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Robust design optimization of dynamic and static manufacturing processes using the stochastic frontier model

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Abstract. The paper discusses a novel method, which addresses robust design optimization of dynamic and static multi-objective processes. For a dynamic process, the optimal setting of the graded signal and input parameters are sought so that it is least sensitive to internal and external noises. In addition to addressing planned and unplanned experiments (cross-sectional and panel data), the method estimates the random and nonrandom variance components variably (i.e., returns a non-constant uncertainty at each combination level or treatment). The stochastic frontier model is utilized to ensure this purpose. For dynamic processes, the method operates in three main steps, (i) data preparation by transforming the outputs to maximization functions, (ii) estimate of the composed variation (random and non-random error components), (iii) and, composition of the process uncertainty array for each output across the signal levels. The robust design optimization solution corresponds to the levels combination of the signal and the input factors, which adds up to the lowest global uncertainty score. The applicability of the approach is then illustrated with a case study that uses one signal factor at two levels and four input factors (x1, x2, x3, and x4) at three levels each. The process responses, Y1, Y2, and Y3 are of types Dynamic Larger the Best (DLB), Dynamic Nominal the Best (DNB), and Dynamic Smaller the Best (DSB), respectively.

Keywords: Robust design optimization / dynamic and static systems/processes / Taguchi method / stochastic frontier model / multi-objective processes / external and internal noises

1 Introduction

Multidisciplinary teams strive to develop designs that comply with uncertainties coming from manufacturing, environmental, deterioration, degradation sources. In literature, studies on sustainable variability have a connection with robust design optimization (RDO) [1–3] and the reliability-based design optimization (RBDO) [4–6] approaches. This research is included in the RDO, especially, robustness for dynamic and static processes. The goal of the research is to determine the input parameter setting that desensitizes the process to the effects of the internal and external noise factores. Moreover, controlling the initiation of lattente effect because of cross interaction. The tolerance design stage is carried after to ensure a trade-off between the functional requirements, the deterioration effect/rate, and the overall cost of the product/process over the operating cycle time [7,8]. The deliverable is design, that accommodates operating factors so that the system response gets closer to the target value(s) with minimum variation in presence of background noise [9]. The Taguchi [10–12] technique – a groundbreaking piece of research in the realm of static and dynamic robust optimization design, involves two consecutive steps: (i) setting of the dispersion factors to minimize a quality loss function or maximize a signal-to-noise ratio, (ii) then, selection of the adjustment factors allowing to shift of the average output onto a process target. However, the Taguchi method can only answer mono-objective problems, moreover, it does not guarantee the same optimum at each stage unless the mean and the variance of the response are uncorrelated [13]. The research efforts to expand the Taguchi method's to address static and dynamic multi-objective processes are fuzzy and elusive to understand [14]. Therefore, different techniques such as the grey function, the desirability function and their hybrids [15], the principal component analysis techniques [15], the fuzzy logic [16], the meta-heuristic methods [17], the multiple regression models [18], and more recently, the

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frontier function [19] have been devised to bring viable alterntives to the Taguchi method for multi-objective processes. Even though these methods are effective in anserwing robust design optimization of static multi-objective processes, they are still in their infancy and to some extent uncapable of addressing dynamic multi-objective processes. Other methods such as the meta-heuristic and data mining techniques have shown success in this regard, but still, they failed to address issues such as producing the ideal mix of control elements in continuous space [20], for instance.

Dynamic multi-objective techniques for robust design optimization can be divided into three main categories, i.e., meta-heuristic [21,22], multiple attribute decision-making [23–26], and mathematical modeling [12,27,28]. Other strategies that take advantage of the quality loss function have also been developed to optimize both static and dynamic problems at the same time [29]. Dynamic robust design optimization consists of monitoring the output (Y) to a target T (min, max, or nominal) under the dynamic control of a signal factor (S). One solution is to determine the setting of the input parameters, which minimizes the average loss function L(Y,S,T). Formally, the dynamic model for multi-objective processes is stated as in equation (1):

$$Y_{js} = f_{js}(X, S_s) + e_{js} \tag{1}$$

where X is a set of control factors, xi, $i = (1,I)$. The response Y_{js} is the output j, $j = (1,J)$ at the signal level s, $s = (1,S)$. The fjs is the transfer function for the jth response while the signal factor S is set at the sth level. The e_{js} is the composed error for the Y_{js} process response.

In general, it is accepted that the signal factor(s) and the system output(s), Y_{js} have a linear relationship, as shown in equation (2):

$$Y_{js} = \beta \cdot S + \varepsilon_{js}. \tag{2}$$

In light of this constation, Taguchi proposes a two-step procedure to optimize dynamic mono-objective processes [12]: (i) maximization of the dynamic signal-to-noise ratio, (ii) then, adjustment of the slope (β); i.e., $\beta = 0$, $0 \leq \beta \leq \infty$, and $\beta = \infty$ upon the objective function, i.e., Dynamic Smaller the Best (DSB), Dynamic Larger the Best (DLB), or Dynamic Nominal the Best (DNB), respectively. However, the two-step procedure is unpractical, as argued earlier since it is unpractical to find a set of control and dispersion parameters, which manage multi-objective responses, simultaneously. To solve the problem, one must first define a global objective function and then optimize it to bring each response variable as close to the target value as feasible. At each of the signal levels, the goal value should vary as little as possible [30].

In literature, many alternatives to the Taguchi dynamic multi-objective systems have been developed [31–33]. However, the frontier production approach has only been considered in a few research works. The article discusses the robust optimization design of dynamic systems while utilizing the stochastic frontier paradigm (SF). It attempts to expand on the ongoing research on the static Robust Design of Products and Processes using the Stochastic Frontier model (RDPP-SF) [34,35].

Most experimental research has adopted the design of experiment (DoE) methodology. Ideally, robust processes are laced with pure symmetrical random variation, solely, thus, they are under statistical control and reliable in time. However, nonrandom variation may initiate not only because of internal and external noises but also because of a violation of the premises of the DoE for industrial processes, i.e., the replication blocking and the "full randomization. In many scenarios such a hard-to-change factors, frequent clamping-unclamping operations, non-homogeneity in raw materials, etc., full randomization is unpractical. The RDPP-SF method uses the stochastic frontier model to estimate composed errors, i.e., random and nonrandom components. The econometric studies, which link technical inefficiency with nonrandom variability among the Decision Making Units (DMUs), are the source of the SF paradigm that the RDPP-SF method. Thus, robustness in the RDPP-SF method to technical inefficiency.

The rest of paper unfolds as follows. Section 2 outlines the concept behind the stochastic frontier method. Section 3 presents the RDPP-SF method for static and dynamic multi-objective systems. In Section 4, a case study is proposed to demonstrate the employability of the RDPP-SF method for dynamic processes, hereinafter, called Dynamic Robust Design of Processes and Products using the Stochastic Frontier model (DRDPP-SF). The computer program, FRONTIER 4.1® is used to estimate the maximum likelihood of the parameters of the stochastic frontier model and tests statistical hypotheses about the functional form of the stochastic frontier, the robustness metric (γ-value), and the distributional forms of the non-random variation component. Section 5 is the conclusion, and it provides the scope and limitations of the RDPP-SF method for static and dynamic robust optimization design.

2 Stochastic frontier model

The stochastic frontier methodology is a widely used econometric technic for estimating the inefficiency of DMUs. The method can define the current state of technology in the industry and also measure the individual performance of the DMUs [36]. Initially, Aigner et al. [37], Meeusen and van Den Broeck [38], and Schmidt and Lovell [39] developed the SF model. Aigner et al. [37] devised the formal representation of a production frontier model is given in equation (3).

$$Y_i = f(x_i; \beta) + (v_i - u_i) \tag{3}$$

where,

- Y_i and x_i are naturally logged variables representing the outputs and the inputs of the DMU_i ($i = 1,N$), respectively. The log transformation is employed to ensure the convexity of the stochastic production function as well as mitigate of the variability in the Y_{is}:

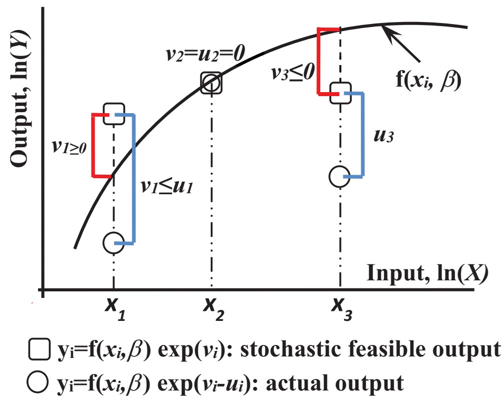

Fig. 1. Output-oriented technical efficiency using the stochastic frontier method (SF).

- $\beta = [\beta_0 \beta_1 \dots \beta_k]^{'}$ is a $(k+1)$ column vector coefficients to be estimated;
- v_i represents the statistical noise distributed as $V_i \sim N(0, \sigma_v^2)$. In production processes, the v_i captures part-to-part variation, gauges repeatability, and the combined effects of unseen factors;
- u_i is a one-sided random term distributed as $U_i \sim N(0, {\sigma_u}^2)$ and represents the technical inefficiency of the DMU, i. In production processes, the u_i stands for the variance brought on by noise factors such as environmental conditions, tool wear, material deviation from specifications, drift of parameter setting from the nominals because of time and/or manufacturing variation, lack of expertise/management performance, etc.

For output-oriented technical inefficiency, the ui term explains why a given DMUi cannot achive the feasible maximum output beyond random noise. As expressend in equation (1) and illustrated in Figure 1, the composed error term is the focus of the model. It allows for the conventional symmetrical random variation, $V_i^{iid} \sim N(0, \sigma_v^2)$, and a positive one-sided disturbance term, $U_i \sim \text{iid}|\text{N}(\mu,\ \sigma^2|$, which represents the nonnatural variation occurring in a process.

3 Static and dynamic RDPP-SF method

3.1 RDPP-SF method for static processes

The static RDPP-SF method, which is devised by Trabelsi and Rezgui [34,35], is dedicated to estimating the inherent random and nonrandom variation in a signal-free process that is rife with internal and external noise sensitivity. The output-oriented production model, which is given in equation (3) is adopted by the RDPP-SF method. Table 1 shows how the static RDPP-SF technique and the econometric stochastic frontier model map out.

The procedural scheme of the RDPP-SF method for static and dynamic multi-objective problems is shown in Figure 2 and it involves four steps.

Step 1 (data preparation): Define the DoE strategy and assign factor levels. Different formattings are acceptable, even though, a $\pm 3\sigma$ coding is preferred. The reason is that the three sigma quality level is adopted by major Design For Six Sigmas (DFSS) procedures. By analogy to the econometric model, every combination of the factor levels (run) in the designed experiment, is seen as a decision-making unit (DMU), which makes use of the resources x_i (process inputs). The process responses, Y_{jer} are scaled and translated depending on whether a nominalization, maximization, or minimization target is sought. The outputs of the types smaller the best (STB) and nominal the best (NTB) should be transformed and scaled beforehand since the stochastic function initially aims to produce the maximum output for each combination of the inputs (x_i).

Equation (4) states the transformation for the NTB outputs.

$$Y_i = \exp[-abs(y_i - y_T)]. \quad (4)$$

For the STB outputs, the transformation is given in equation (5).

$$Y_i = \frac{1}{y_i}. \quad (5)$$

The original interval of the raw data for the STB and NTB cases is recovered using equation (6).

$$Y_{scaled} = (U_b - L_b)\frac{Y_i - Y_{min}}{Y_{Max} - Y_{min}}, \quad (6)$$

where y_i, y_T, Y_i, Y_{scaled}, U_b, *and* L_b represent, in that order, the actual output (not transformed), the target value, the transformed output (not scaled), the transformed and scaled output, and the upper and lower bounds of the original data interval.

Step 2 (data analysis): The input-output function is used to account for both the interaction and the individual effects of the input parameters, x_i. The following hypotheses are tested at a 95% confidence level for each output (Y_{jer}) using the FRONTIER 4.1® program.

- (H_0: $\beta_k = 0$ *vs.* H_1: $\beta_k \neq 0$). The goal is to test the statistical significance of the two-way interactions among the process parameters.
- (H_0: $\gamma = 0$ *vs.* H_1: $\gamma > 0$). The test checks, for each Y_{jer}, the statistical significance of non-random variations in a process, which could be brought on by both internal and external noises. In any case, the test establishes whether there is confounding between the stochastic frontier (SF) and the average line model (RSM).
- The test (H_0: U_i~half normal *vs.* H_1: U_i ~ truncated normal distribution) ascertains the distributional form of the non-random variation component. The RDPP-SF method for static and dynamic processes assumes that the $u_i(s)$ are following a half-normal distribution as suggested in Aigner et al. [37].

Step 3 (constitute the uncertainty arrays for each Y_{jer}): The components of the composed error term (u_i *and* v_i) are estimated using the test on the γ-value (test b in step 2). For each Y_{jer}, the uncertainty array is created as follows. If $\gamma \geq 95\%$ then the $v_i \approx 0$, and the $u_i(s)$ estimates compose the uncertainty array (ε_i). The variations in the

Table 1. Mapping between the econometric and the RDPP-SF approach.

Econometric model	RDPP-SF method
Set of DMUs	Design plan (DoE)
DMU(i)	Run, i, in the DoE
Panel/cross data	Replicated/non-replicated dataset
vi: random variation	Statistical random variation
ui: technical inefficiency	Non-random variation (measurement of robustness in the process)

Metric for the RDPP-SF*: ($H_0 : \gamma = 0$ *vs.* $H_1 : \gamma > 0$) where $\gamma = \frac{\sigma_u^2}{\sigma_v^2+\sigma_u^2}$, Rejected H_0 at $\alpha = 5\%$ level if the Likelihood Ratio (LR) exceeds $\chi^2_{1,2\alpha=10\%}$.

* Hypothesis test on robustness. If it is proven statistically significant we should reject the hypothesis saying that the process is fully robust, i.e., under statistical control, solely.

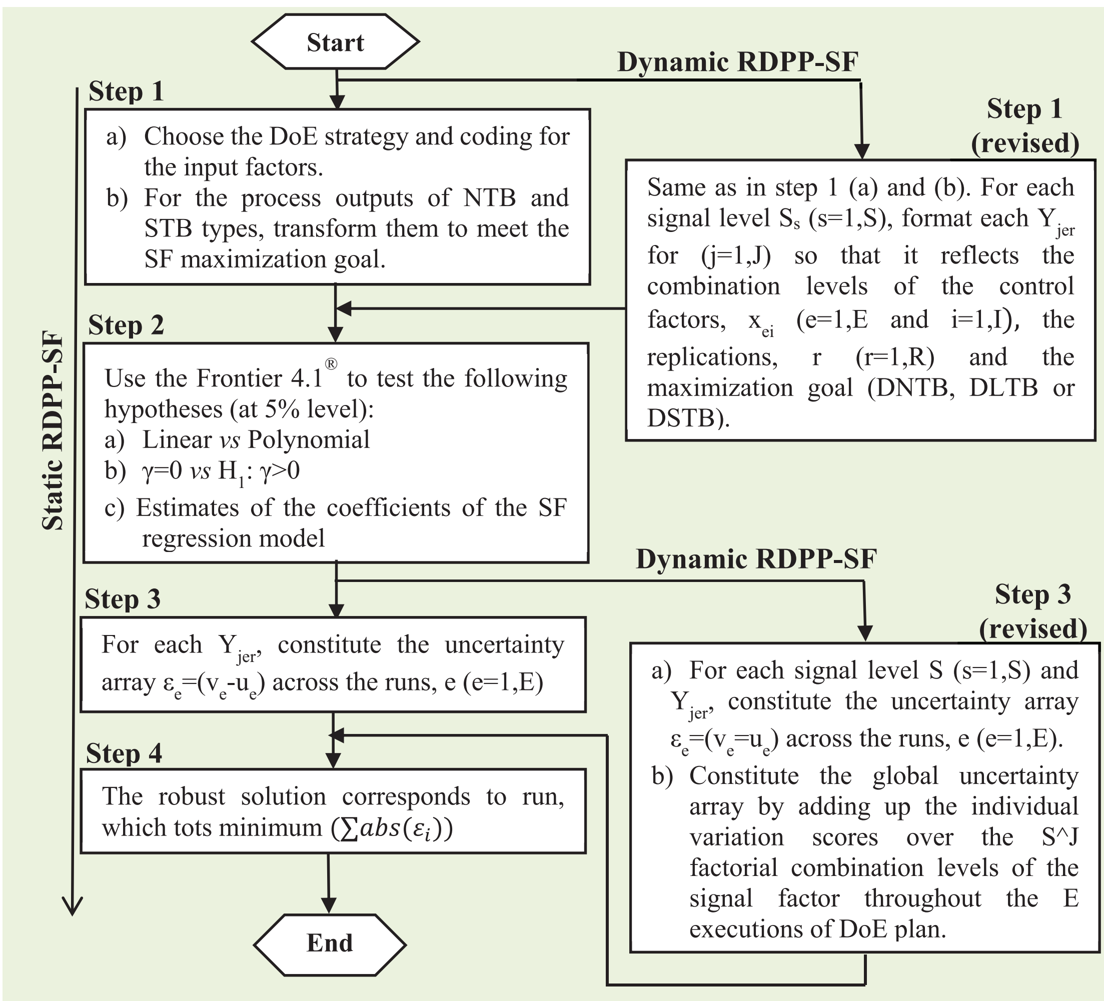

Fig. 2. Procedural scheme of the RDPP-SF method for static and dynamic processes.

process is entirely non-random. If $\gamma \leq 5\%$ then the $u_i \approx 0$ and the process is experiencing pure random unit-to-unit variations, solely. The uncertainty array (ε_i) is then composed of the $v_i(s)$ estimates at each run. The average line model (RSM) is confounding with the SF model in this case. If $5\% \leq \gamma \leq 95\%$, the two types of variations-random unit-to-unit and nonrandom, are there and should be accounted for. The uncertainty array (ε_i) is composed of the estimates ($\varepsilon_i = v_i - u_i$) for each run. In the FRONTIER4.1® program, the individual inefficiency ($\exp(-u)$) is used to calculate the $u_i(s)$ terms for each run.

Step 4 (determination of the robust design solution): The robust optimization design solution correlates with the run in the DoE layout, which adds

Table 2. Data formatting for Frontier 4.1® at a signal level $S = s$.

	Y_{jer}	x_1	x_2	………	x_I
DMU_1	Y_{j11s}	x_{11}	x_{21}	………	x_{I1}
………	………	………	………	………	………
………	………	………	………	………	………
DMU_E	Y_{jE1s}	x_{1E}	x_{2E}	………	x_{IE}
………	………	………	………	………	………
………	………	………	………	………	………
DMU_1	Y_{j1Rs}	x_{11}	x_{21}	………	x_{I1}
…….	………	………	………	………	………
…….	………	………	………	………	………
DMU_E	Y_{jERs}	x_{1E}	x_{2E}	………	x_{IE}

*i: process inputs, j: process output, e: execution (run), and r: replication.

up to a minimum (Σ abs(ε_i)) across the $Y_{jer}(s)$. Trabelsi and Rezgui [34,35] have compiled applications of the RDPP-SF method for static processes.

3.2 RDPP-SF method for dynamic processes

In dynamic multi-objective processes, the control factors should be set at suitable levels so that the signal-output relationship is insensitive to internal and external noise. The dynamic DRDPP-SF process, which still adheres to the functional scheme depicted in Figure 2, modifies steps 1 and 3.

Step 1 revised (data preparation): In dynamic processes, each output, Y_{jer}, is correlated to a signal factor level ($S = s$). The designed experiment is formatted so that it reflects the combination levels of the control factors, x_{ei} ($e = 1$, E and $i = 1$, I), the replication, r ($r = 1$, R), and the signal set, S ($s = 1$, S). The apdated DoE layout formatting for a dynamic output, Y_{jer}, at a signal level, $S = s$ is displayed in Table 2.

Step 3 revised (determination of the robust design solution): The uncertainty array for evry output, Y_{jer}, and signal level, ($S = s$) is composed is composed using the estimate of the γ-value (refer to step 3 above). The global uncertainty array is obtained by totaling the individual uncertainty scores for the outputs Y_{jer} over the $S\hat{}J$ factorial combinations across the E executions (runs) of the designed experiment runs. The level combination of the input factors (x_{ie}, $i = 1$, I, and $e = 1$, E) and the signal level ($S = s$), giving the minimum global uncertainty score corresponds to the robust solution.

4 The DRDPP-SF method: an illustrative example

A simulated dataset, which is retrieved from Chang and Chen [20] is used to demonstrate the employability of the DRDPP-SF method for dynamic processes. The DoE plan is an L18 OA, which is replicated once. The process has four control factors x_i, ($i = 1$, 4); each at three discrete levels (1, 2, and 3). The process outputs Y_{jer} ($j = 1$, 3, $e = 1$, 18, and $r = 1$, 2) are of DLB, DNB, and DSB types, respectively. The signal factor has two levels, $s = 0.1$, and $s = 0.2$. Table 3 shows the L18 dataset. The DRDPP-SF is executed in the same manner as the static RDPP-SF, except for steps 1 and 3, which are revised in the following section.

Step 1 revised: The dataset is templated as in Table 2 for each signal level ($S = s$). Because the rationale behind the stochastic frontier function (SF) drew on the maximization of the production level, the dynamic outputs Y_{2er} (DNB) and Y_{3er} (DSB) are transformed and mapped to their original bounds using equations (4), (5), and (6). The target value for the output, Y_{2er}, is decided to be the average of the raw data, i.e., $\bar{Y}_2 = 0.987$ at $s = 0.1$ and $s = 0.2$. Table 4 shows the prepared dataset with $s = 0.1$ and the original and transformed outputs (Y_{2er}) and (Y_{3er}).

Step 2 (data analysis): The x_i and Y_{jer} values given in Table 4 should be logged before the FRONTIER4.1® program starts processing. The Neperian log transformation is applied to eliminate extrema, correct skewness, and assure convexity of the stochastic frontier. The DRDPP-SF method considers two input-output models: the Translog, a quadratic regression in log(x_i), which takes interactions into account, and the Cobb Douglas, a multiple linear regression in log(x_i), which does not. The hypotheses tests (H_0: CD vs TL) in Table 5 show the Cobb-Douglas model is the best fit for Y_{1er}, and Y_{3er} when the signal levels are set at $s = 0.1$ or $s = 0.2$, indistinctly. This is because the LR statistics (13.23, 14.50, −49.10, and 7.20, respectively) are smaller than the critical value, $\chi^2_{10} = 18.31$. However, a TL model provides a superior fit for Y_{2er} at signal levels, $s = 0.1$ and $s = 0.2$ (LR 18.96, and 41.48, respectively). In the remainder, we apply the TL model to all outputs. For the robustness index (γ_estimate), Table 5 indicates that the test (H_0: $\gamma = 0$ *vs* H_1: $\gamma > 0$) is statistically significant at a 95% confidence interval regardless of the signal levels, i.e., the Log Ratios are higher than the critical value

Table 3. The layout of the simulated L18 designed experiment [20].

Run (DMUi)	Factors				Y_{1er}				Y_{2er}				Y_{3er}			
					s = 0.1		s = 0.2		s = 0.1		s = 0.2		s = 0.1		s = 0.2	
	x_1	x_2	x_3	x_4	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
1	1	1	1	1	7.80	8.13	14.22	14.92	0.98	1.09	1.63	1.42	15.00	16.56	32.24	41.81
2	1	2	2	2	8.63	7.53	17.01	16.52	1.02	1.05	2.22	1.73	16.00	16.30	42.91	29.17
3	1	3	3	3	8.12	7.28	16.65	15.84	1.05	0.94	2.17	2.15	23.00	23.29	40.63	48.37
4	2	1	1	2	8.18	8.07	18.29	15.92	0.68	0.72	1.46	1.50	25.00	15.98	37.74	41.75
5	2	2	2	3	7.04	7.58	13.11	16.53	1.14	1.23	2.64	2.27	18.00	14.40	23.80	44.36
6	2	3	3	1	8.32	9.79	16.80	14.74	1.00	0.96	2.49	1.97	26.00	10.28	40.45	30.69
7	3	1	2	1	8.02	8.30	14.46	15.42	1.22	1.20	2.29	2.39	28.00	19.68	40.57	50.66
8	3	2	3	2	6.36	8.24	18.23	17.48	0.73	0.86	1.43	2.13	12.00	26.70	31.01	32.74
9	3	3	1	3	5.93	8.65	16.51	13.43	1.12	0.91	1.92	1.77	17.00	19.78	49.92	28.39
10	1	1	3	3	8.56	8.88	17.57	19.17	0.80	0.75	1.45	1.62	21.00	28.16	39.08	47.59
11	1	2	1	1	7.61	9.85	17.34	16.31	0.92	1.23	2.55	2.54	20.00	16.24	43.19	28.68
12	1	3	2	2	7.88	8.07	16.89	12.55	1.08	1.05	2.28	2.22	18.00	11.38	46.14	22.51
13	2	1	2	3	8.73	6.82	18.22	15.64	0.95	0.99	2.00	2.00	26.00	22.32	64.67	40.40
14	2	2	3	1	7.97	9.72	16.72	11.98	1.17	1.14	1.95	2.35	16.00	23.16	24.82	44.13
15	2	3	1	2	9.16	8.77	16.72	15.86	0.85	0.79	1.42	1.75	14.00	12.88	40.57	33.27
16	3	1	3	2	9.32	8.71	14.86	15.67	1.05	1.10	2.01	2.26	22.00	15.90	51.58	43.90
17	3	2	1	3	8.32	6.91	16.03	14.10	0.80	0.85	2.07	1.99	23.00	20.34	42.91	32.95
18	3	3	2	1	8.71	6.37	14.87	18.74	1.14	0.98	1.92	1.58	19.00	12.43	37.70	38.89

Table 4. Data formatting (non-logged) at s = 0.1.

Run (DMUi)	Repl.	Factors				Y_{1er} (DLB)	Y_{2er} (DNB)		Y_{3er} (DSB)	
		x_1	x_2	x_3	x_4	Orig.	Orig.	Transf.	Orig.	Transf.
1	1	1	1	1	1	7.800	0.980	0.993	15.000	0.067
2	1	1	2	2	2	8.630	1.020	0.968	16.000	0.063
......	......	......	......	......	......	......	......	......	......	......
18	1	3	3	2	1	8.710	1.140	0.858	19.000	0.053
1	2	1	1	1	1	8.130	1.090	0.902	16.560	0.060
2	2	1	2	2	2	7.530	1.050	0.939	16.300	0.061
......	......	......	......	......	......	......	......	......	......	......
18	2	3	3	2	1	6.370	0.980	0.993	12.430	0.080

Orig.: Original Data, and Transf.: Transformed Data.

Table 5. Hypothesis tests for the regressional model (Y_{1er}, Y_{2er}, and Y_{3er}) and significance of non-random variation.

Tests	Y_{1er}		Y_{2er}		Y_{3er}	
	s = 0.1	s = 0.2	s = 0.1	s = 0.2	s = 0.1	s = 0.2
*(H_0: CD vs H_1: TL)	(CD)	(CD)	(TL)	(TL)	(CD)	(CD)
LR statistic	13.22	14.50	18.96	41.48	–49.10	7.20
**(H_0: $\gamma = 0$ vs H_1: $\gamma > 0$)	Significant	Significant	Significant	Significant	Significant	Significant
LR statistic	153.1	184.32	53.63	105.40	83.03	156.44

*At the 5% level, (H_0: CD *vs* H_1: TL) test has $\chi_c^2 = \chi_{10}^2 = 18.31$ and (H_0: $\gamma = 0$ *vs* H_1: $\gamma > 0$) has $\chi_c^2 = \chi_1^2(10\%) = 2.71$.
**TL (TransLog): Quadratic model, and CD (Cobb-Douglas): Linear model.

Table 6. Estimates of the TL SF models' parameters for the responses Y_{1er}, Y_{2er}, and Y_{3er}.

Var.	Param.	Y_{1er}		Y_{2er}		Y_{3er}	
		s = 0.1	s = 0.2	s = 0.1	s = 0.2	s = 0.1	s = 0.2
Cte.	β_0	2.059	2.704	0.181	0.181	–2.457	–3.583
ln(x1)	β_1	2.914*	4.020*	0.235	0.235	–4.213	–4.785*
ln(x2)	β_2	0.123	0.219	–0.133	–0.133	–0.584	0.313
ln(x3)	β_3	2.614*	3.577*	0.563	0.563	–2.744	–4.437*
ln(x4)	β_4	–2.652*	–3.638*	–0.492	–0.492	3.572	4.867*
ln(x1)^2	β_5	–1.268	–1.515	–0.107	–0.107	0.972	1.739
ln(x1)*ln(x2)	β_6	–0.123	0.048	–0.006	–0.006	0.935	0.292
ln(x1)*ln(x3)	β_7	–4.000*	–5.789*	–0.585	–0.585	5.635	6.752*
ln(x1)*ln(x4)	β_8	2.680*	3.725*	0.546	0.546	–2.953	–4.483*
ln(x2)^2	β_9	0.063	–0.123	0.151	0.151	0.501	–0.449
ln(x2)*Ln(x3)	β_{10}	–0.204	0.045	0.019	0.019	–0.409	0.074
ln(x2)*Ln(x4)	β_{11}	–0.074	–0.232	–0.025	–0.025	–0.019	0.162
ln(x3)^2	β_{12}	–0.823	–1.448	–0.260	–0.260	0.957	1.746
ln(x3)*Ln(x4)	β_{13}	2.622*	4.161*	0.371	0.371	–3.478	–4.679*
ln(x4)^2	β_{14}	–1.675	–2.482*	–0.205	–0.205	2.102	2.343*
$\sigma^2 = \sigma_v^2 + \sigma_u^2$		$2.85.10^{-2}$	$0.76.10^{-2}$	$0.60.10^{-3}$	$0.32.10^{-2}$	2,54	$-0.37.10^{-1}$
γ		10^{-8}	10^{-8}	0.14	$0.16.10^{-7}$	0.98	10^{-8}
Log (likelihood)		34.72	36.73	84.32	52.04	–23.50	8.03

t-test critical value (5% level)=2.12.

Table 7. Composition of the uncertainty arrays for the outputs Y_{1er}, Y_{2er}, and Y_{3er}

Run (DMU)	Y_{1er}		Y_{2er}		Y_{3er}	
	$\varepsilon_e=(v_e-u_e)$		$\varepsilon_e=(v_e-u_e)$		$\varepsilon_e=(v_e-u_e)$	
	s=0.1	s=0.2	s=0.1	s=0.2	s=0.1	s=0.2
1	0.016	–0.050	0.004	0.745	–0.299	–0.011
2	0.016	0.058	0.025	0.600	–0.328	–0.094
3	0.001	0.031	0.011	0.521	–0.537	–0.053
4	–0.019	0.068	–0.024	0.829	–0.127	0.039
5	0.022	–0.165	–0.029	0.467	–0.766	0.125
6	0.031	0.102	–0.007	0.477	–0.392	0.023
7	–0.001	0.004	–0.015	0.516	–0.076	0.028
8	–0.016	0.073	–0.017	0.698	–0.651	0.017
9	–0.015	0.140	–0.016	0.618	–0.959	0.037
10	0.001	–0.050	–0.025	0.776	–1.128	0.064
11	–0.008	0.056	–0.018	0.436	–0.265	0.042
12	–0.014	0.105	–0.042	0.453	–0.189	0.177
13	–0.014	0.125	0.014	0.558	–1.098	–0.048
14	–0.023	0.130	–0.007	0.559	–0.435	0.033
15	0.019	0.027	0.010	0.776	–0.414	–0.028
16	0.025	–0.079	–2.608	0.537	–0.532	–0.022
17	0.037	0.028	–2.608	0.597	–0.590	0.011
18	0.013	–0.152	–2.677	0.664	–0.426	–0.020

Table 8. Global error as per signal combination for the outputs Y_{1er}, Y_{2er}, and Y_{3er}.

Run (DMUi)	Sum (Abs(ε_i))							
	[S_1, S_1, S_1]	[S_2, S_1, S_1]	[S_1, S_2, S_1]	[S_2, S_2, S_1]	[S_1, S_1, S_2]	[S_2, S_1, S_2]	[S_1, S_2, S_2]	[S_2, S_2, S_2]
1	0.320	0.353	1.061	1.095	**0.032**	0.065	0.773	0.807
2	0.370	0.412	0.944	0.986	0.136	0.177	0.710	0.752
3	0.548	0.578	1.058	1.088	0.064	0.094	0.574	0.604
4	0.170	0.220	0.975	1.024	0.082	0.132	0.887	0.937
5	0.817	0.959	1.256	1.398	0.176	0.318	0.615	0.757
6	0.430	0.501	0.900	0.971	0.061	0.132	0.531	0.602
7	**0.091**	0.095	0.592	0.596	0.044	0.047	0.545	0.549
8	0.685	0.742	1.366	1.423	0.051	0.108	0.732	0.789
9	0.990	1.116	1.592	1.717	0.068	0.193	0.670	0.795
10	1.154	1.203	1.905	1.954	0.090	0.139	0.841	0.890
11	0.291	0.339	0.708	0.757	0.067	0.116	0.485	0.534
12	0.245	0.335	0.655	0.746	0.233	0.323	0.643	0.734
13	1.125	1.236	1.669	1.780	0.075	0.186	0.619	0.730
14	0.464	0.572	1.017	1.124	0.063	0.171	0.615	0.723
15	0.442	0.451	1.209	1.217	0.057	0.065	0.823	0.832
16	0.561	0.614	1.095	1.148	0.051	0.105	0.585	0.639
17	0.629	0.620	1.224	1.214	0.050	**0.040**	0.644	0.635
18	0.440	0.579	1.102	1.241	0.034	0.173	0.697	0.836
Min	**0.091**	**0.095**	**0.592**	**0.596**	**0.032**	**0.040**	**0.485**	**0.534**
Exp(Min)	1.096	1.100	1.808	1.815	1.032	1.041	1.624	1.705

$\chi_c^2 = \chi_1^2(10\%) = 2.71$. Therefore, the RSM model does not confound with the stochastic frontier for all outputs Y_{1er}, Y_{2er}, and Y_{3er}.

The regressional models for the process responses, Y_{1er}, Y_{2er}, and Y_{3er} are provided in Table 6.

Step 3 revised (constitute the uncertainty arrays for the outputs Y_{1er}, Y_{2er}, and Y_{3er}): Using $\varepsilon_i = (v_i - u_i)$ for each run, i, the uncertainty arrays for the dynamic outputs, Y_{1er}, Y_{2er}, and Y_{3er} at the signal levels $s = 0.1$ and $s = 0.2$ are obtained. Table 7 shows the uncertainty scores (ε_i) as generated by the FRONTIER® 4.1 program.

There are six uncertainty arrays in total since the uncertainty arrays are arranged based on the signal level for each output. The global error at each run, i, of the L18 layout is calculated by totting the uncertainty scores over the factorial combinations of the signal levels for the outputs Y_{1er}, Y_{2er}, and Y_{3er} as shown in Table 8. It represents the number. of applications from the set of $\{Y_{1er}, Y_{2er}, \text{and } Y_{3er}\}$ to the set of two signal levels $\{s = 0.1$ and $s = 0.2\}$, i.e., 2^3 combinations. Because the purpose is to determine the control and signal factor settings, which produce the least amount of variation in magnitude rather than direction, the absolute values of the uncertainty scores are considered.

The global uncertainty of 0.986 (line 2 in Tab. 8), for instance, is obtained when summing up the absolute errors in run 2 while setting Y_{1er} at $s = s_2 = 0.2$ ($\varepsilon_2 = |0.058|$), Y_{2er} at $s = s_2 = 0.2$ ($\varepsilon_2 = |0.600|$), and Y_{3er} at $s = s_1 = 0.1$ ($\varepsilon_2 = |-0.328|$). According to Table 8, when the signal factor is set at $s = 0.1$ for Y_{1er} and Y_{2er} and $s = 0.2$ for Y_{3er}, respectively, the minimum global error of 0.032 is met. This corresponds to run 1 ($x1 = x2 = x3 = x4 = 1$), which is deemed as a robust optimization solution. Another workable solution is run 17 ($x1 = 3$, $x2 = 2$, $x3 = 1$, and $x4 = 3$), which has a global uncertainty score of 0.040. In certain operational scenarios, it is often advised to consider the same signal sitting for all outputs. In this scenario, run 7 ($x1 = 3$, $x2 = 1$, $x3 = 2$, and $x4 = 1$) with a signal setting of $s = 0.1$ for Y_{1er}, Y_{2er}, and Y_{3er} is the proper alternative. The global uncertainty error is 0.091.

5 Conclusion

The RDPP-SF approach for dynamic systems is covered in this article. The method's novelty may be seen in how the stochastic frontier composed error is decomposed, i.e., random and nonrandom components. So, the control of dynamic processes, which are vulnerable to environmental noise, unseen control factors, and violation of the DoE premises such as the randomization, replication, and blocking principles is affordable. A major application of the static and dynamic RDPP-SF method is estimating the design and manufacturing process maturity. This is performed using the test on the γ-value (H0: $\gamma = 0$ *vs.* H1: $\gamma > 0$). So, at a 5% level, for instance, if $\gamma \leq 5\%$, the process is mature since it only experiences random variance and may be tested for long-term statistical process control. If the γ-value falls between 5% and 95%, then, the sensitivity is marginal, and both natural and non-natural sources of variation are accountable for. If the $\gamma \geq 95\%$, the nonrandom sources of variation are predominating, meaning the process is immature and should be improved/redesigned. The presence of non-random variation is the main cause of short-term drift and bias in a process. The additional advantages and limitations of the DRDPP-SF are outlined below.

a. The RSM optimization technique, which uses the statistical Ordinary Least Square method, yields a constant uncertainty estimate. On the other hand, the stochastic frontier model provides a variable (and often better) estimate of the uncertainty; i.e., each pairing of the input and signal parameters has its uncertainty estimate.
b. The stochastic frontier model is parametric, uses flexible functional models, can estimate the standard errors, and makes use of the hypotheses tests to ascertain the statistical significance of the non-natural variation using the Maximum Likelihood Method. In that regard, it is superior to other frontier-based methods such as Data Envelopment Analysis [19], which incorporates noise as part of the efficiency score.
c. The proposed DRDPP-SF method can provide the link between robustness and the signal-to-noise metric, which is used in many robust design and reliability methods. The estimate of the γ-value in the DRDPP-SF method can be used to do this. Furthermore, the method holds the most promise for engineering fields, such as reliability, versatility, resilience, adaptability, and flexibility.
d. The DRDPP-SF method uses a Translog as a transfer function to account for the interactions among the control factors. Nevertheless, more research is still needed to figure out other functional forms that are more suitable to manufacturing processes.
e. The DRDPP-SF technique has considered robust optimization design of processes but in discrete space. Research using Latin hypercube sampling and ANN-GA algorithm is launched to address optimization in continuous space.

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Conflicts of interest

The authors declare no potential conflicts of interest concerning the research, authorship, and/or publication of this article.

Data availability statement

The data that support the findings of this study are available from the corresponding second author (Mohamed-Ali Rezgui) upon reasonable request.

Author contribution statement

Conceptualization, A.T. and M.A.R.; Methodology, A.T. and M. A.R.; Software, A.T., M.A. and A.D.; Validation, A.T. and A.D.;

Formal Analysis, A.T. and M.A.R.; Investigation, A.T. and M.A. R.; Resources, A.T. and M.A.R.; Data Curation, A.T., M.A.R.; Writing – Original Draft Preparation, A.T. and M.A.R.; Writing – Review & Editing, A.T., M.A.R. and J.H.; Visualization, A.T., M.A.R. and H.J.; Supervision, A.T. and M.A. and H.J.; Project Administration, M.A.R. and H.J.; Funding Acquisition, H.J. All authors have read and agreed to the published version of the manuscript.

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