

# A backstepping sliding mode control algorithm of electro-hydraulic position servo system of valve-controlled symmetric cylinder based on extended state observer

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**Abstract.** Electro-hydraulic position servo system of valve-controlled symmetrical cylinder (EPSSVCS) is a very important and widely used electro-hydraulic servo system, the performance of which is often affected by friction nonlinearity and external random disturbances together with unmodeled dynamic factors such as parameter uncertainty in practical work. To improve the tracking performance of the system, a backstepping sliding mode control algorithm (BSMC) based on an extended state observer was presented in this paper. Firstly, a nonlinear mathematical model of the EPSSVCS was established which takes into account the composite disturbance induced by the systematic friction nonlinearity, external disturbances and unmodeled dynamic factors. Then, an extended state observer (ESO) was designed which can effectively estimate the velocity, acceleration, and composite disturbance of the valve-controlled cylinder online. Furthermore, a kind of BSMC was presented based on the online ESO estimates and the displacement feedback signals. The control law was given and the system stability was proved. The analysis results showed that the output signal of the system can effectively track the input signal under the influence of typical external disturbances and unmodeled dynamic factors, which illustrated the effectiveness of the established nonlinear mathematical model and the stability of the designed control system. Moreover, by taking the parameters of the electro-hydraulic servo system of a certain type of CNC machine tool as a real example, the proposed algorithm (BSMC) was compared with the conventional PID control algorithm, the backstepping sliding mode control algorithm (BSC), and the adaptive robust control algorithm (ARC), and the results verified the superiority of the BSMC. This work may provide a useful reference for the research in the field of related control systems.

**Keywords:** Electro-hydraulic position servo system / valve-controlled symmetrical cylinder / extended state observer (ESO) / backstepping sliding mode control algorithm (BSMC) / composite disturbance

## 1 Introduction

Electro-hydraulic servo system has the characteristics of strong bearing capacity, fast response speed and high control accuracy [1,2], which are widely used in various fields such as numerical control systems [3,4], vehicle hydraulic turbines [5], robots [6–8] and automobile suspensions [9]. EPSSVCS is an important type of electro-hydraulic servo system. Designing a high-quality controller is crucial, as it directly determines the rapidity and accuracy of the output signal when tracking the expected signal and hence determining the actual performance of the control system.

In recent years, many scholars have conducted a series of research on this issue by means of linear or nonlinear modeling. In the study of linear modeling, people generally viewed the system friction as purely viscous friction, ignoring the friction nonlinearity and the external random disturbances together with unmodeled dynamic factors such as parameter uncertainty. For example, Kunshan et al. [10] designed a backstepping controller based on the construction of an online linearization model in which the uncertain factors that exist in real system were not considered. To improve the tracking performance of the valve-controlled symmetrical cylinder electro-hydraulic position servo system, it is necessary to establish a nonlinear mathematical model of the electro-hydraulic servo system, laying the foundation for the design of high-performance controllers for the system. Huang et al. [11], Liu et al. [12] and Wen et al. [13] all studied a fuzzy control algorithm by constructing fuzzy disturbance observer,

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optimizing parameters of the system, using fuzzy disturbance observer to estimate the value of the system and designed a controller combined with a backstepping sliding mode control algorithm. Yang et al. [14] established a mathematical model of nonlinear system, optimizing the data collected of the system by using a backstepping control algorithm (BSC), however, it was more idealistic about the form of system data collection and ignored the unmodeled dynamic characteristics. Yao et al. [15] proposed a wavelet neural network system to track the output signal of valve-controlled cylinder, but the neural network system model constructed was relatively complex and had a slow convergence rate. Wang et al. [16] adopted an adaptive fuzzy PID control algorithm to control the output signal of the valve control cylinder, and the adaptive fuzzy PID controller could dynamically adjust parameters in a timely manner, although the uncertainties of the system were considered, accurate control of the system required a large amount of data and experience, otherwise the control accuracy would be affected. Yue et al. [17] and Hu et al. [18] both proposed an adaptive robust sliding mode control algorithm which combined adaptive robust control with a friction model, reduced the impact of external disturbances and system friction, improved the control accuracy and dynamic tracking performance of the electro-hydraulic position servo system, although these algorithms could suppress model parameter uncertainties and external disturbances, robust controllers had a high order. Sun et al. [19], Jing et al. [20] and Luo et al. [21] all studied an extended state adaptive observer, considered the influence of nonlinear friction factors on the dynamic and static performance of the system, reduced the error of tracking the desired signal by the output signal of the valve-controlled cylinder. Zhang et al. [22] and Du et al. [23] both proposed an adaptive sliding mode control algorithm which shortened the time to reach the sliding mode surface and reduced the chattering problem of sliding mode variable structure, by introducing exponential reaching law and power reaching law, however, it was not sensitive to the disturbance of the system and parameter changes, and needed the upper bound of the system uncertainty factors, otherwise, the system was prone to buffeting. Overall, the existing research on nonlinear models is still being deepened in order to continuously improve the control performance of electro-hydraulic servo systems.

In this paper, a new control algorithm is proposed. Firstly, a nonlinear mathematical model of the EPSSVCS is established in which the composite disturbance produced by the friction nonlinearity and external random disturbances together with unmodeled dynamic factors are embodied. Then, an ESO is designed to effectively estimate the velocity, acceleration, and the composite disturbance of the valve-controlled cylinder online. Furthermore, a kind of BSMC is presented based on the online ESO estimates and the displacement feedback signals and the control law is given. The stability proof is also provided for the control system. To verify the rationality of the nonlinear model and the algorithm, the influences of several typical external disturbances and unmodeled dynamic factors are investigated and the signal tracking ability of the system is

illustrated. Moreover, the proposed algorithm (BSMC) is further compared with three kinds of existing algorithms, namely, the conventional PID control algorithm, the backstepping sliding mode control algorithm (BSC), and the adaptive robust control algorithm (ARC). The comparative results can verify the superiority of the BSMC, hence providing a beneficial reference for related researches of control systems.

The paper is organized as follows. The nonlinear mathematical model of the system is given in Section 2. The ESO and BSMC are presented in Sections 3.1 and 3.2 respectively. Section 4 gives the proof of the system stability. Section 5 conducts the analysis of several examples and Section 6 concludes the work.

## 2 Nonlinear mathematical model of the system

The structural diagram of the considered EPSSVCS is shown in Figure 1.

The expected signal is input to the controller, which acts on the electro-hydraulic servo valve through the servo amplifier. The hydraulic cylinder generates output signal of position under the action of electro-hydraulic servo valve and this signal is fed back to the controller by using the position sensor.

According to Newton's second law, the dynamic equation of the inertial load can be obtained as

$$m\ddot{y} = (p_1 - p_2)A - B\dot{y} - A_f S_f - f, \quad (1)$$

where  $m$  and  $y$  are the load mass and displacement respectively,  $p_1$  and  $p_2$  represent the working pressure of the left and right cavities respectively,  $A$  is the effective area of the two cavities,  $B$  is the viscous friction coefficient,  $A_f$  represents the Coulomb friction amplitude,  $S_f$  is the shape function of the Coulomb friction,  $f$  represents the external disturbance and unmodeled dynamics of the system.

The dynamic equation for the pressure of the two chambers of the valve controlled symmetrical cylinder is:

$$\begin{cases} \dot{p}_1 = \frac{\beta_e}{V_1} (-A\dot{y} - C_\tau(p_1 - p_2) + Q_1) \\ \dot{p}_2 = \frac{\beta_e}{V_2} (-A\dot{y} - C_\tau(p_1 - p_2) - Q_2) \end{cases}, \quad (2)$$

where  $\beta_e$  is the oil bulk modulus,  $V_1 = V_{01} + Ay$  and  $V_2 = V_{02} - Ay$  represent the effective volumes of the two cavities respectively,  $V_{01}$  and  $V_{02}$  denote the initial volumes of the two cavities respectively, and  $C_\tau$  represents the internal leakage coefficient,  $Q_1$  and  $Q_2$  represent the inlet and return oil flow rates of the hydraulic cylinder, respectively.

Assuming that the frequency bandwidth of the electro-hydraulic position servo system is much smaller than that of the electro-hydraulic servo valve, and the electro-hydraulic servo valve is a proportional control link, the flow equation of the electro-hydraulic servo valve can be given as

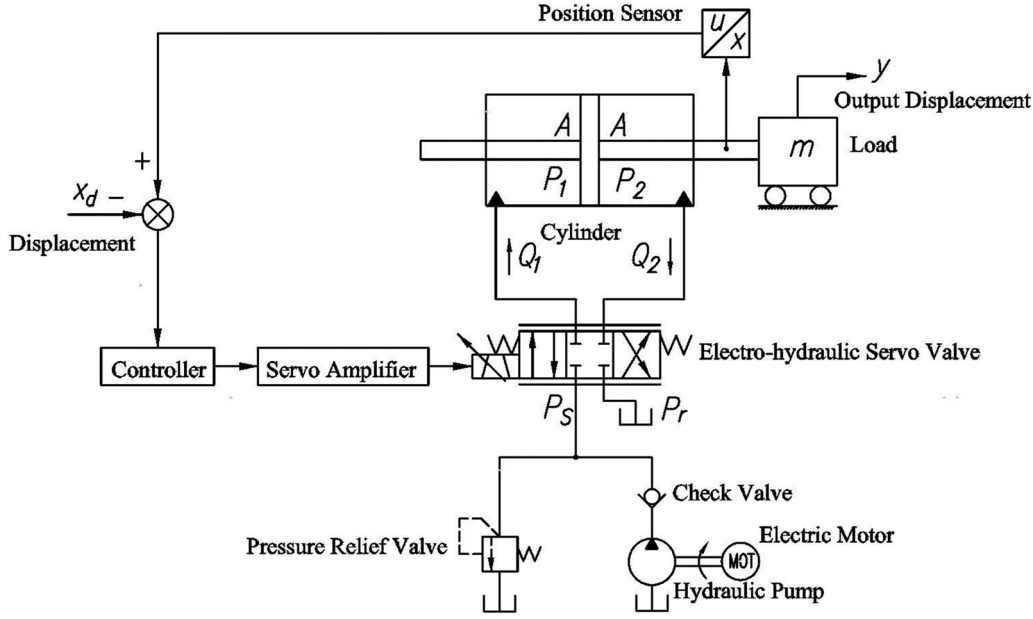


Fig. 1. The structural diagram of the considered EPSSVCS.

$$\begin{cases} Q_1 = \sqrt{\frac{2}{\rho}} C_d K_i \omega u [s(u) \sqrt{p_s - p_1} + s(-u) \sqrt{p_1 - p_r}] \\ Q_2 = \sqrt{\frac{2}{\rho}} C_d K_i \omega u [s(u) \sqrt{p_2 - p_r} + s(-u) \sqrt{p_s - p_2}] \end{cases}, \quad (3)$$

or

$$\ddot{y} = \frac{1}{m} \left\{ \left( -\frac{\beta_e A^2}{V_1} - \frac{\beta_e A^2}{V_2} \right) \dot{y} + \left( -\frac{\beta_e A}{V_1} - \frac{\beta_e A}{V_2} \right) (p_1 - p_2) C_\tau + \frac{\beta_e A}{V_1} g R_{11} u + \frac{\beta_e A}{V_2} g R_{22} u - B \ddot{y} - A_f \dot{S}_f - \dot{f} \right\}. \quad (7)$$

where  $\rho$  is the hydraulic oil density,  $C_d$  is the discharge coefficient for the orifice of servo valve,  $k_i$  is the current gain of the servo valve spool,  $\omega$  is the orifice area gradient,  $u$  is the control output,  $p_s$  is the pressure source of the system and  $p_r$  represents the return oil pressure.

Defining function  $s(u)$  as:

$$s(u) = \begin{cases} 1, & u \geq 0 \\ 0, & u < 0 \end{cases}, \quad (4)$$

and setting

$$g = \sqrt{\frac{2}{\rho}} C_d \omega K_i, R_{11} = s(u) \sqrt{p_s - p_1} + s(-u) \sqrt{p_1 - p_r}, R_{22} = s(u) \sqrt{p_2 - p_r} + s(-u) \sqrt{p_s - p_2}$$

then equation (3) can be simplified as

$$\begin{cases} Q_1 = g R_{11} u \\ Q_2 = g R_{22} u \end{cases}. \quad (5)$$

To facilitate the analysis of the tracking accuracy of the output signal, the dynamic equation of the inertial load, the pressure dynamic equation of the valve-controlled cylinder and the flow equation of the electro-hydraulic servo valve is combined to establish a third order differential equation that can describe the system in the following form

$$m \ddot{y} = (\dot{p}_1 - \dot{p}_2) A - B \ddot{y} - A_f \dot{S}_f - \dot{f}, \quad (6)$$

### 3 Controller design

The block diagram of the proposed BSMC based on ESO is shown in Figure 2. The controller design includes two parts: (1) Building an ESO and (2) Designing a BSMC based on the ESO.

#### 3.1 ESO design

Setting the state variable as  $x = [x_1, x_2, x_3]^T = [y, \dot{y}, \ddot{y}]^T$ , where  $x_1, x_2, x_3$  represent the load displacement, speed and acceleration, respectively, the state space equation of the system can be written as follows according to equation (7):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = R_1 x_2 + R_2 x_3 + R_3 + \zeta \end{cases}, \quad (8)$$

where the term  $\zeta = \frac{1}{m} \left( \left( -\frac{\beta_e A}{V_1} - \frac{\beta_e A}{V_2} \right) C_\tau (p_1 - p_2) - A_f \dot{S}_f - \dot{f} \right)$  represents the composite disturbance brought by the friction nonlinearity, external disturbances and unmodeled dynamic factors, and  $R_1 = \frac{1}{m} \left( -\frac{\beta_e A^2}{V_1} - \frac{\beta_e A^2}{V_2} \right)$ ,  $R_2 = -\frac{B}{m}$ ,  $R_3 = \frac{1}{m} \left( \frac{\beta_e A}{V_1} g R_{11} + \frac{\beta_e A}{V_2} g R_{22} \right)$ .

Introducing another state variable  $x_4$  as the extended state of  $\zeta$ , then the state variable  $x = [x_1, x_2, x_3]^T$  can be extended as  $x = [x_1, x_2, x_3, x_4]^T$  and equation (8) can be expanded as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = R_1 x_2 + R_2 x_3 + R_3 u + x_4 \\ \dot{x}_4 = d(t) \end{cases}, \quad (9)$$

where  $d(t)$  represents a bounded uncertain function.

Assuming that  $\hat{x}_i (i = 1, 2, 3, 4)$  is the estimated value of the ESO for state  $x_i$ , the online estimated value can be obtained as:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + 4\omega_0(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + 6\omega_0^2(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_3 = R_1 \hat{x}_2 + R_2 \hat{x}_3 + R_3 u + \hat{x}_4 + 4\omega_0^3(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_4 = \omega_0^4(x_1 - \hat{x}_1) \end{cases}, \quad (10)$$

where  $\omega_0$  is the bandwidth of ESO [24].

Therefore, the online estimation error  $\tilde{x}_i = x_i - \hat{x}_i$  of the ESO can be expressed as

$$\begin{cases} \dot{\tilde{x}}_1 = -4\omega_0 \tilde{x}_1 + \tilde{x}_2 \\ \dot{\tilde{x}}_2 = -6\omega_0^2 \tilde{x}_1 + \tilde{x}_3 \\ \dot{\tilde{x}}_3 = -4\omega_0^3 \tilde{x}_1 + R_1 \tilde{x}_2 + R_2 \tilde{x}_3 + \tilde{x}_4 \\ \dot{\tilde{x}}_4 = -\omega_0^4 \tilde{x}_1 + d(t) \end{cases}, \quad (11)$$

which can also be described in matrix form, i.e.,  $\dot{\tilde{x}} = A\tilde{x} + D$ ,

$$\text{and } A = \begin{bmatrix} -4\omega_0 & 1 & 0 & 0 \\ -6\omega_0^2 & 0 & 1 & 0 \\ -4\omega_0^3 & R_1 & R_2 & 1 \\ -\omega_0^4 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d(t) \end{bmatrix}$$

The constructed ESO (Eq. (11)) can then be used to perform online observation and estimation of load speed, acceleration and composite disturbance.

### 3.2 BSMC based on ESO

Based on the ESO described above, a BSMC can be further designed by using the Lyapunov function and the control law can be obtained. The detailed derivation process is as follows where  $k_i > 0 (i = 1, 2, 3)$ ,  $c_1 > 0$ ,  $c_2 > 0$  are control parameters.

**Step 1:** Setting  $x_{1d}$  as the displacement tracking value of  $x_1$ , the displacement tracking error is  $e_1 = x_1 - x_{1d}$ . We introduce the Lyapunov function of the primary system as  $V_1 = e_1^2/2$ . Obviously, the system is stable only when  $\dot{V}_1 < 0$ . If setting  $\dot{e}_1 = -k_1 e_1$ , then  $\dot{V}_1 = -k_1 e_1^2 < 0 (k_1 > 0)$ , meaning that the system is stable. We have the relation of  $x_2 = -k_1 e_1 + \dot{x}_{1d}$  since  $x_2 - \dot{x}_{1d} = -k_1 e_1$ , hence the value of  $x_2$  is  $-k_1 e_1 + \dot{x}_{1d}$ . Further, setting  $x_{2d}$  as the displacement tracking value of  $x_2$ , the displacement tracking error is  $e_2 = x_2 - x_{2d}$  from

which it can be found that:

$$\begin{aligned} \dot{V}_1 &= e_1 \dot{e}_1 \\ &= -k_1 e_1^2 + e_1 e_2. \end{aligned} \quad (12)$$

**Step 2:** To ensure that  $e_1$  and  $e_2$  in equation (12) tend to 0, a Lyapunov function of the secondary system is introduced which should include  $e_1$  and  $e_2$ . The Lyapunov function is set as

$$V_2 = V_1 + \frac{1}{2} e_2^2. \quad (13)$$

From equations (12) and (13), it can be obtained that

$$\dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 = -k_1 e_1^2 + e_2(e_1 + x_3 - \dot{x}_{2d}). \quad (14)$$

If this secondary system is to be stable, then  $\dot{V}_2 < 0$  should be satisfied. Since the primary system is already stable, it is only necessary to ensure  $e_1 + x_3 - \dot{x}_{2d} = -k_2 e_2 (k_2 > 0)$  to keep the system stable. It can be seen that the tracking value of  $x_3$  is  $-k_2 e_2 + \dot{x}_{2d} - e_1$ . Further, by setting  $x_{3d}$  as the displacement tracking value of  $x_3$ , the displacement tracking error is  $e_3 = x_3 - x_{3d}$ . Since  $x_{2d} = -k_1 e_1 + \dot{x}_{1d}$ , we have  $\dot{x}_{2d} = -k_1 \dot{e}_1 + \ddot{x}_{1d} = -k_1(x_2 - \dot{x}_{1d}) + \ddot{x}_{1d}$  in equation (14).

In the design of ESO, the variables  $\hat{x}_i (i = 1, 2, 3, 4)$  have been defined as the estimated values of state  $x_i (i = 1, 2, 3, 4)$ , then  $\tilde{x}_i = x_i - \hat{x}_i$  represents the error online estimated by the ESO. Hence, the following relationship can be obtained:

$$\begin{cases} \hat{x}_{2d} = -k_1(\hat{x}_2 - \dot{\hat{x}}_{1d}) + \ddot{\hat{x}}_{1d} \\ \hat{e}_2 = \hat{x}_2 - x_{2d} \\ x_{3d} = -k_2 \hat{e}_2 + \hat{x}_{2d} - e_1 \dot{x}_{2d} - \hat{x}_{2d} = -k_1 \tilde{x}_2 \\ e_2 - \hat{e}_2 = \tilde{x}_2 \end{cases}. \quad (15)$$

From equation (15), it can be acquired that:

$$\begin{aligned} x_3 &= e_3 + x_{3d} \\ &= e_3 - k_2 e_2 - e_1 + \dot{x}_{2d} + (k_1 + k_2) \tilde{x}_2 \end{aligned} \quad (16)$$

and the following relation can be obtained according to equations (14)–(16):

$$\begin{aligned} \dot{V}_2 &= -k_1 e_1^2 + e_2(e_1 + x_3 - \dot{x}_{2d}) \\ &= -k_1 e_1^2 + e_2(e_3 - k_2 e_2 + (k_1 + k_2) \tilde{x}_2), \end{aligned} \quad (17)$$

**Step 3:** Defining a sliding mode function and deriving the control law of the BSMC based on ESO.

Let the sliding mode function be:

$$s = c_1 e_1 + c_2 e_2 + e_3, \quad (18)$$

From equations (9) and (18), it is not difficult to found that:

$$\begin{aligned} \dot{s} &= c_1 \dot{e}_1 + c_2 \dot{e}_2 + \dot{e}_3 \\ &= c_1 \dot{e}_1 + c_2 \dot{e}_2 + R_1 x_2 + R_2 x_3 + R_3 u + x_4 - \dot{x}_{3d}. \end{aligned} \quad (19)$$

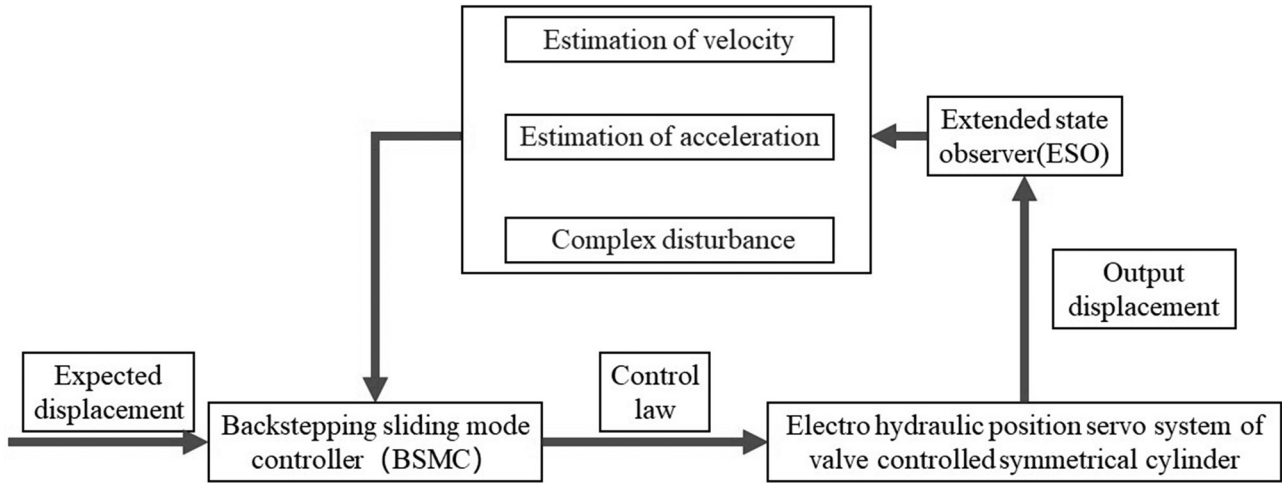


Fig. 2. Block diagram of the BSMC based on ESO.

From the first and second steps, it can be seen that

$$\begin{aligned}\dot{x}_{3d} &= -k_2(\hat{x}_2 - \dot{x}_{2d}) - k_1(\hat{x}_2 - \ddot{x}_{1d}) + \ddot{x}_{1d} - (\dot{x}_1 - \dot{x}_{1d}) \\ &= -k_2(\hat{x}_2 + k_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}) \\ &\quad - k_1(\hat{x}_2 - \ddot{x}_{1d}) + \ddot{x}_{1d} - (x_2 - \dot{x}_{1d}).\end{aligned}\quad (20)$$

Setting  $\hat{x}_3 - x_{3d} = \hat{e}_3$ , equations (18) and (20) can be transformed as:

$$\begin{cases} \hat{s} = c_1 e_1 + c_2 \hat{e}_2 + \hat{e}_3 \\ \hat{x}_{3d} = -k_2(\hat{x}_2 + k_1(\hat{x}_2 - \dot{x}_{1d}) - \ddot{x}_{1d}) \\ -k_1(\hat{x}_2 - \ddot{x}_{1d}) + \ddot{x}_{1d} - (\hat{x}_2 - \dot{x}_{1d}) \end{cases} \quad (21)$$

According to equations (19)–(21), the control law  $u$  of the BSMC based on ESO is acquired as following where  $\dot{s} = -k_3 \hat{s}$  is used:

$$\begin{aligned}u &= -\frac{1}{R_3}(c_1 \dot{e}_1 + c_2 \dot{e}_2 + R_1 x_2 + R_2 x_3 + x_4 - \dot{x}_{3d} + k_3 \hat{s}) \\ &= -\frac{1}{R_3} \left( c_1(\hat{x}_2 - \dot{x}_{1d}) + c_2(\hat{x}_3 + k_1(\hat{x}_2 - \dot{x}_{1d}) - \ddot{x}_{1d}) \right. \\ &\quad \left. + R_1 \hat{x}_2 + R_2 \hat{x}_3 + \hat{x}_4 - \hat{x}_{3d} + k_3 \hat{s} \right).\end{aligned}\quad (22)$$

## 4 Stability proof of the system

Defining  $e_i - \hat{e}_i = x_i - \hat{x}_i = \tilde{x}_i (i = 2, 3)$ , following equation can be obtained according to equations (18)–(21):

$$s - \hat{s} = c_2 \tilde{x}_2 + \tilde{x}_3. \quad (23)$$

According to equations (17), (22) and (23), it can be found that:

$$\begin{aligned}\dot{s} &= (c_1 + k_1 c_2 + k_3 c_2 + R_1 - k_1 k_2 - 1) \tilde{x}_2 \\ &\quad + (c_2 + k_3 + R_2) \tilde{x}_3 + \tilde{x}_4 - k_3 s.\end{aligned}\quad (24)$$

Setting the Lyapunov function as

$$V = V_2 + \frac{1}{2} s^2 + \tilde{x}^T P \tilde{x}, \quad (25)$$

and taking the derivative of  $V$ , and then substituting equations (17), (18) and (24) into equation (25), we have

$$\begin{aligned}\dot{V} &= \dot{V}_2 + s \dot{s} + \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} \\ &= -k_1 e_1^2 - (k_2 + c_2) e_2^2 - k_3 s^2 - c_1 e_1 e_2 + (k_1 + k_2) e_2 \tilde{x}_2 \\ &\quad + s e_2 + s[(c_1 + k_1 c_2 + k_3 c_2 + R_1 - k_1 k_2 - 1) \tilde{x}_2 \\ &\quad + (c_2 + k_3 + R_2) \tilde{x}_3 + \tilde{x}_4] - \tilde{x}^T Q \tilde{x},\end{aligned}\quad (26)$$

where  $A^T P + P A = -Q$  and  $P$  is a real symmetric positive definite matrix.

By setting  $|\tilde{x}_i| \leq \alpha$ , we have

$$\begin{aligned}\dot{V} &\leq -k_1 e_1^2 - (k_2 + c_2) e_2^2 - k_3 s^2 - c_1 |e_1| |e_2| \\ &\quad + (k_1 + k_2) \alpha e_2 + s e_2 + \beta s - \|Q\|_F \|\tilde{x}\|_2^2,\end{aligned}\quad (27)$$

where

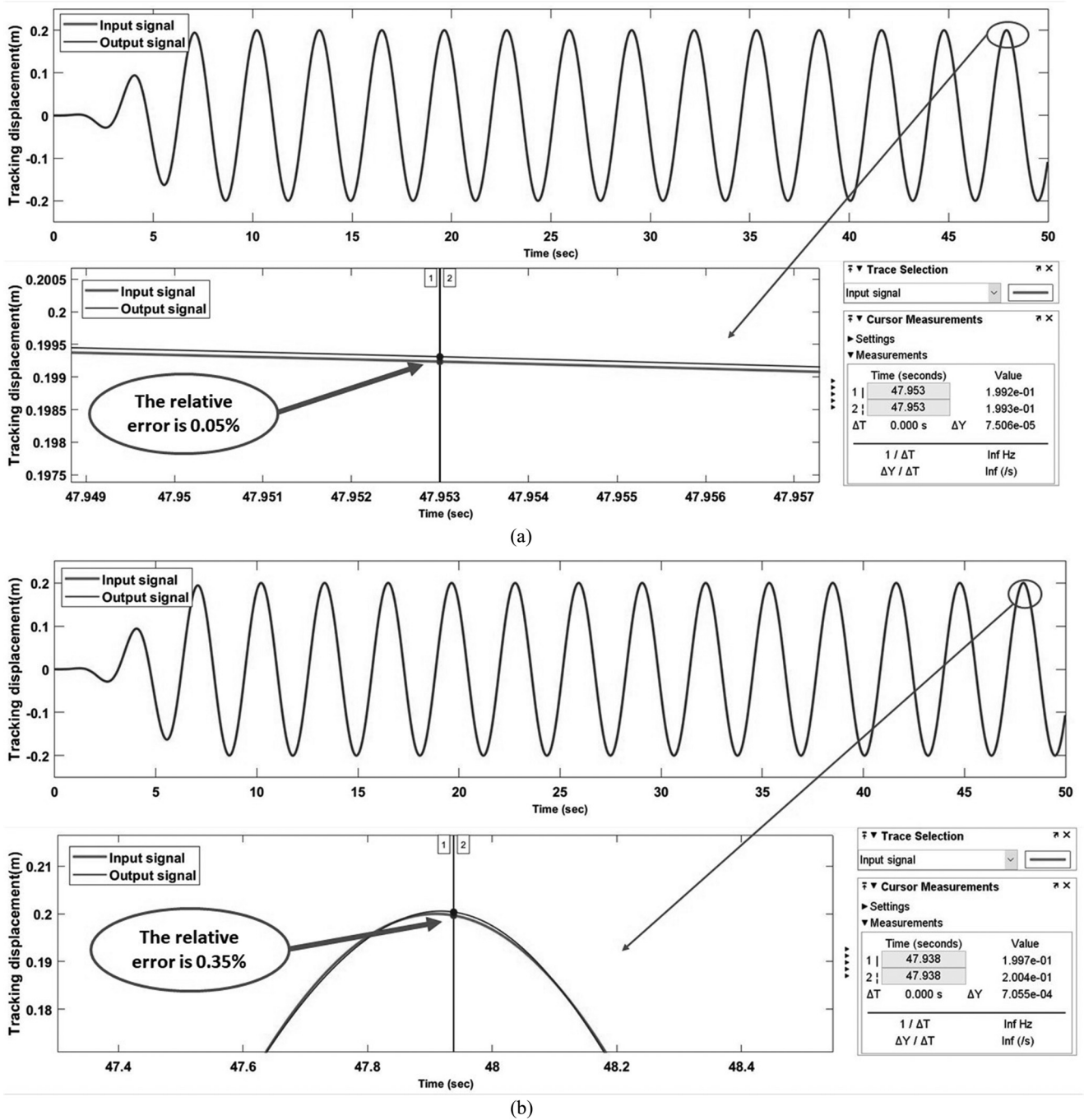
$$\beta = \max\{c_1 + k_1 c_2 + k_3 c_2 + R_1 - k_1 k_2 - 1, c_2 + k_3 + R_2, 1\} \alpha.$$

According to the Young's inequality  $2xy \leq x^2 + y^2$ , it is easy to found that

$$\begin{aligned}\dot{V} &\leq -\left(k_1 - \frac{1}{2\eta} c_1\right) e_1^2 \\ &\quad - \left[k_2 + c_2 - \frac{1}{2} c_1 \eta - \frac{1}{2} (k_1 + k_2) \eta - \frac{1}{2} \eta\right] e_2^2 \\ &\quad - \left(k_3 - \frac{1}{2\eta} - \frac{1}{2} \eta\right) s^2 + \frac{1}{2\eta} (k_1 + k_2) \alpha^2 + \frac{1}{2\eta} \beta^2 - \|Q\|_F \|\tilde{x}\|_2^2,\end{aligned}\quad (28)$$

where  $\eta \in R^+$ .

If the selected control parameters satisfy  $k_1 \geq \frac{c_1}{2\eta}$ ,  $k_2 \geq \frac{c_1 \eta + k_1 \eta - \eta - 2c_2}{2 + \eta}$ ,  $k_3 \geq \frac{1}{2\eta} - \frac{1}{2} \eta$ , then all signals of the valve controlled symmetrical cylinder electro-hydraulic



**Fig. 3.** Tracking performance of system signals under five typical external disturbances and unmodeled dynamic factors: (a)~(e) corresponds to the case of sine function, unit step function, unit ramp function, unit pulse function and unit parabolic function, respectively.

position servo system are consistent and ultimately bounded, and the tracking error of the system can be arbitrarily small by adjusting the parameters  $k_1, k_2, k_3, c_1, c_2$ .

## 5 Example analysis and performance comparison

To investigate the effectiveness of the constructed nonlinear model and control algorithm, the signal tracking performances of the system under five typical external

disturbances and unmodeled dynamic factors are first analyzed where the input signal is set as  $x_d = 0.2\sin(2t)(1 - e^{-0.01t^3})$ , and  $f$  is taken as sine function, unit step function, unit ramp function, unit pulse function and unit parabolic function, respectively. The analysis results are shown in Figure 3.

Figure 3a corresponds to the case of  $f = 20\sin(4\pi t)$ , and it can be seen that the signal tracking performance is the best. The maximum displacement of the output signal is  $1.993e-01m$  and the relative error of the output signal to the

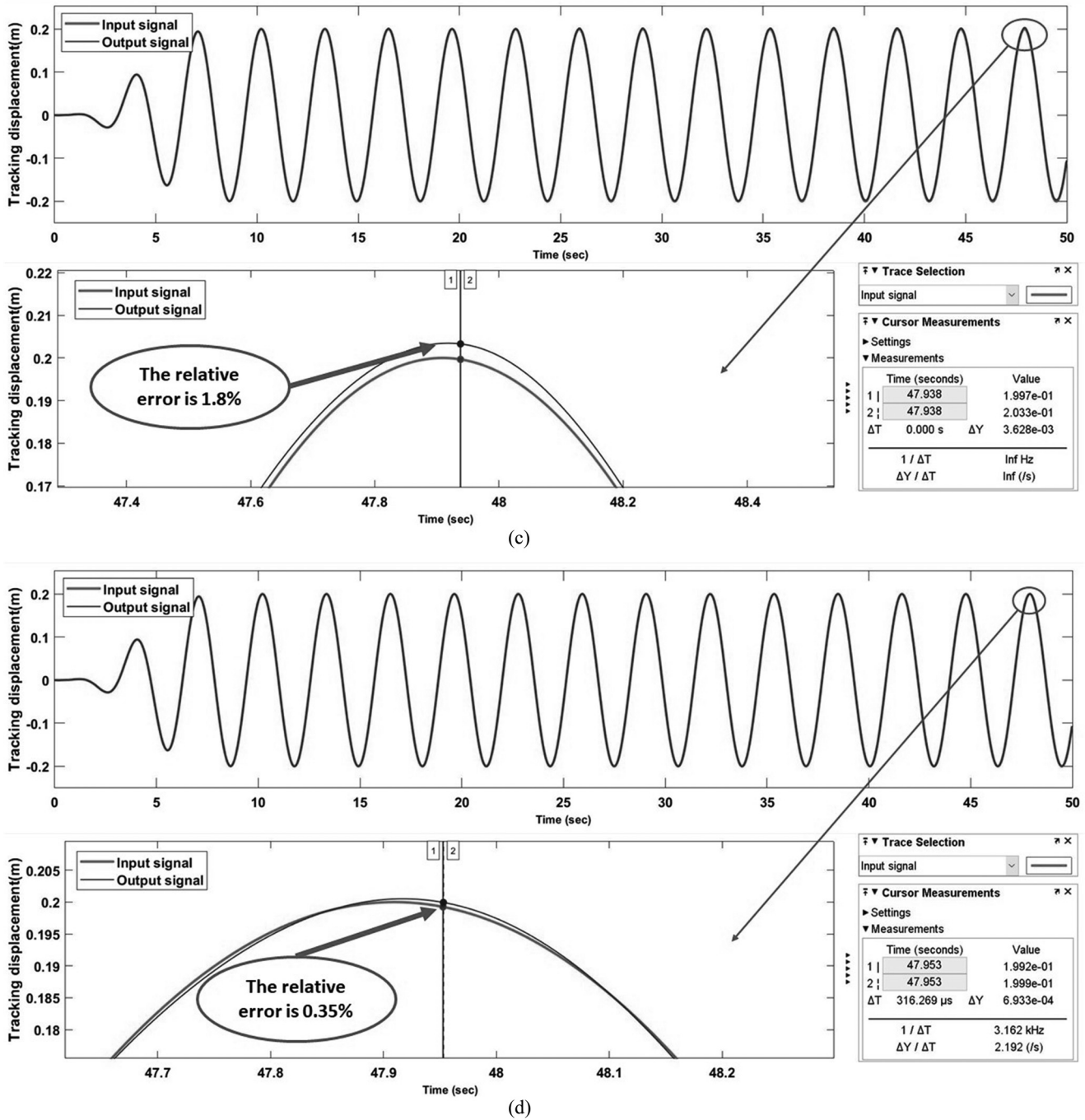


Fig. 3. (Continued).

input signal is 0.05% only. Figures 3b and 3d present the cases of unit step signal and unit pulse signal, respectively. The maximum displacement of the output signal is  $2.004 \times 10^{-1}$ m and  $1.999 \times 10^{-1}$ m respectively from which the relative errors are obtained as both 0.35%. Figure 3c is for the case of unit slope signal and the maximum displacement is found to be  $2.033 \times 10^{-1}$ m with an error of 1.8% relative to the input signal. The case of unit parabolic signal is shown in Figure 3e with a maximum displacement of  $2.044 \times 10^{-1}$ m and relative error of 2.6%. These analysis results show that under the influences of typical external disturbances and

unmodeled dynamic factors, the output signal of the system can still effectively track the input signal with small errors which verifies the rationality of the nonlinear model and the control system.

To further illustrate the effectiveness of the proposed algorithm, the electro-hydraulic servo system of a certain type of CNC machine tool is considered here as an example and the related parameters are shown in Table 1. The proposed algorithm (BSMC) is analyzed and calculated, and compared with existing three algorithms, namely, conventional PID control algorithm, backstepping control

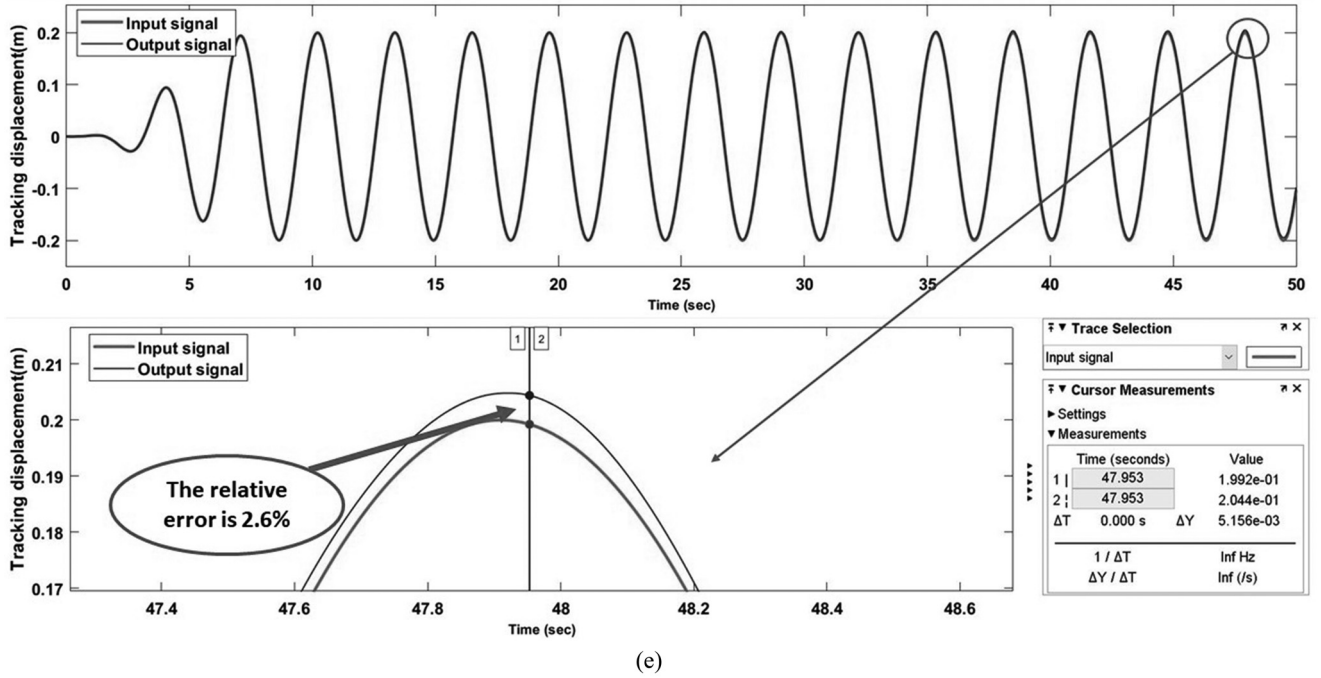


Fig. 3. (Continued).

Table 1. Relevant parameters of electrohydraulic servo system of a certain type of CNC machine tool.

| Parameter                                                           | Value              | Parameter                                                                    | Value               |
|---------------------------------------------------------------------|--------------------|------------------------------------------------------------------------------|---------------------|
| Pressure source of the system $p_s/P_a$                             | $7 \times 10^6$    | Oil bulk modulus $\beta_e/P_a$                                               | $2 \times 10^8$     |
| Return oil pressure of the system $p_r/P_a$                         | 0                  | Load mass $m / \text{kg}$                                                    | 180                 |
| Viscous friction coefficient $B / (\text{N}\cdot\text{s}/\text{m})$ | 80                 | Current gain of servo valve $k_i / (\text{m}/\text{V})$                      | $4 \times 10^{-8}$  |
| Initial volume of left cavity $V_{01}/\text{m}^3$                   | $1 \times 10^{-3}$ | Internal leakage coefficient $C_\tau / (\text{m}^5/(\text{N}\cdot\text{s}))$ | $7 \times 10^{-12}$ |
| Initial volume of right cavity $V_{02}/\text{m}^3$                  | $1 \times 10^{-3}$ | Discharge coefficient $C_d$                                                  | 0.6                 |
| Effective area of two cavities $A / \text{m}^2$                     | $2 \times 10^{-4}$ | Orifice area gradient $\omega/\text{m}$                                      | 0.025               |

algorithm (BSC), and adaptive robust control algorithm (ARC). In the example analysis, the coulomb friction shape function  $S_f = 2\arctan(1000x^2/\pi)$  and coulomb friction amplitude  $A_f = 10$  are adopted, and the external disturbance of the system and unmodeled dynamics are set as an equivalent composite disturbance ( $f = 20\sin(4\pi t)$ ) for illustration. Furthermore, the expected displacement signal is set as  $x_d = 0.2\sin(2t)(1 - e^{-0.01t^3})$  and the simulation step size is taken as 0.0002s.

The tuning process and results of relevant parameters of the controller are as follows:

–For the proposed control algorithm (BSMC): ESO Bandwidth is set as  $\omega_0 = 100$  [24] and the selection of the controller parameters can refer to Section 4, which means that  $k_1 \geq \frac{c_1}{2\eta}$ ,  $k_2 \geq \frac{c_1\eta + k_1\eta - \eta - 2c_2}{2 + \eta}$ ,  $k_3 \geq \frac{1}{2\eta} - \frac{1}{2}\eta$  should be satisfied and the system tracking error is tuned to be rather small by adjusting  $k_1, k_2, k_3, c_1, c_2$ . Accordingly, the controller parameters are finally set as  $k_1 = 10, k_2 = 10, k_3 = 1000, c_1 = 100, c_2 = 1000$ .

–For the conventional PID control algorithm [1]: The parameter selection of PID control algorithm is based on the trial debugging of the output displacement signal. The

trial debugging is conducted from a smaller value.  $k_p$  is adjusted firstly to reduce the error to appropriate value and  $k_i$  is tuned further to eliminate the error, and  $k_d$  is adjusted finally to reduce the overshoot and oscillation, hence enhancing the stability and the dynamic performance of the system. By comprehensively balancing the transient response and steady-state performance of the system, the controller parameters are finally determined through debugging as  $k_p = 15, k_i = 1, k_d = 3$ .

–For the Backstepping Sliding Mode Control Algorithm (BSC) [25]: According to equation (7), it is set that

$$f_c = \left(\frac{A^2}{V_1} + \frac{A^2}{V_2}\right)\beta_e x_2, f_\mu = \left(\frac{A}{V_1} + \frac{A}{V_2}\right)(P_1 - P_2)\beta_e, g_3 = \left(\frac{AR_1}{V_1} + \frac{AR_2}{V_2}\right)g\beta_e$$

$\dot{x}_{3d} = mk_2 e_2 + mk_1(\ddot{x}_{1d} - \ddot{x}_2) + m\ddot{x}_{1d} + me_1 + \theta_1 \dot{x}_2 + \theta_2 s_f + \theta_3 + f, \theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T = [B, A_f, f, C_\tau]^T$ . The control

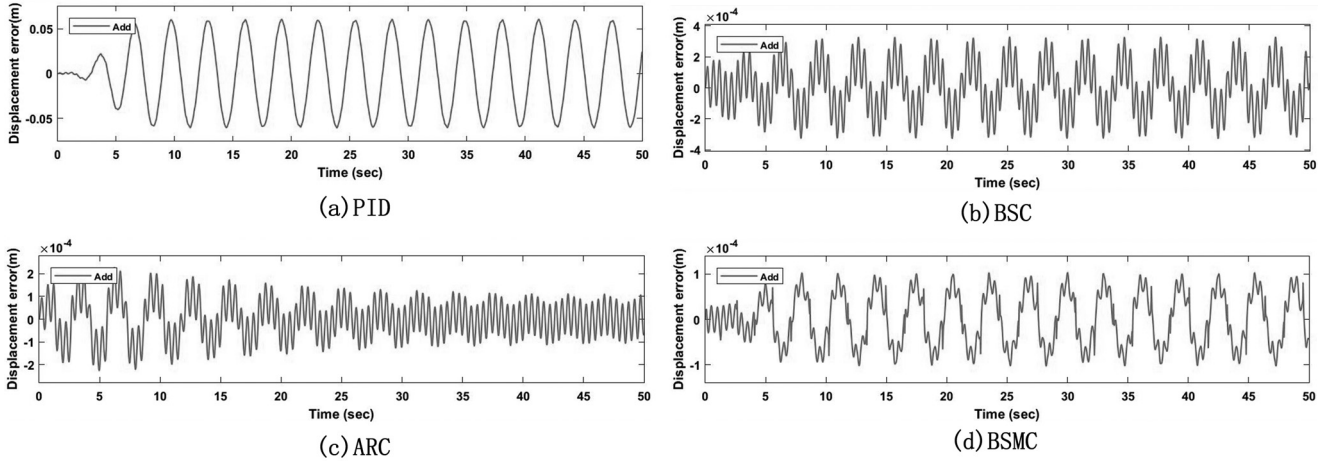


Fig. 4. Displacement error of four control algorithms.

law of BSC is  $u' = (\dot{x}_{3d} + f_c + \theta_4 f_\mu + e_2/m + k_3 e_3)/g_3$ . The parameters ( $k_1, k_2$  and  $k_3$ ) should be selected to meet the system stability requirements. By consideration of the transient and steady-state performance of the system, the control parameters are finally set as  $k_1 = 30, k_2 = 100, k_3 = 1000$ .

For the Adaptive Robust Control Algorithm (ARC) [17]: According to equation (7), it is set that  $f_c = \left(\frac{A^2}{V_1} + \frac{A^2}{V_2}\right)\beta_e x_2$ ,  $f_\mu = \left(\frac{A}{V_1} + \frac{A}{V_2}\right)(P_1 - P_2)\beta_e$ ,  $g_3 = \left(\frac{AR_1}{V_1} + \frac{AR_2}{V_2}\right)g\beta_e$ ,  $\dot{x}_{3d} = mk_2 e_2 + mk_1(\ddot{x}_{1d} - x_2) + m\ddot{x}_{1d} + me_1 + \theta_1 \dot{x}_2 + \theta_2 s_f + \theta_3 + f$ ,  $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T = [B, A_f, f, C_\tau]^T$ . The controllaw of ARC is  $u'' = (\dot{x}_{3d} + f_c + \theta_4 f_\mu - k_3 e_3)/g_3$ .  $k_1, k_2$  and  $k_3$  should be also selected to meet the stability requirements. Similarly, they are finally set as  $k_1 = 400, k_2 = 260, k_3 = 50$  by tuning the system responses.

The displacement errors of four algorithms are calculated and the results are depicted in Figure 4. It can be clearly seen that BSMC and ARC have smaller displacement errors which are  $-1.027 \times 10^{-4}$  and  $-2.261 \times 10^{-4}$  respectively, compared with BSC ( $-3.266 \times 10^{-4}$ ) and conventional PID control algorithm ( $-6.050 \times 10^{-2}$ ).

The displacement tracking performance of the four algorithms are compared as shown in Figure 5.

The results show that all of the four algorithms have good tracking effect within the period of  $t < 5s$ . However, when  $t > 5s$ , the conventional PID control algorithm exhibits the worst tracking effect with slow time response and low accuracy in tracking displacement while BSMC and ARC possess much better tracking performance. Moreover, the tracking displacement curve of BSMC coincides with the expected displacement curve with higher accuracy, compared with ARC and BSC which are slightly overshoot.

The control law  $u$  response curves of four algorithms are shown in Figure 6 from which it is observed that BSMC can still ensure stable output with fast time response under composite disturbance. Figures 7–9 show the velocity, acceleration and composite disturbance estimations of valve-controlled cylinder by using the designed ESO. It can be seen that the overall estimation effect is rather good and the estimated values are bounded.

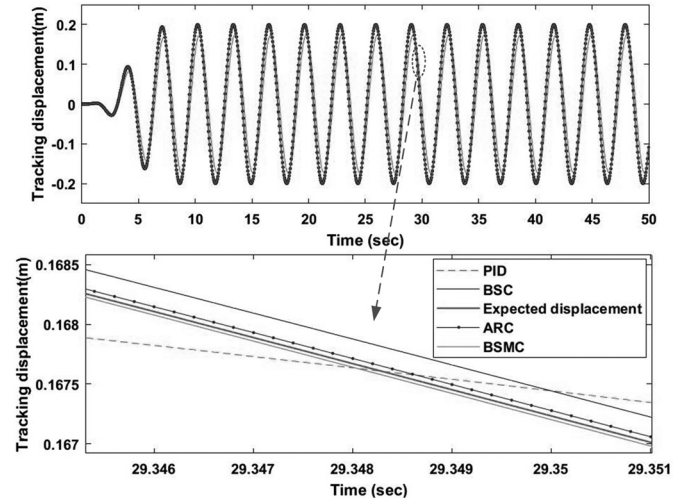


Fig. 5. Displacement tracking performance comparison of the four algorithms.

To evaluate the effectiveness of BSMC more clearly, three performance indexes are selected here for comparison, namely the maximum tracking displacement error (Max), the average tracking displacement error (Mean) and the standard deviation of tracking displacement error (Rms), as listed in Table 2.

It is found from Table 2 that the maximum tracking error of BSMC is the smallest one ( $1.030 \times 10^{-4}$ ) and the accuracy enhancements relative to ARC and BSC achieve 54.75% and 68.28% respectively. Due to the disability to compensate for composite disturbance in the forward feedback channel, the PID control algorithm has the largest tracking displacement error compared to others.

The average value of tracking displacement error reflects the average level of control error. The results show that the average level of BSMC is also the best one followed by ARC and the PID control is the worst one. The standard deviation of tracking displacement error reflects the degree of dispersion of control error. Similarly, it can be found that the degree of dispersion of BSMC is the smallest one followed by ARC, and the PID control is the largest one.

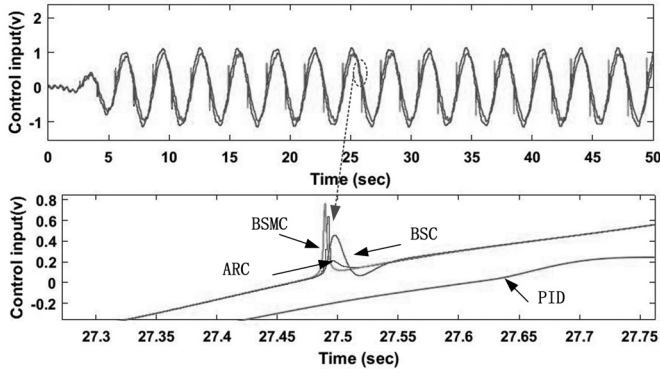


Fig. 6. The control law  $u$  response curves of four control algorithms.

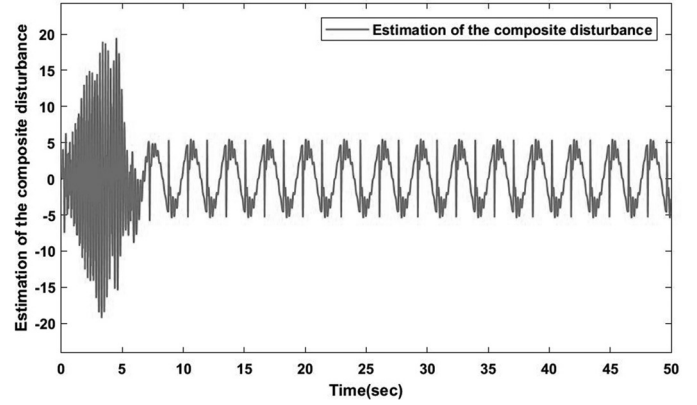


Fig. 9. Estimation of the composite disturbance

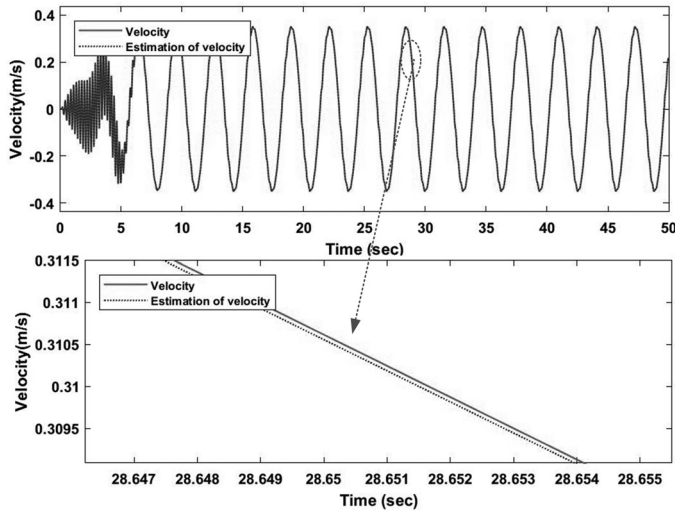


Fig. 7. Velocity and the estimation of velocity.

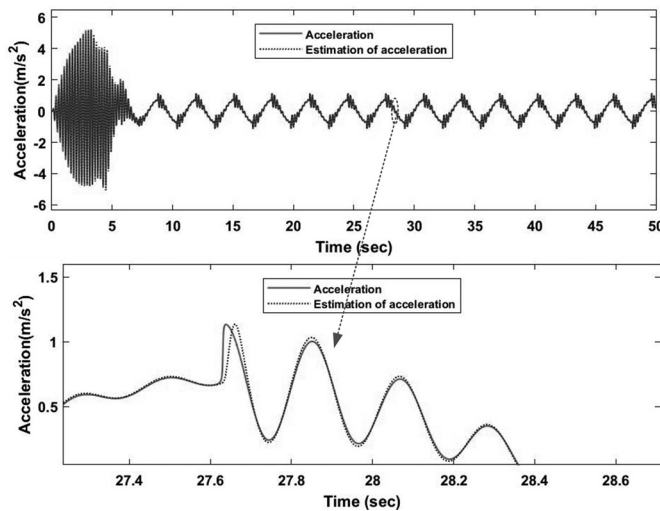


Fig. 8. Acceleration and the estimation of acceleration.

Table 2. Performance indexes of four algorithms.

| Algorithms | Max       | Min        | Mean       | Rms       |
|------------|-----------|------------|------------|-----------|
| PID        | 6.047e-02 | -6.050e-02 | -5.376e-04 | 3.966e-02 |
| BSC        | 3.267e-04 | -3.266e-04 | -9.060e-07 | 1.657e-04 |
| ARC        | 2.276e-04 | -2.261e-04 | 2.791e-07  | 8.577e-05 |
| BSMC       | 1.030e-04 | -1.027e-04 | 2.105e-07  | 5.747e-05 |

## 6 Conclusions

In this paper, a backstepping sliding mode control algorithm (BSMC) based on ESO is proposed for the EPSSVCS with composite disturbance induced by friction nonlinearity and external disturbances together with unmodeled dynamic factors. An ESO is designed to effectively estimate the velocity, acceleration and the composite disturbance of the valve-controlled cylinder online. A kind of BSMC is presented based on the online ESO estimates and the displacement feedback signals, the control law is given and the system stability is proved. Through case studies, the proposed nonlinear model and algorithm are found to be effective and the advantage of BSMC is also shown by comparison with other algorithms. The main results are as follows:—Under the influence of typical external disturbances and unmodeled dynamic factors, the output signal of the system can still effectively track the input signal, and the error of the output signal relative to the input signal remains small, which shows the effectiveness of the established nonlinear model and the correctness of the designed control system.

—The designed ESO shows good tracking performance and can effectively estimate the values of velocity, acceleration and composite disturbances online, which not only illustrates the stability of the system and may also provide certain reference for the measurements of velocity, acceleration and composite disturbances that are difficult to measure in practical engineering fields.

–The presented algorithm has smaller tracking displacement error, higher control accuracy and smaller dispersion compared to PID algorithm, BSC and ARC. The control accuracy of BSMC has been improved by 54.75% and 68.28% respectively by comparison with ARC and BSC, hence providing a feasible way for the design of EPSSVCS and related control systems.

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### Conflict of interest

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Author contribution statement

This article is a sole author, and the entire content is written by the author personally.

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