

# Wavelet-based high order spectrum for local damage diagnosis of gears

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Received: 10 March 2024 / Accepted: 20 September 2024

**Abstract.** Gears play an important role in transmission systems, allowing for high performance in terms of load capacity and efficiency. Common failures, such as spalling and pitting of gear teeth, can occur as a result of contact fatigue, excessive load, or sudden impact. Starting from an initial stage, their steady growth can lead to irreparable damage and unexpected breakdowns. Therefore, condition monitoring of gears is critical for smooth operation of drivelines and transmissions. Local tooth damage produces transient impacts in the vibration signals, exhibiting non-linear characteristics. Taking into account its capacity to characterize the phase coupling between signal components caused by non-linearities, wavelet-based high order spectrum is considered to be effective to attain reliable fault-related features. Among others, wavelet bicoherence has been successfully estimated to detect artificially created gear faults. However, a number of challenges still remain in selecting the most informative bi-frequency bands and extracting instantaneous diagnostic features. Moreover, the adaptability of the technique in identifying natural defects on multiple teeth still needs further investigation. This paper presents a novel strategy for selecting informative bi-frequency bands and extracting instantaneous diagnostic features in the time bi-frequency domain. The performance of the proposed methodology is applied, evaluated and compared with a state-of-the-art method on scenarios that include diagnosis of single-tooth and multiple-teeth damage. To validate its effectiveness, the methodology is tested using three datasets: two featuring artificially induced pitting damage on one gear tooth, and a third one containing naturally developed spalling defects on two gear teeth.

**Keywords:** Condition monitoring / gear diagnosis / wavelet-based high order spectrum / vibrations

## 1 Introduction

Gearboxes are widely used across numerous industries for power transmission. However, the occurrence of unforeseen gear failures, such as local teeth defects like pitting, spalling, and cracks, can lead to costly breakdowns and considerable economic losses. Consequently, implementing condition monitoring is crucial to ensure operational safety and reduce costs. Over the past several years, vibration-based diagnosis has emerged as a robust and effective tool [1–3].

When local damage such as pitting occurs on a gear tooth, it results in transient impulses each time the damaged surface meshes with other teeth [4]. The vibration signals generated during the meshing process of a gear tooth with local tooth damage exhibit characteristics of non-linearity [5]. As a result, conventional linear

representations, such as the Short-Time Fourier Transform (STFT) and the Continuous Wavelet Transform (CWT), prove less effective in capturing the complex nature of these signals.

The application of high-order spectra, such as bispectrum and bicoherence, has proven successful in identifying phase coupling and nonlinear behavior [6] across different disciplines, including ocean engineering [7] and biomedical engineering [8]. These techniques have been also introduced in the domain of condition monitoring and fault diagnosis of mechanical systems. For instance, Rivola et al. [9] employed the normalized bispectrum to detect cracks in beams, presenting high sensitivity to fatigue cracks. Bicoherence was applied in [10] to analyse vibration and acoustic signals acquired from a gearbox with artificially created broken teeth, yielding reliable fault-related information. Additionally, the combination of wavelet transform and bicoherence analysis, known as wavelet

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bicoherence (WB), has been proposed and utilized. More specifically, WB was employed in [11] for the detection of naturally-developing gear faults, where the integrated band was chosen by comparing WB maps at different stages. To enhance accuracy, Li et al. [12] incorporated a biphas randomization step in WB and extracted two diagnostic features for bearing diagnostics of inner race defects. Nevertheless, at the abovementioned papers the WB was estimated by integrating over finite-time intervals, which may result in information loss in the time domain. To address this issue, the concept of instantaneous wavelet bicoherence/bispectrum (IWBC/IWBS) was introduced, enabling analysis in the time-bi-frequency domain. One notable application can be found in [5], where IWBC was employed for detecting multiple ‘like natural’ pitting faults in a back-to-back industrial spur gearbox system. Furthermore, to preserve phase information, the instantaneous biphas randomization wavelet bicoherence was further developed in [13], successfully detecting chipped gear teeth and broken teeth.

However, the selection of informative bi-frequency bands to extract instantaneous diagnostic features in the time-bi-frequency domain remains an open challenge. Currently, researchers mainly rely on information provided by the scalogram of CWT [5], or make comparisons between healthy and faulty cases to select the informative bi-frequency band [11]. For example, the selection of the integration band in [5] involves analyzing the scalogram of the residual signal extracted from a pitted gear case, where the 200–400 order range was selected due to the presence of strong peaks. However, this method may not be convenient or accurate in certain scenarios. Thus, there is no mature approach for selecting/optimizing the informative bi-frequency bands specifically related to faults in the time-bi-frequency domain in order to better extract instantaneous diagnostic features. Additionally, the application and evaluation of the performance and effectiveness of these techniques in scenarios involving defects on multiple teeth are still underexplored.

The primary objective of this paper is to propose a novel diagnostic feature, the Enhanced Instantaneous Wavelet Bispectrum Feature (EIWBSF), for gear diagnostics. This is achieved by initially extracting the residual signal from the raw vibration signal, which involves removing the dominant periodic components, such as the mesh harmonics. Subsequently, the instantaneous wavelet bispectrum is estimated on the residual signal. An optimization procedure is then developed to assign weights to the informative frequency band, which are integrated to obtain the diagnostic feature. The validation of the proposed methodology employs three datasets. The first and the second one consist of vibration signals acquired from two experimental setups at KU Leuven, targeted at single-tooth damage diagnostics. The third dataset, which is publicly available, contains naturally developed spalling defects on two gear teeth,

intended for multiple-teeth damage diagnostics. The performance of the methodology is compared with the classical Wavelet-based Instantaneous Feature (WIF) method [14–16]. The rest of the paper is organized as follows. In Section 2, the theoretical background and the EIWBSF methodology are described. In Section 3 the proposed EIWBSF is applied on two datasets and the results are discussed. The paper’s conclusions are then summarized in Section 4.

## 2 Theoretical background and proposed methodology

### 2.1 Basic signal processing theory

Wavelet-based higher-order spectra rely on the utilization of CWT to process signals. The CWT is a commonly used technique for characterizing the properties of non-stationary signals in the time-frequency domain. The CWT of a signal  $x(t)$  can be calculated by:

$$W_{\psi}(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t - \tau}{a} \right) dt, \quad (1)$$

where  $\psi(t)$  is the mother wavelet function,  $a$  is the scale,  $\tau$  is the time shift variable, and  $*$  represents the complex conjugation form. The scale  $a$  can be converted to frequency  $f$  using the equation  $a = \frac{f_0}{f}$ , where  $f_0$  is the frequency of the mother wavelet function [5]. The bispectrum, defined as a higher-order spectrum, has been successfully applied to analyze the nonlinear components present in the signals with high accuracy in short duration [17]. It is calculated as a triple product cumulant:

$$B_{xx}(f_1, f_2) = E\{X(f_1)X(f_2)X^*(f_3)\}, \quad (2)$$

where  $X(f)$  represents the Fourier transform of the signal. The variables  $f_1$ ,  $f_2$  and  $f_3$  denote three different frequency components.  $E\{\cdot\}$  denote the expectation operator and  $*$  represents the complex conjugation.

Analogous to the bispectrum, the wavelet bispectrum (WBS) was initially proposed and applied in the field of turbulence analysis to investigate nonlinear interactions. It offers the advantage of preserving temporal information. The WBS is defined as follows [18]:

$$B_{W,T}(f_1, f_2) = E \left\{ \int_0^T W_{\psi}(f_1, \tau) W_{\psi}(f_2, \tau) W_{\psi}^*(f_3, \tau) d\tau \right\}, \quad (3)$$

where  $W_{\psi}(f, \tau)$  is the wavelet coefficient calculated by the CWT of the signals,  $T$  is the time interval of the signal and  $E\{\cdot\}$  is the expectation operator. Due to the expectation operator, equation (3) involves ensemble average, which will be explained in detail in the step 5 of the next section. Additionally, the frequencies in equation (3) satisfy the relationship  $f_3 = f_1 + f_2$ .

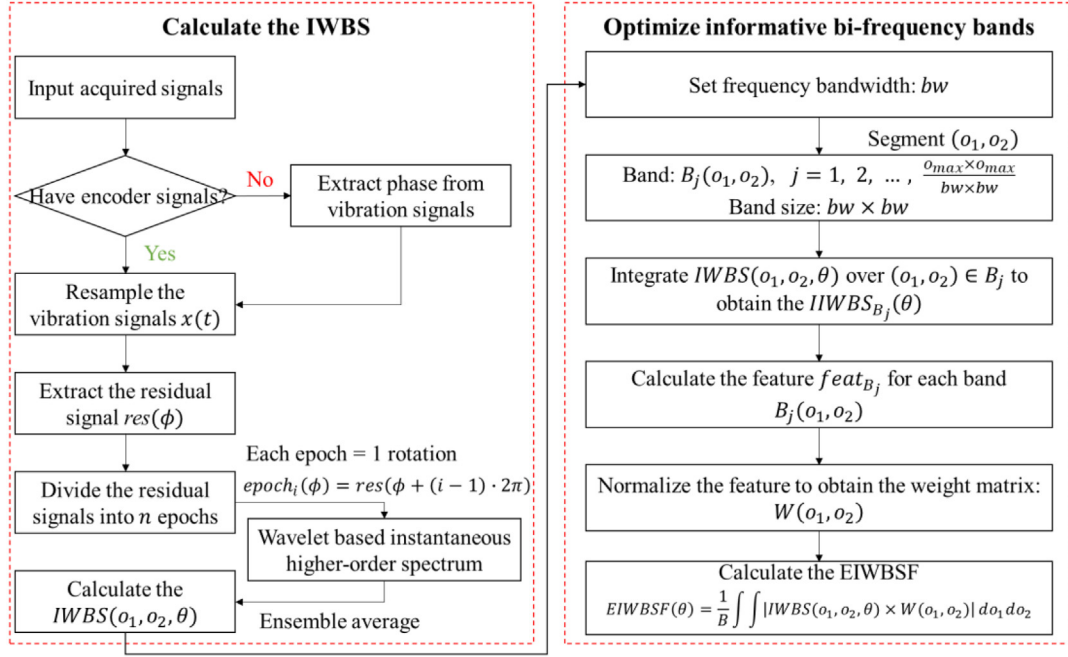


Fig. 1. Schematic description of the EIWBSF.

Considering that the values of WBS are complex, WBS can be expressed in the form of amplitude  $Amp(f_1, f_2)$  and biphas  $\phi(f_1, f_2)$  as following:

$$B_{W,T}(f_1, f_2) = E\{Amp(f_1, f_2)e^{i\phi(f_1, f_2)}\}, \quad (4)$$

$$\phi(f_1, f_2) = \phi(f_1) + \phi(f_2) - \phi(f_3), \quad (5)$$

where,  $\phi(f_1)$ ,  $\phi(f_2)$ , and  $\phi(f_3)$  are respectively the phases of the frequencies  $f_1, f_2, f_3$

If a phase coupling exists in the signal, the phase component will satisfy the relationship:

$$\phi(f_3) = \phi(f_1) + \phi(f_2), \quad (6)$$

and therefore the biphas  $\phi(f_1, f_2)$  will equal to 0. Thus  $B_{W,T}(f_1, f_2) = E\{Amp(f_1, f_2)\}$ , and the normalized version of the wavelet bicoherence (WBC) will be equal to 1.

In the case of non-phase coupling in the signal, the biphas  $\phi(f_1, f_2)$  is normally randomly distributed within  $(-\pi, \pi]$ , which means that  $e^{i\phi(f_1, f_2)}$  is distributed within  $(-1, 1]$ . Consequently, after taking the expectation operator,  $B_{W,T}(f_1, f_2)$  tends to approach 0. As a result, the WBC approximates to 0. Due to the fact that WBS and WBC need to be integrated over a finite-time interval, which may bring time information loss of the nonstationary signal, the IWBS has been proposed [5]. The IWBS aims to extend the bi-frequency domain to the time bi-frequency domain and is defined as follows:

$$IWBS_{W,T}(f_1, f_2, \tau) = E\{W_\psi(f_1, \tau)W_\psi(f_2, \tau)W_\psi^*(f_3, \tau)\}. \quad (7)$$

The IWBS can be further expressed by its instantaneous amplitude and biphas components, similar to the WBS, following the same rule for detecting the phase coupling and non-phase coupling part in the signals.

## 2.2 Proposed methodology

As mentioned before, the selection/optimization of informative bi-frequency bands to extract instantaneous diagnostic features in the time-bi-frequency domain is still challenging. To address this, the EIWBSF is proposed in this paper and presented in Figure 1. The steps to estimate the EIWBSF are detailed below:

**Step 1:** The raw vibration signal  $x(t)$  is input and resampled in the angular domain to obtain the  $x(\varphi)$ . This can be performed by using the speed signal of an encoder if available, or equally using a methodology based on the phase demodulation of a mesh harmonic of the vibration signal [19], [20].

**Step 2:** The periodic part is estimated and removed from the vibration signals to obtain the residual signal  $res(\varphi)$ . The impacts produced by small gear faults are normally quite weak and may be masked by the strong periodic components (e.g. mesh harmonics). Consequently, removing the periodic components is a common practice for analyzing the impacts of small gear faults. The implementation of residual signals can be realized using Fast Fourier Transform and Inverse Fast Fourier Transform [21].

**Step 3:** The residual signal  $res(\varphi)$  is divided into  $n$  epochs based on the rotating period of the target gear. Each epoch equals to one rotation period of the gear:

$$epoch_i(\varphi) = res(\varphi + (i - 1) \cdot 2\pi), \quad \varphi \in [0, 2\pi) \text{ and } i = 1, 2, \dots, n. \quad (8)$$

**Step 4:** The IWBS is computed for each epoch using equations (1) and (7). When using equation (1), it should be noted that since the signal has been resampled and represented in the angular domain, instead of the time-frequency plane  $W_\psi(f, \tau)$ , the obtained result is in the angular-order  $W_\psi(o, \theta)$ . In this representation, the frequency is converted to order  $o$ , and the time shift variable  $\tau$  is transformed into the angular shift variable  $\theta$ . Therefore, the IWBS can be expressed as follows:

$$IWBS(o_1, o_2, \theta) = E\{W_\psi(o_1, \theta)W_\psi(o_2, \theta)W_\psi^*(o_3, \theta)\}. \quad (9)$$

**Step 5:** In equation (9), the use of the expectation operator  $E\{\cdot\}$  necessitates an ensemble average. As suggested in [5,22], a cyclostationary hypothesis assumption is made, meaning that the statistical properties of the acquired signals are supposed to vary periodically in time. Specifically, for gears, this period is considered as the rotation period of the gear. Consequently, the expectation operator means the ensemble averaging over the gear rotation period. Therefore, the  $IWBS(o_1, o_2, \theta)$  is obtained by taking the ensemble average of the IWBS of  $n$  epochs.

**Step 6:** The obtained  $IWBS(o_1, o_2, \theta)$  has three variables, the bi-frequency plane in orders  $(o_1, o_2)$  and the angle  $(\theta)$ . The bi-frequency plane  $B(o_1, o_2)$  is segmented into a series of bands  $B_j(o_1, o_2)$ , which have the band size of  $bw \times bw$ , where  $bw$  represents the bandwidth. A smaller  $bw$  increases the precision of the weight matrix calculated in Step 9 but at the cost of increased computational time. Conversely, a broader  $bw$  decreases the computational time but may reduce the detail in the weight matrix. This study recommends setting the  $bw$  relative to the order resolution, provided that the computational efficiency remains acceptable.

**Step 7:** In each bi-frequency band  $B_j$ , the  $IWBS(o_1, o_2, \theta)$  modulus is integrated over both  $o_1$  and  $o_2$  and the  $IIWBS_{B_j}(\theta)$  is calculated by equation (10):

$$IIWBS_{B_j}(\theta) = \frac{1}{B_j} \iint_{B_j} |IWBS(o_1, o_2, \theta)| do_1 do_2, \quad (10)$$

$$feat_{B_j} = \frac{\int_{\theta_{max}-\Delta\theta/2}^{\theta_{max}+\Delta\theta/2} IIWBS_{B_j}(\theta) d\theta}{\int_{\theta_0}^{\theta_{end}} IIWBS_{B_j}(\theta) d\theta - \int_{\theta_{max}-\Delta\theta/2}^{\theta_{max}+\Delta\theta/2} IIWBS_{B_j}(\theta) d\theta}. \quad (11)$$

**Step 8:** The feature  $feat_{B_j}$  of each processed  $IIWBS_{B_j}(\theta)$  is extracted using equation (11). The feature is based on the assumption that if a gear has a damaged tooth, then in one rotation period of the gear, there will be a high-amplitude impulse during the time interval when the damaged tooth meshes other teeth. Therefore, the numerator of equation (11) calculates this high-amplitude impulse by searching for the maximum value of  $IIWBS_{B_j}(\theta)$  and integrating around the position of the maximum value  $\theta_{max}$  with the interval of  $\Delta\theta$  ( $\Delta\theta$  can be set via dividing one rotation by the tooth number). The denominator of equation (11) is calculated by integrating  $IIWBS_{B_j}(\theta)$  over the full range of  $\theta$  and then subtracting the numerator. Therefore the denominator is considered as the noise level.

**Step 9:** The  $feat_{B_j}$  is then normalized between 0 and 1 to generate a weight matrix as shown in equation (12). Within the  $W(o_1, o_2)$ , each bi-frequency has a weight value.

$$W(o_1, o_2) = \frac{feat_{B_j(o_1, o_2)} - \min(feat_{B_j(o_1, o_2)})}{\max(feat_{B_j(o_1, o_2)}) - \min(feat_{B_j(o_1, o_2)})}. \quad (12)$$

**Step 10:** The final step is to weight the  $IWBS(o_1, o_2, \theta)$  based on the weight matrix and to integrate the weighted  $IWBS(o_1, o_2, \theta)$  modulus over both frequencies  $o_1$  and  $o_2$  in the full frequency range to obtain the EIWSF:

$$EIWSF(\theta) = \frac{1}{B} \iint |IWBS(o_1, o_2, \theta) \cdot W(o_1, o_2)| do_1 do_2. \quad (13)$$

## 3 Application of the methodology and analysis of the results

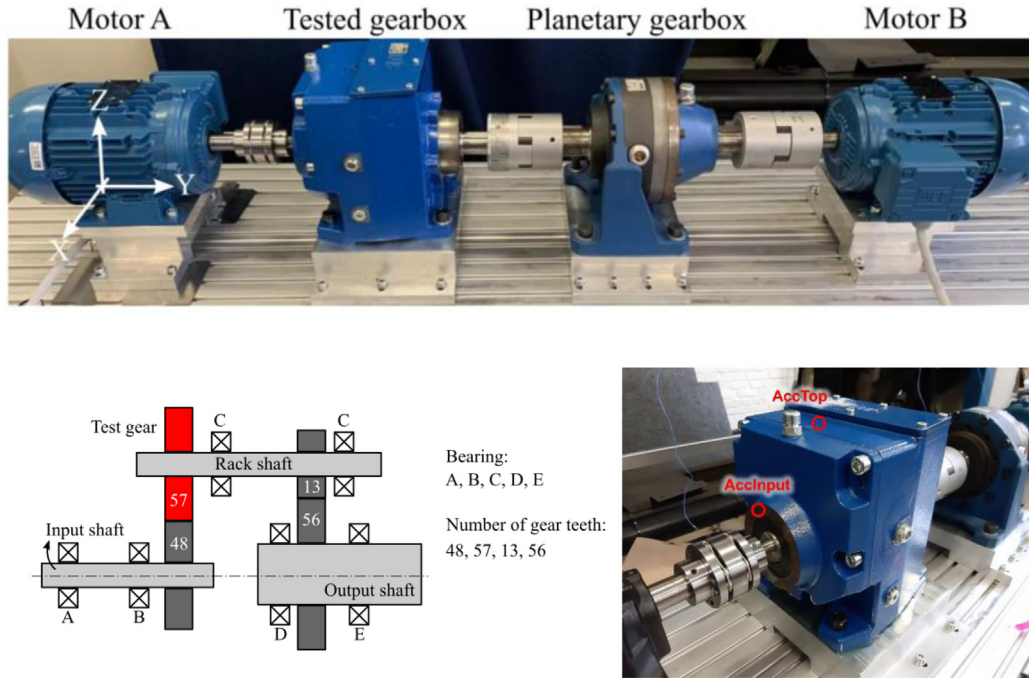
### 3.1 Case 1—Single-tooth damage diagnostics

In the first case study of this paper, experiments were conducted using a back-to-back gearbox setup at KU Leuven. The experimental setup consists of several components, including an electric motor A, the test gearbox, a planetary gearbox, and a loading electric motor B, as shown in Figure 2 (Up). The kinematic diagram of the test gearbox is shown in Figure 2 (Left), providing the gear information. Speed signals are obtained by an encoder installed on the back side of motor A, while two accelerometers (PCB 352A24) are placed respectively on the top surface of the test gearbox and on the input shaft side, as depicted in Figure 2 (Right). Both motors are induction motors of 1.5 kW power with rated load and speed of 5 Nm and 2880 rpm, respectively.

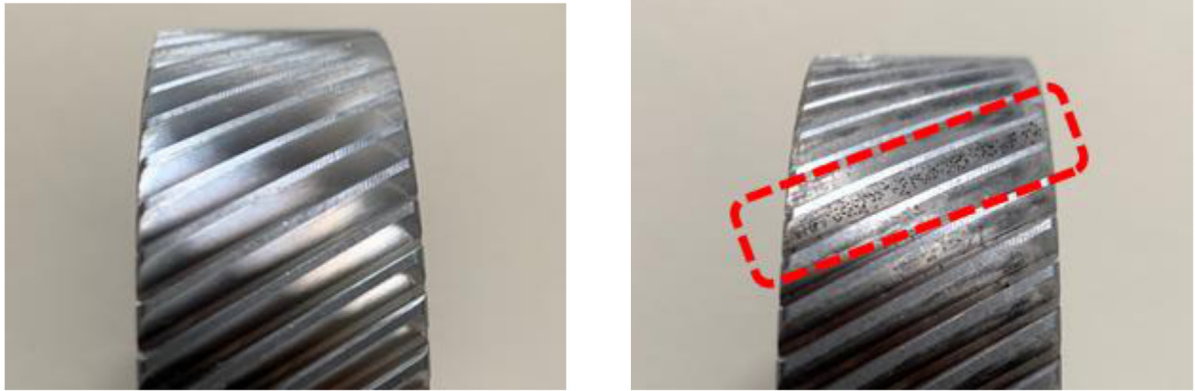
Healthy and faulty gears are tested on the setup (Fig. 3). The faulty gear has one tooth with 50% pitting damage which is artificially created using electrical discharge machining (EDM). Specifically, a preselected number of pits with a diameter of 250  $\mu\text{m}$  were induced on the tooth surface, collectively occupying 50% of the gear tooth area. During the tests, two rotational speeds (speed of the motor A that is equal to the input shaft speed of the test gearbox) have been considered, 1620 rpm ( $\sim 27$  Hz) and 2200 rpm ( $\sim 37$  Hz). Additionally, the torque applied by motor B can be estimated by the torque percentage (the ratio of the torque developed by the motor and the nominal torque) from the drive, which is around 5 Nm. The signals were sampled at a sampling frequency of 51200 Hz and the duration of each acquisition was 5 s. More details can be found in [15,16].

For the case of speed 1620 rpm, raw vibration signals acquired from the accelerometer that is positioned close to the input shaft of the gearbox are used for analysis. The raw signals under healthy and faulty conditions are presented in Figure 4 (Left). Besides, the residual signals extracted from the resampled signals by removing the





**Fig. 2.** (Up) The test rig setup, (Left) the kinematic diagram of the test gearbox, (Right) the accelerometers' position.

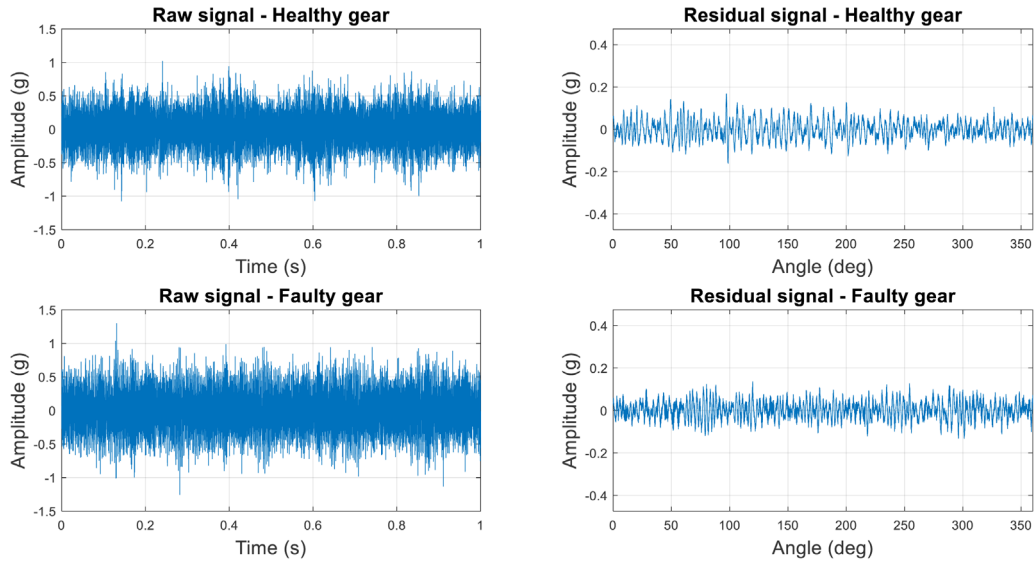


**Fig. 3.** (Left) Photo of the healthy gear, (Right) photo of the faulty gear.

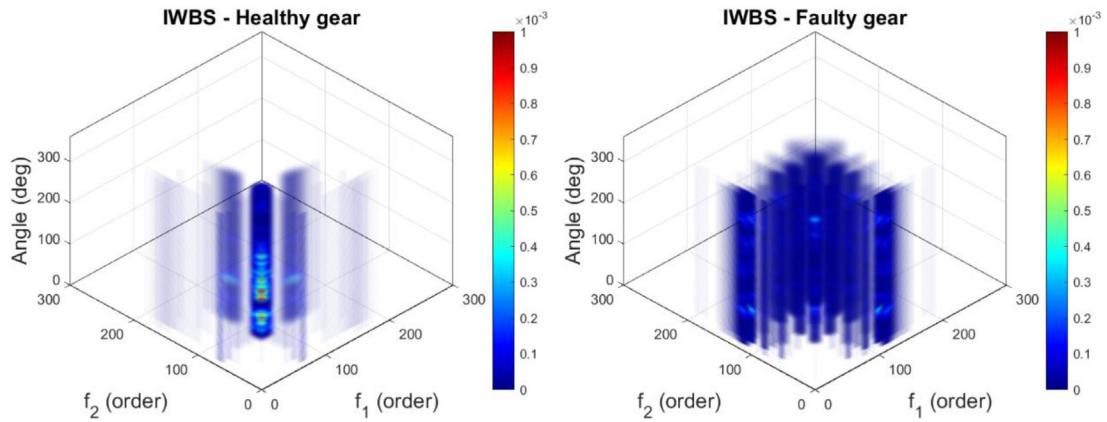
periodic components are shown in Figure 4 (Right). No apparent differences can be observed between the raw signals or the residual signals, as the fault information related to the impact produced by the tooth damage is very weak and is easy to be masked by the background noise.

Following the procedure described in Section 2, the IWBSs of the healthy gear and the faulty gear are obtained by taking the ensemble average on the IWBSs of the residual signals of all revolutions. As shown in Figure 5, the detected phase coupling of frequencies mainly exist within 200 orders. Comparing with the IWBS of the healthy gear, it is evident that the IWBS of the faulty gear exhibits a greater number of coupled frequencies in the time bi-frequency domain. This increase in coupled frequencies is attributed to the presence of pitting damage on the contact surface of the helical gear tooth.

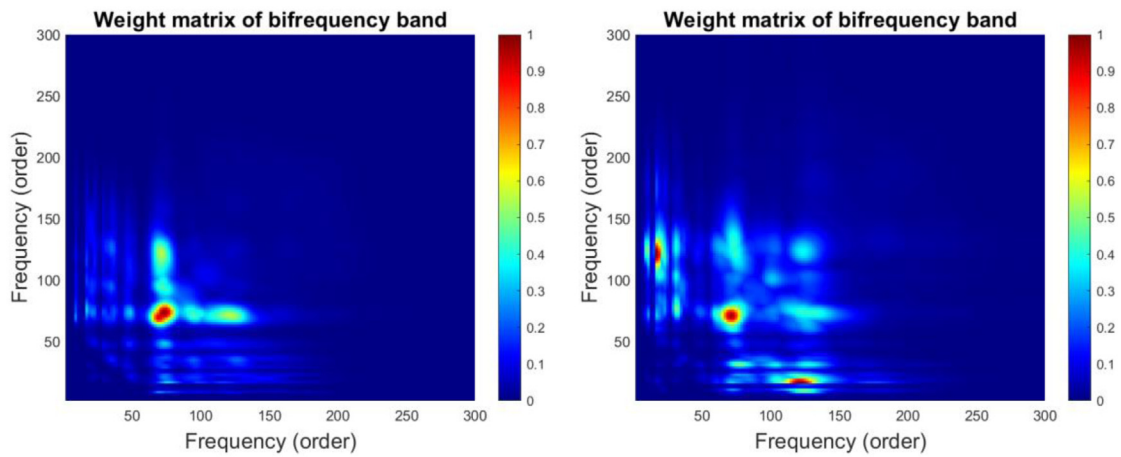
Considering the computational efficiency and the accuracy, the bandwidth was set based on the frequency resolution of the IWBS, thereafter the feature  $feat_{Bj(f1,f2)}$  of each band was extracted, and the weight matrices of the bi-frequency band of the healthy and the faulty gear are displayed in Figure 6. The highest weight values can be observed at around [70 order, 70 order] for both healthy and faulty gears, corresponding to the sum of the meshing orders of the two stages, which are the 57 order and the 13 order respectively. By comparison, in the case of the faulty gear condition, additional high weight values can be observed in a different region, specifically within the range of 100–150 order and 10–30 order. The presence of high weight values in the specific frequency region for the faulty gear case indicates that these regions are related to the pitting damage. This finding highlights the capability of the proposed feature to identify the fault-related informative band.



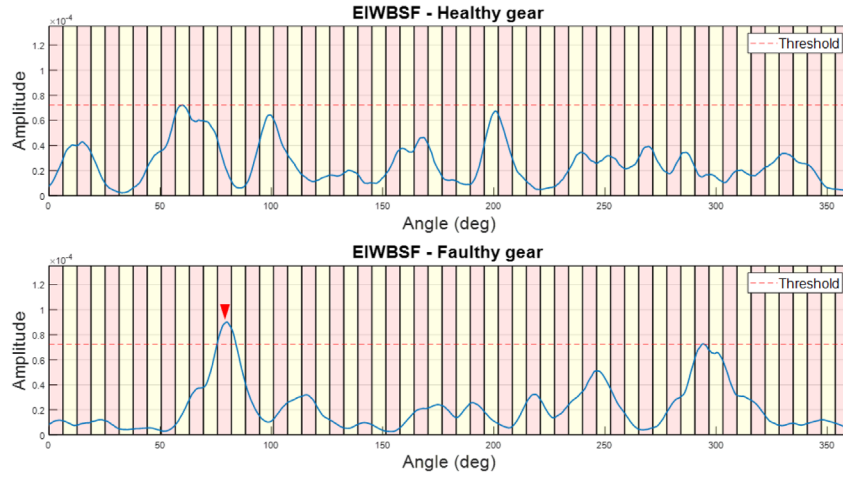
**Fig. 4.** (Left) Raw vibration signals, (Right) residual signals of the healthy and faulty gears for 1620 rpm.



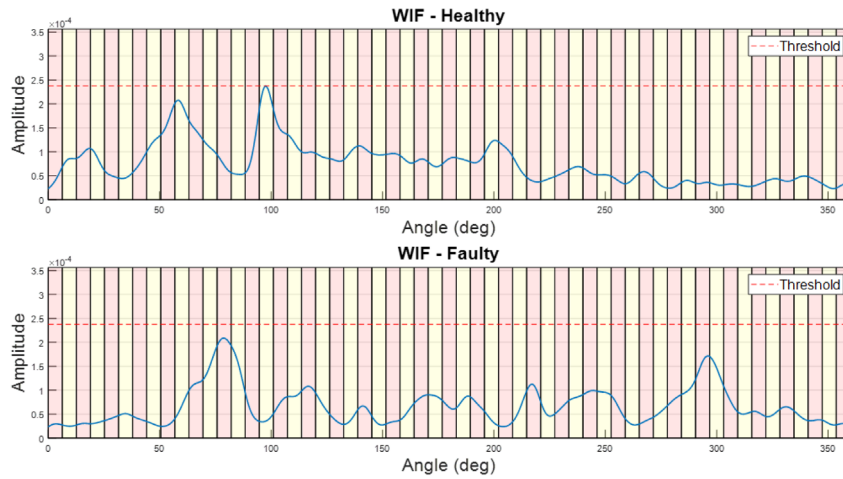
**Fig. 5.** (Left) IWBS of the healthy gear, (Right) IWBS of the faulty gear for 1620 rpm.



**Fig. 6.** Weight matrix of IWBS (Left) of the healthy gear, (Right) of the faulty gear for 1620 rpm.



**Fig. 7.** The EIWBSF of the healthy and the faulty gear for 1620 rpm.



**Fig. 8.** The WIF of the healthy and the faulty gear for 1620 rpm.

Based on the results of Figures 5 and 6, the EIWBSF is extracted using equation (13). The corresponding EIWBSF results are presented in Figure 7, where the X-axis represents the angular position of the gear, and the Y-axis represents the amplitude of the new feature. For better evaluation, the maximum EIWBSF value of the healthy gear is set as the threshold to check if the faulty gear tooth can be detected. The vertical sections presented in the plots are drawn based on the assumption that there is no random fluctuation during the meshing process, thus, one rotation is divided into several intervals, each one corresponding to a specific gear tooth. From the results, it is clear that the EIWBSF of the faulty gear exceeds the threshold within a specific angle interval with the peak in the middle of the interval, which is around 80 deg.

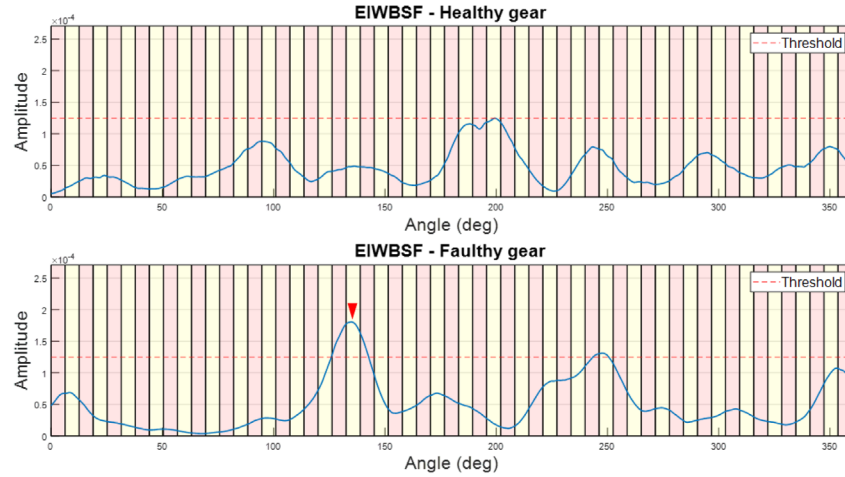
In order to evaluate the proposed EIWBSF, it is compared with the WIF results presented in [16]. The threshold of the WIF is selected in the same way as in the EIWBSF case. From the results shown in Figure 8, the WIF of the faulty gear is always lower than the threshold extracted from the healthy gear, and it is concluded that

the WIF methodology does not provide any valuable diagnostic information in this case, and consequently it cannot detect the pitting defect on the faulty gear.

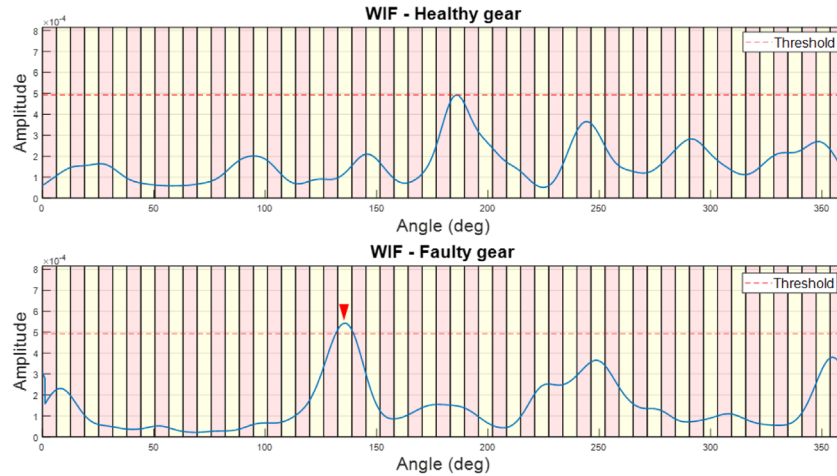
Next, the case of operating speed at 2200 rpm is investigated. Figures 9 and 10 illustrate the WIF and the EIWBSF results obtained for both the healthy and faulty gears. The results indicate that both methodologies are capable of detecting the faulty gear tooth, as the maximum amplitude value of the diagnostic features exceed the threshold and peaked at around 135 deg. This is reasonable, as due to the higher speed conditions, the impulses intensity caused by the mesh of the pitted tooth become stronger, which consequently makes it easier to successfully detect the pitted tooth.

### 3.2 Case 2—Single-tooth damage diagnostics

A second dataset is employed for single tooth damage detection to provide more concrete validation. Measurements were conducted on a one-stage gearbox setup at KU Leuven. The setup consists of several components, including an electric motor A, a torque sensor, the test



**Fig. 9.** The EIWSF of the healthy and the faulty gear for 2220 rpm



**Fig. 10.** The WIF of the healthy and the faulty gear for 2220 rpm

gearbox, a drive gearbox, and a loading electric motor B, as shown in Figure 11. In the measurements, the gear on the output shaft of the test gearbox is tested with both healthy and faulty gears. The faulty gear has one tooth with pitting damage, which was artificially created using a Dremel tool, as shown in Figure 12.

During the tests, the position of the faulty gear tooth is defined by its relationship to the zebra tape installed on the output shaft. Specifically, a ‘butt joint’ is created where the two ends of the zebra tape overlap, resulting in an uneven distribution of the stripes over the shaft. This often leads to a variation in the angle between subsequent pulse moments, as illustrated in the Figure 13. Consequently, the pulse moment generated by the ‘butt joint’ acts as a reference point. If this is established as the starting point for each revolution in the analysis, it becomes only necessary to verify whether the position of the detected peak aligns with the angular difference between the faulty gear tooth and the ‘butt joint’.

During the measurements, signals were sampled at a sampling frequency of 102,400 Hz with the acquisition duration of 10 s. A load corresponding to 75% of the test

gearbox nominal load (2 Nm) was applied, and the input shaft’s rotating speed was 2500 rpm. Raw vibration signals, acquired from the accelerometer positioned near the output shaft of the gearbox, are used for analysis. The raw signals under both healthy and faulty conditions are presented in Figure 14. No apparent differences can be observed between the raw signals.

The results of EIWSF and WIF for both the healthy and faulty gears are displayed in Figure 15 (Left) and (Right). The X and Y axes represent the angular position of the gear and the amplitude of the feature respectively. Similar to the previous case, the maximum feature value of the healthy gear is established as the threshold. The results demonstrate that both EIWSF and WIF for the healthy gear show significantly lower amplitudes compared to those of the faulty gear. For the latter, a clear peak is detected around 170 deg by both methodologies, which is believed to be caused by the damaged gear tooth. The results indicate that both methodologies are capable of detecting the faulty gear tooth. This is reasonable, as the tooth damage is very significant making the impulses caused by the mesh of pitted tooth more detectable.



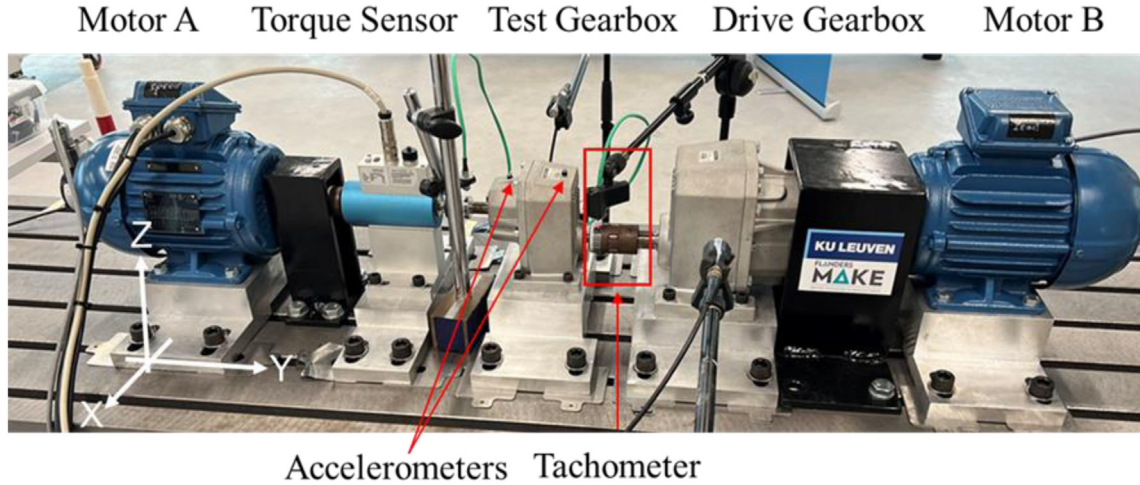


Fig. 11. The one-stage gearbox test rig



Fig. 12. The faulty gear used in the test of Case 2

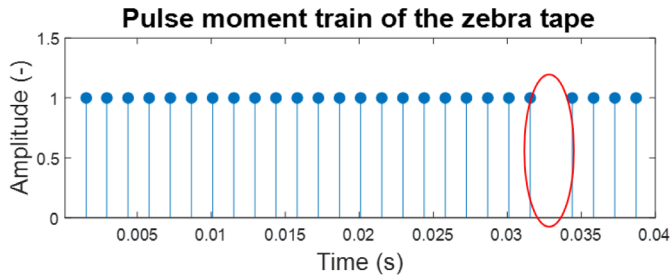
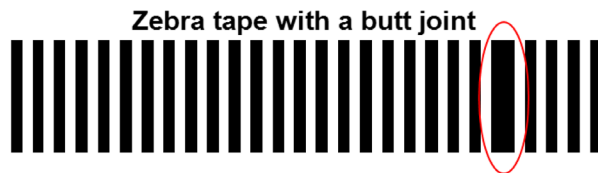


Fig. 13. An example of the pulse moment train of the zebra tape with a butt joint.

To further validate the proposed method, the angular difference between the faulty gear tooth and the 'butt joint' is measured when the 'butt joint' is in the vertical up position as displayed in Figure 16. The measured angle is 173 degrees, closely aligning with the position of the detected peak of the EIWSF and the WIF. A slight deviation in this alignment may be attributed to the precision of the tachometer measurements, the transmission

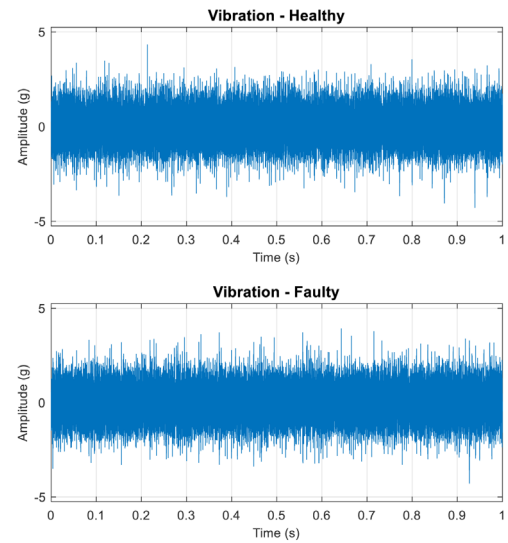
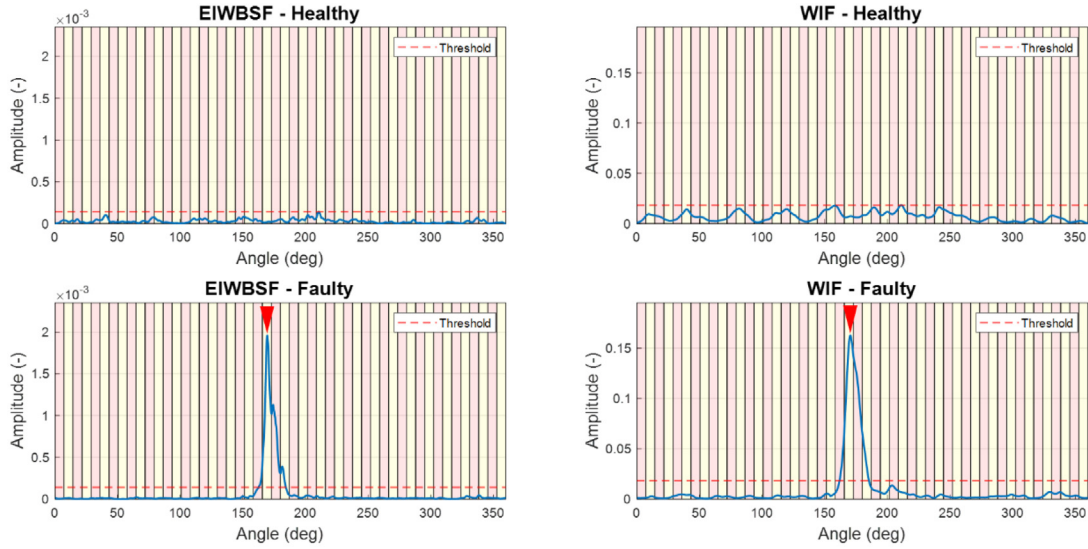


Fig. 14. Raw vibration signals of the healthy and faulty gear.

errors arising from the tooth defect or the manufacturing errors of the gear. The EIWSF and the WIF results are consistent with the expectations regarding the detected peak and its position, affirming the effectiveness of the proposed methodology.

### 3.3 Case 3—Multi-teeth damage diagnostics

The signals of the second case were acquired from a single-stage spur gearbox test rig at CETIM, as shown in Figure 17. The gearbox consists of a 20-tooth driving pinion and a 21-tooth driven gear, with the driving pinion rotating at a speed of 1000 rpm. This results in rotating frequencies of 16.67 Hz for the pinion and 15.87 Hz for the gear. Vibration signals were recorded daily over a 12-day period, with the sampling frequency of 20 kHz and a sampling duration of 3 s. During the test, the pinion went from a healthy to a deteriorated condition, with daily examinations of the pinion teeth's condition documented in [23,24].



**Fig. 15.** (Left) The EIWSF and (Right) the WIF of the healthy and the faulty gear.



**Fig. 16.** The angular difference between the faulty gear tooth and the 'butt joint'.

The daily examinations reveal that the pinion was in a healthy condition on day 1, while by day 12, two teeth were damaged: tooth 2 showed minor spalling, and tooth 16 experienced extensive spalling covering the entire tooth. Photos of teeth 2 and 16 during the test are presented in Figure 18 [25]. To evaluate the capability of EIWSF in detecting multi-teeth damage, measurements from day 1, day 9 and day 12 were selected to represent a healthy case, an initial fault on both tooth 2 and tooth 16, and a severe fault on tooth 2 and 16, respectively.

Figure 19 displays the raw vibration signals of the healthy and faulty gears. The signal measured on day 12 exhibits strong impacts due to the meshing of the damaged gear teeth. However, the amplitude of the signals measured on day 1 and 9 remain relatively low and stable, even

though gear teeth 2 and 16 began to show spalling damage on day 9. Identifying the specific number of the damaged teeth from the vibration signals in the time domain is challenging. Therefore, the EIWSF is applied on the healthy and the faulty signals. As outlined in Section 2, due to the fact that no speed signals have been acquired during the test, the raw vibration signals are firstly resampled based on the phase demodulation of the 2<sup>nd</sup> gear mesh harmonic of the vibration signal since the 2<sup>nd</sup> gear mesh harmonic is the most dominant frequency in the spectrum. This is followed by extracting the residual signals by removing the periodic components. The obtained residual signals are then segmented into individual rotations, with the IWBS being calculated for each rotation and averaged. Subsequently, optimization of the bi-frequency band is conducted to automatically enhance the fault related bi-frequency bands, thereby aiding the feature extraction.

The results of EIWSF are shown in Figure 20. The X and the Y axes represent respectively the angular position of the pinion and the amplitude of the feature. For better evaluation, the maximum EIWSF value of the healthy gear is set as the threshold. The EIWSF of the healthy one clearly demonstrate lower feature amplitudes compared to the faulty ones. On day 9, as expected, two distinct peaks are detected: one located around 45 deg and a second one at around 320 deg, which are believed to be caused by the initial damage on the gear teeth. On day 12, two significant peaks are detected: one around 65 deg, positioned in the middle of an interval, and the other near 340 deg, with the peak at around 65 deg being more prominent. The higher amplitude peak is believed to correspond to tooth 16, which exhibits extensive spalling across its surface, while the lower peak aligns with tooth 2, characterized by minor spalling. Moreover, the amplitude of the peaks detected on day 9 is significantly lower than that detected on day 12, indicating the evolution of the spalling on the gear teeth. Furthermore, the angular separation between these two peaks aligns with the physical positioning of tooth 2 and 16.

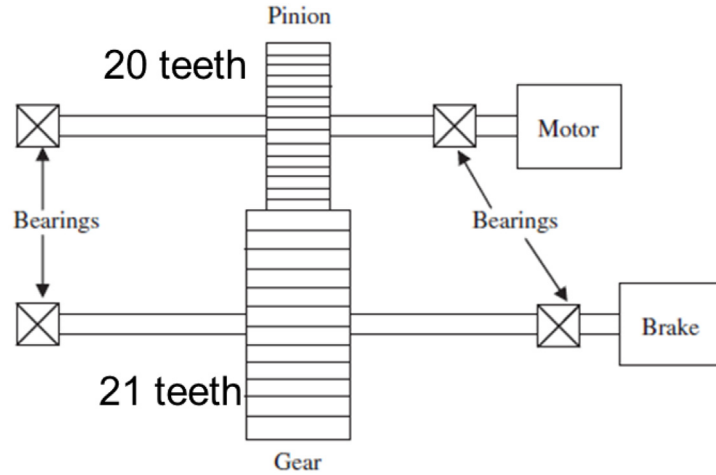


Fig. 17. The single-stage spur gearbox test rig.

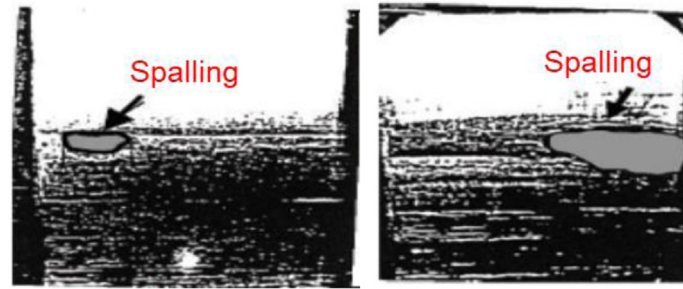


Fig. 18. (Left) Tooth 2 in the 10<sup>th</sup> day, (Right) tooth 16 in the 11<sup>th</sup> day.

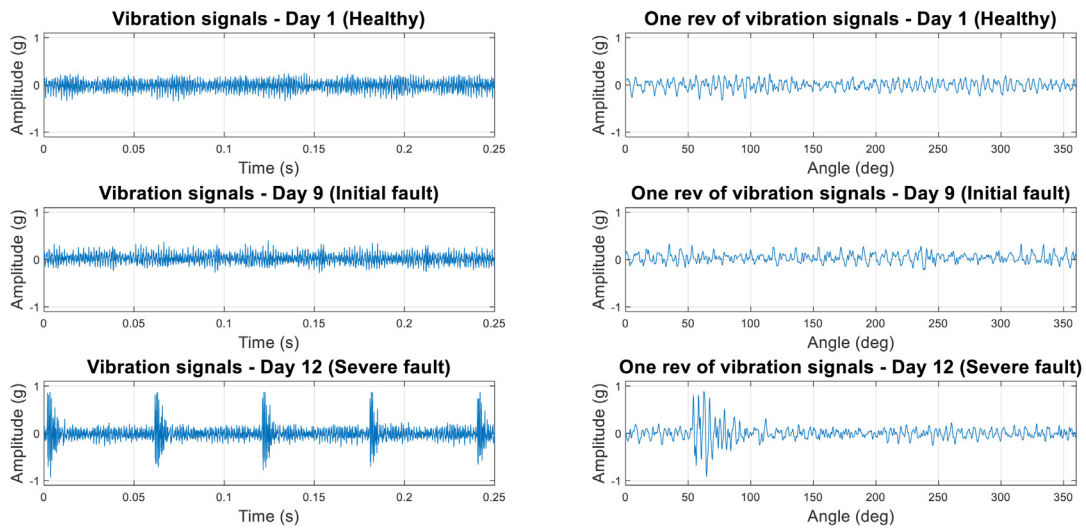
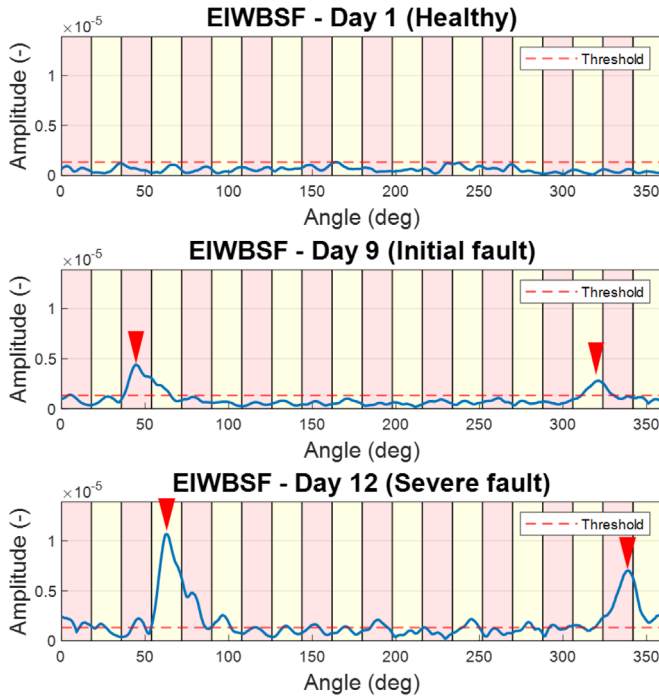


Fig. 19. (Left) the raw vibration signals of day 1, 9 and 12 (Right) one revolution of the signals.

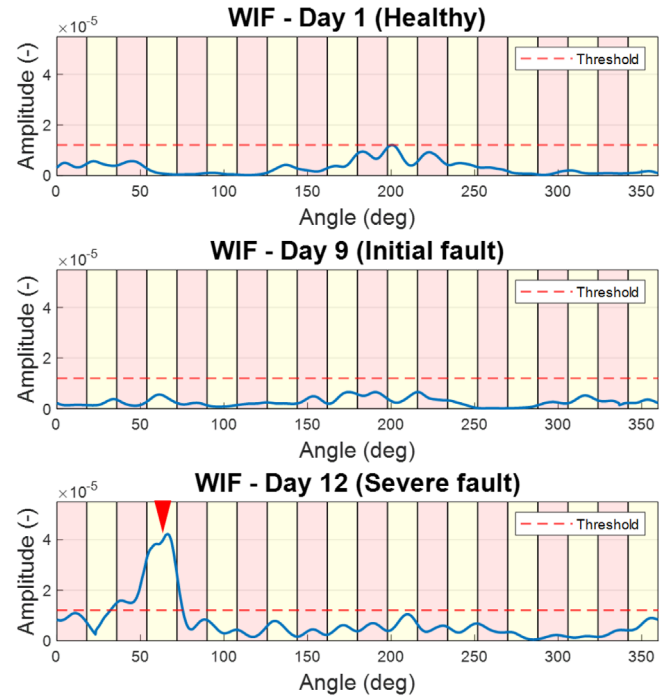
A slight deviation in this alignment might be attributed to the gear contact ratio and the transmission error arose from the defects. The EIWBSF results are consistent with the description of the gear status daily examinations, affirming the effectiveness of the proposed methodology.

For comparison, the WIF is also applied on the healthy and faulty signals. As illustrated in Figure 21, the WIF for the healthy one remains at a relatively lower amplitude

than those of the severe faulty case. In the latter case, on day 9, the amplitude is entirely below the threshold without any peaks detected. On day 12, a significant peak is observed around 65 degrees, aligning with the prominent peak detected in the result of the EIWBSF, indicating a strong impulse from tooth 16, which is also clearly visible in the time domain vibration signal shown in Figure 19. However, this method fails to detect the damaged teeth on



**Fig. 20.** The EIWBSF of the healthy and the faulty pinion.



**Fig. 21.** The WIF of the healthy and the faulty pinion.

day 9 and the second damaged tooth on day 12, as no other peak can be observed. This can be explained as following: on day 9, the damage-related impacts are weak since this is the initial fault, which is not easy to be detected. On day 12, the impacts produced by a minor spalling on tooth 2 are still very weak and may be masked by the strong impacts of the more severe defect on tooth 16. Consequently, the WIF does not yield critical diagnostic information for the damaged teeth, which underscores the superiority of the EIWBSF in the diagnosis of multi teeth damage.

## 4 Conclusion

A new diagnostic feature is proposed in this paper for the diagnosis of local damage in gears. Initially, the acquired vibration signals are resampled in the angular domain. Afterwards the residual signals are extracted by removing the periodical component from the resampled signals. Then, after segmenting the residual signal, the IWBS is obtained by taking ensemble average of IWBS of each epoch. An optimization of the informative bi-frequency band based on a feature is proposed and the EIWBSF is obtained based on the weight matrix and integrations over the bi-frequency.

The proposed methodology has been validated using three datasets. The first one is an experimental dataset, focusing on the diagnosis of single-tooth damage, where a healthy gear and a faulty gear (one tooth with artificially generated 50% pitting damage) are tested under two operating speeds. Compared with the classical WIF, EIWBSF shows good performance in detecting the faulty gear tooth. Although WIF also works when operating the drivetrain at higher speeds, the EIWBSF is able to extract the fault peaks more effectively, even under low speed

conditions. The second dataset also focuses on single tooth damage detection in order to provide more concrete validation. In this case, a healthy gear and a faulty gear are tested. Unlike the first case, here the position of the faulty tooth is indicated by the 'butt joint' on a zebra tape. The results of both methodologies are consistent with the expectations regarding the detected peak and its position. The third dataset, which is publicly available, targets at the diagnosis of multiple-teeth damage and contains naturally developed spalling defects on two gear teeth. The EIWBSF shows good performance in the detection and localisation of the two damaged teeth, with results that are consistent with the description of the gear status from daily examinations. Meanwhile, the WIF is only able to detect the significant peak corresponding to tooth 16 on day 12, which has a more severe defect. In the future, more tests will be done in order to validate and further improve the applicability of the method by considering various operating conditions, damage evolution, as well as various gear defect types.

## Acknowledgements

Rui Zhu would like to gratefully acknowledge the support from the China Scholarship Council. This work was supported by Flanders Make, the strategic research center for the manufacturing industry, in the context of the QED project.

## Funding

This research was funded by VLAIO, via Flanders Make, the strategic research center for the manufacturing industry, in the context of the QED project. The Article Processing Charges for this article are taken in charge by the French Association of Mechanics (AFM).



## Conflicts of interest

The authors have nothing to disclose.

## Data availability statement

Datasets 1 and 2 will be made publicly available in the future. Dataset 3 is publicly available.

## Author contribution statement

Conceptualization, K.G., R.Zh.; Methodology, K.G., R.Zh.; Software, R.Zh.; Validation, R.Zh.; Formal Analysis, R.Zh.; Investigation, K.G., G.M., R.Zh.; Resources, K.G.; Data Curation, R.Zh.; Writing – Original Draft Preparation, R.Zh.; Writing – Review & Editing, R.Zh., G.M., K.G.; Visualization, R.Zh.; Supervision, K.G.; Project Administration, K.G., G.M.; Funding Acquisition, K.G.

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