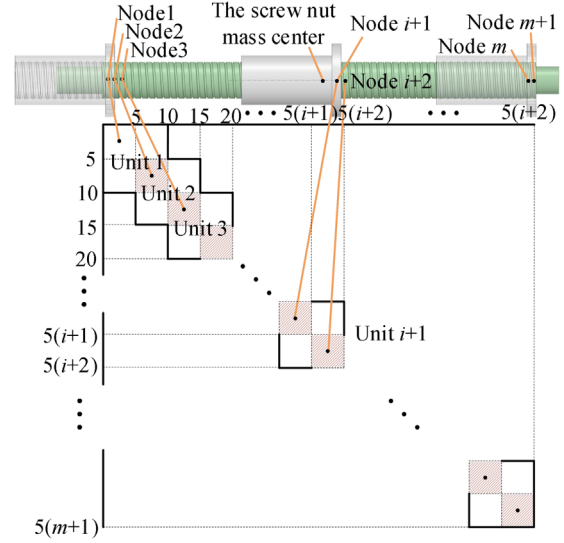


Fig. 5. The stiffness matrix K_e of the effective travel L_1 part for the high-speed ball screw.

common node of the adjacent units, the m axial segment units are sequentially superimposed and integrated to produce the effective stroke of the high-speed ball screw. The contribution of the L_1 segment.

X is defined as the location of the mass center of screw nut along the ball screw movement, as shown in Figure 5. When the screw nut mass center is located in the first $1/2$ segment of the unit axis segment, X can be expressed as equation (7).

$$X \leq (2i+1) \times b/2, \quad (7)$$

where $i=0, 1, 2, \dots, n-1$ and i is the smallest integer. The 5th order beam stiffness matrix at node $i+1$ in Figure 5 should be replaced by the unit stiffness matrix K_{SN} of the nut interface. Conversely, if the mass center of screw nut is located in the second $1/2$ of the arbitrarily unit shaft segment, X can be expressed as in equation (8)

$$X > (2i+1) \times b/2, \quad (8)$$

where $i=0, 1, 2, \dots, n-1$, where i is the largest integer. The fifth order beam stiffness matrix at node $i+2$ in Figure 5 should be replaced by the stiffness matrix K_{SN} of the nut interface assembly. By introducing the boundary condition of the nut center of mass position X , the nut interface unit stiffness matrix can be coupled to the constitutive stiffness matrix of the high-speed ball screw beam unit, which provides theoretical support for solving the consideration of the dynamic stiffness of the high-speed ball screw as a function of the nut center of mass.

3 Simulation calculation

After establishing the dynamic stiffness model of the high-speed ball screw, this paper verifies the accuracy of the model through experiments. The flow chart for calculating the dynamic stiffness K of a high-speed ball screw is shown

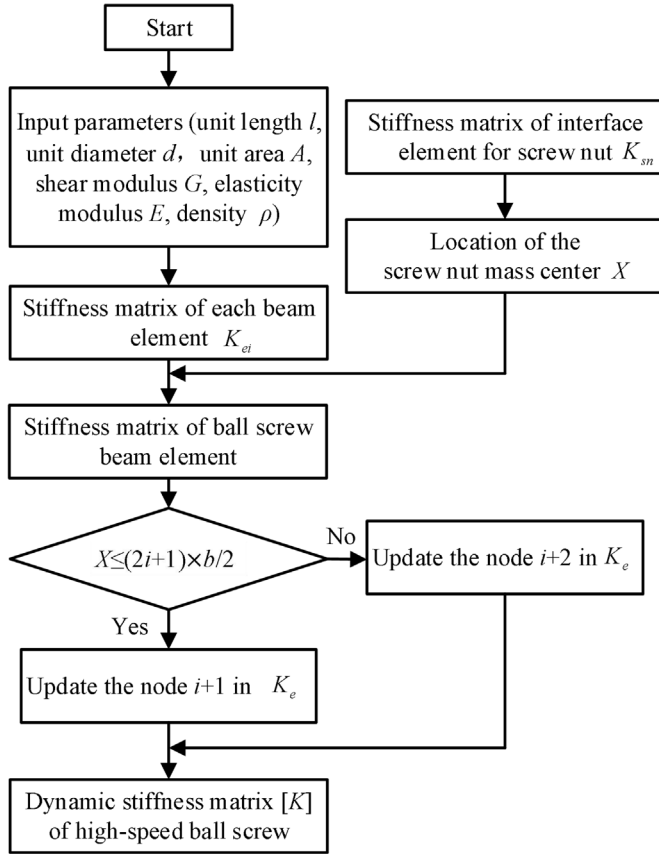


Fig. 6. Calculation flow chart of dynamic stiffness for high-speed ball screw.

in Figure 6. In this flowchart, the stiffness matrix of each beam unit in this paper is constructed, and then the obtained stiffness matrix of each beam unit is superimposed to form the stiffness matrix of the ball screw beam unit. Through the arbitrariness of the screw-nut center of mass position X , the screw-nut interface unit stiffness matrix is coupled to the appropriate position to complete the calculation of the dynamic stiffness matrix K of the ball screw. The main parameters of ball screw are shown in Table 1.

The accuracy of the dynamic stiffness model is verified by comparing the calculated and experimental analysis of the eigen frequencies of the Ball Screw in the free state. The differential equation of motion of the ball screw can be simplified to an undamped free vibration equation by solving the inherent frequency of each order of the ball screw. The differential equation of motion of the ball screw can be simplified to the undamped free vibration equation, which can be expressed as

$$[M]x(t) + [K]x(t) = 0, \quad (9)$$

where $[K]$ is the dynamic stiffness matrix of the high-speed ball screw; $x(t)$ is the acceleration vector; $x(t)$ is the displacement vector; $[M]$ is the mass matrix of the high-speed ball screw formed by superposing the mass matrix $[M_{ei}]$ of each beam element. And $[M_{ei}]$ can be expressed as [14]

Table 1. Main parameters of ball screw.

The parameter	Value
Young's modulus E (MPa)	2,100,000
Ball screw nominal lead p_h (mm)	10
Ball screw nominal diameter d_0 (mm)	25
Density ρ (kg/m ³)	7850
Poisson's ratio μ	0.3
Screw length l_0 (mm)	600
Outer diameter of screw nut D_1 (mm)	40

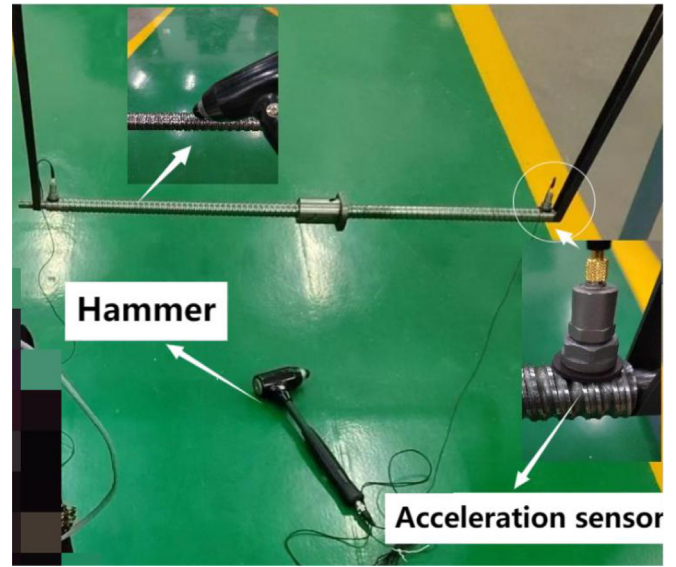


Fig. 7. Construction of the experimental platform.

$$[M_{ei}] = [M_{tei}] + [M_{sei}] + [M_{cei}], \quad (10)$$

where $[M_{tei}]$ is mass matrix; $[M_{sei}]$ is the stiffness matrix of condensation clustering additional parts. $[M_{cei}]$ is a mass matrix treated as a centralized mass.

The ball screw can be divided into 32 units and 33 nodes by dividing and numbering the shaft segments. The stiffness matrix and mass matrix of each beam element are superimposed, and the stiffness matrix and mass matrix of the ball screw beam element are obtained as matrices of 165th order. Then, when the centroid of the nut is located at the centroid of the effective travel of the ball screw, the stiffness matrix of the nut interface element is updated to the 17th node of the stiffness matrix of the ball screw beam element by calculation. The calculated natural frequencies of each order for the ball screw in the free state are shown in Table 2.

4 Experimental verification

In order to obtain the intrinsic frequency of the screw, experiments were carried out in this paper using an LMS (vibration noise test system) and the intrinsic frequency of

Table 2. Comparison of simulation and experimental result.

Order numb error	The experimental data/Hz	Calculate data/Hz	Error 1
1	276.833	317.552	14.71%
3	725.291	812.395	12.01%
5	1482.096	1658.275	11.89%

Table 3. Comparison of simulation and experimental result of reference [21].

Order numb error	The experimental data /Hz	Calculate data /Hz	Error 2
1	213.46	180.46	15.50%
3	538.56	618.36	14.80%
5	1208.20	1368.7	13.30%

the system was obtained experimentally and compared with the simulation results. As shown in Figure 7, an acceleration sensor (YMC121A100-IEPE) was fixed to both ends of the screw and connected to the LMS system. The screw is suspended by two belt ropes and eight hammering points are evenly selected over the full length of the screw, hammering excitation is applied by means of a force hammer (YMC1H-10PE, sensitivity 10.08 mV/N), which and is connected to the LMS system and finally, as shown in Figure 7, the data is collected by the LMS system.

The results of the comparative analysis of the modal experiments and numerical simulations in this paper are shown in Table 2. The results of the analysis of the classical method (numerical comparison of LMS modal experiments and finite element simulations) in the literature [21] for the derived frequency values are shown in Table 3. Since adjacent modes are the same, only the first, third and fifth order modes are extracted here.

The error of the simulated value of the experimental value measured in this paper is about 12.9%, and it can be assumed that the simulated and experimental data are basically correct. The main reason for the error is that during the calculation process, the nut, which is treated according to the concentrated mass, constantly changes its working position, and the center of mass of the nut is assigned to only one node of the split screw shaft segment unit, which causes the calculated value to be slightly larger than the experimentally measured value. In addition, neglecting the effect of the screw shaft thread also contributes to the error. When comparing the data in the literature [21] with the experimental LMS modal measurements, the average error between the two sets of frequency values is approximately 14.5%. The comparison shows that the computational method used in this paper is closer to the experimental values than the classical finite element method in the literature [21], which shows that the simulation algorithm in this paper is more in line with the actual working conditions.

The dynamic stiffness matrix of the assembled ball screw is considered reasonable by analysing the inherent frequency error of the ball screw in the free state. Since the

operating speed of the five-axis machining center is 24,000 rpm, which corresponds to a frequency of 400 Hz [21], all the inherent frequencies of the ball screw calculated in Tables 2 and 3 are outside this frequency range. The natural frequency of the ball screw pair is not the same as the operating frequency, so resonance does not occur during operation. It can be assumed that the dynamic stiffness analysis model of the high-speed ball screw established in this paper is reasonable and feasible with the change of the mass center position of the screw nut.

5 Conclusion

- This paper establishes a dynamic stiffness analysis model for a high-speed ball screw pair based on Timoshenko beam theory and the concentrated mass parameter method, which is characterised by the changing position of the center of mass of the screw nut.
- Under the condition that the screw nut position changes at any time, the stiffness matrix of the ball screw beam unit and the stiffness matrix of the lead screw nut interface unit are coupled. The first three orders of natural frequencies of the ball screw pair in the free state are obtained by experiment and simulation. The feasibility of the dynamic stiffness matrix is verified, providing a theoretical basis for the dynamic design and structural optimisation of high-speed ball screws.

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